BOOK 1 – ETHICS AND QUANTITATIVE METHODS

Thanks for trusting Schweser. We are committed to providing you with Study Notes that are comprehensive, up to date, and clearly written. Work hard, stay motivated, and stick to your study plan. Above all, put in the significant *time and effort* necessary to learn the material in the Level 1 Schweser Study Notes.

Best regards,

Doug Van Eaton, CFA
Vice President and Level 1 Manager
Schweser Study Program

WELCOME TO THE 2006 SCHWESER STUDY NOTES

Thank you for purchasing the Schweser Study Notes. It is our hope that you find our product effective and easy to use. The following comments are designed to get you started on your studies and help you get the most out of the Notes.

Overview of the CFA® Program

The CFA program and the CFA curriculum are very carefully structured to help you learn exactly what you need to know to pass each level and earn your Charter. The 2006 CFA Level 1 Study Guide that you received when you registered for the Level 1 exam has a complete description of the CFA program. Please read the study guide in its entirety before you start studying. Here are a few key points to remember.

Curriculum Structure

- Reading assignments are the foundation of the study program and the basis for the exam questions.
- Unless the end-of-chapter questions and appendices from textbook-based reading assignments are specifically assigned, you need not study them.

Topic-Level Learning Objectives

- The primary purpose of what CFA Institute refers to as topic-level learning objectives is to give you a broad overview of the material in a topic area (e.g., asset valuation) and indicate the depth of knowledge that is expected. The topic-level learning objectives appear at the beginning of each of the four topic areas in the Level 1 curriculum.
- The topic-level learning objectives combined with reading-specific learning outcome statements constitute the study sessions for each topic area.

Learning Outcome Statements (LOS)

- The LOS command words (e.g., calculate, define, describe) indicate what you are expected to be able to do after you have read and studied the material in each assigned reading.
- The LOS will direct you toward specific sections of the reading assignment and away from other material that is not intended to be part of the curriculum. You should use the LOS to guide your study program.
- It is possible that you will be required to demonstrate a knowledge of more than one LOS in a single exam question.
- The LOS should not be viewed as a proxy for exam questions, but mastery of the LOS is necessary for your success.

How Do the Levels Differ?

- The Level 1 study program emphasizes tools and inputs.
- The Level 2 study program emphasizes asset valuation and applications of the tools.
- The Level 3 study program emphasizes portfolio management and strategies for applying those tools.

Why the CFA Exam Is Unique

The CFA exam is *different* from most examinations that you have taken. For example, if you've taken the Series 7 exam, it is likely that you extensively studied sample questions to get a feel for the actual exam. For exams like the GMAT, SAT, Series 7, Series 63, and many insurance exams, it is relatively easy for a study course provider to

estimate the structure, style, and content of the actual test due to its narrow curriculum focus and the frequency with which the exam is offered. Consequently, studying *questions* may be an effective approach to prepare for these types of exams.

This is not the case with the CFA exam. Due to the broad nature of the CFA curriculum, plus the fact that recent exams are not available for public review, it is virtually impossible for a CFA prep-course provider to write sample examination questions that will, with any significant degree of certainty, mimic the actual CFA exam experience. The point is that you *must learn the underlying material*, and you must learn it well enough to be able to transport that knowledge to the new and innovative questions that will be asked on exam day. You should *never* expect to see the questions asked in the CFA Institute sample exams, or in any prep course provider's sample exams, on the actual exam.

A trap that some CFA candidates fall into is to focus their study efforts on practicing questions from sample exams and/or test banks and not on rigorously learning the underlying concepts. This type of study approach makes it extremely difficult for candidates to apply their knowledge to actual exam questions that are related in focus to practice questions, but presented from a different perspective. Compound the inability to transport rote knowledge of specific questions with the tremendous stress of exam day, and you have a formula for disaster.

The bottom line is that you must *learn the material!*. The CFA exam is grueling, both physically and mentally, and it should not to be taken lightly. **Your diligent effort combined with Schweser's high quality study products are the keys to your exam-day success.**

Designing an Effective Study Program

If you are reading this in January or February (July or August for the December exam), you should have time to study each reading in full detail, review our commentary, and work all of the concept checkers in the Study Notes. As you get closer to the exam date, your focus should shift away from the narrative material and toward the exam flashbacks embedded in the reviews of Books 1–5. Then in the week(s) preceding the exam you should devote most of your time to the sample exams (SchweserPro™ Question Bank, Online Practice Exams in SchweserOnline™, the Practice Exam Book, and Book 6 of the Study Notes). While you are working through the sample exams, you should also clear up any problem areas that you may have with the material by continuing to review Books 1–5.

Stay on schedule. CFA Institute estimates that you should commit 10 to 15 hours per week to your exam preparation over an 18-week period, leaving one month before the exam for review. Regardless of the exact nature of your study program, approaching the material in a systematic and thorough manner is the key to passing the exam. A suggested study schedule is contained in the following table. Additionally, signing up for SchweserOnline™ will help you structure your study efforts and maintain an effective and manageable pace.

Level 1 Schedule (December 2006 in parentheses)		
Week of:	**Topic (Study Session)**	**Comments**
January 23 (July 24)	Ethics & Standards (1)	A thorough first read of ethics is appropriate here.
January 30 (July 31)	Quantitative Methods (2)	Although we've only assigned two weeks for quant, you will be using these concepts
Feb 6 (Aug 7)	Quantitative Methods (3)	throughout the remainder of the spring (fall). You'll need to continually review this material as you progress.
February 13 (August 14)	Economics (4 & 5)	Economics can be relatively straightforward (except foreign currency issues).
February 20 (August 21)	Economics (5 & 6)	It can seem like the foreign currency material dominates the questions on the actual exam. Study this material carefully.
February 27 (August 28)	Financial Statement Analysis (7)	These can be the most brutal weeks in your study plan. There will be some topics that you
March 6 (September 4)	Financial Statement Analysis (8)	will have a hard time grasping. Do as much as you can here—accounting and corporate finance are 28 percent of your test. However, don't be
March 13 (September 11)	Financial Statement Analysis (9)	disappointed if you don't get 100 percent in this area on the exam. The actual accounting questions can be killers!
March 20 (September 18)	Catch up week	Catch up, or take a break from accounting and review ethics and quantitative methods.
March 27 (September 25)	Financial Statement Analysis (10)	One last push to finish the accounting sections. Much of the financial statement analysis material here builds on the earlier study sessions.
April 3 (October 2)	Corporate Finance (11)	There could be a greater emphasis on corporate finance on the actual test. Please study this material carefully.
April 10 (October 9)	Equity Investments (12 & 13)	The equity material here is relatively light-weight. Study it carefully and get some "easy" points.
April 17 (October 16)	Equity Investments (13) & Alternative Investments (17)	Markets, equity securities, and alternative investments represent pretty easy exam points. You will need to memorize many factoids here.
April 24 (October 23)	Portfolio Management (18)	Portfolio management is five percent of the exam. The concepts here are relatively straightforward and will help you understand the asset valuation material.

Level 1 Schedule Continued (December 2006 in parentheses)		
May 1 (October 30)	Debt Investments (14 & 15)	So many LOSs, so little time. Note that in terms of exam exposure per LOS, this area carries the least weight. Keep your focus and worry primarily about the big issues.
May 8 (November 6)	Debt Investments (14 & 15)	
May 15 (November 13)	Derivative Investments (16)	Derivatives may seem like a drag, but the material is pretty straightforward at Level 1 (enjoy it while you can—Levels 2 and 3 are much more difficult when it comes to the derivatives material).
May 22 (November 27)	Ethics review & sample exams	Find the areas where you are still having trouble and focus additional review on those. Remember ethics and GIPS® are 15 percent of the exam. Try to take each exam under actual exam conditions, timed and with only one break.

Use the Schweser Study Planner. All notes purchasers can use our online Study Planner. The online Schweser Study Planner creates study plans that keep you on track daily, weekly and monthly. The Study Planner will also help you track your progress and suggest alternative study tools and activities.

Read the Schweser Study Notes first. If you have purchased the original CFA Institute-assigned materials, which we encourage you to do, I recommend that you read the Schweser Notes before you read the source reading. This will give you an overview of what is in the assigned readings. If the material is new to you, carefully and completely study the assigned reading. If you are already familiar with some of the material in the assigned reading, read it, but concentrate on the sections that you are not fully comfortable with. The Notes are keyed to the assigned readings via the LOS for quick cross-referencing. Even if you feel you really know the material, read through every reading quickly just to make sure you understand the terminology, the approach to the material taken by the author, and how to address the LOS.

Develop your own notes. I recommend jotting down your personal study notes in the margins of the Schweser Notes. By doing this, all your notes will be together in one place when you have completed the reading assignments. You can now review the material quickly without the confusion of flipping between notebooks. I also recommend having a book of Schweser Notes with you at all times. If you have five or ten minutes on a plane, train or bus, take out the Notes and review a few chapters.

Pay attention to details and definitions. The exam is about understanding the concepts. Remember that you not only have to know the material, but you also have to be able to demonstrate your knowledge within the framework of the exam.

Use the problem sets for practice. When you finish reading a chapter, work the concept checkers and any comprehensive problems at the end of each topic review in the Study Notes. If you can't work these problems, you don't know the material well enough to go on. Go back and study the Notes and reading again. The concept checkers are there to test your understanding of the concepts and, in general, are less difficult than exam-type questions. If you need more questions, try our SchweserPro™ Question Bank available online, on CD, or as a download, or try SchweserOnline™, our online package of study tools that contains an online version of SchweserPro™ QBank and Online Practice Exams - two alternatives that provide thousands of additional questions.

Study the ethics material more than once. Ethics is one of the keys to passing the exam. It would be optimal if you could study ethics in January or February and once again in April or May. Also, buy and read the original *Standards of Practice Handbook*. Although we are very proud of our summaries of the ethics material, there are two reasons why we recommend that you buy the CFA Institute's Standards of Practice Handbook. First, as a CFA candidate, you have pledged to abide by the CFA Institute Standards. Second, the ethics questions on the actual exam can be an exercise in "trivial pursuit." You will be much better off if you read both our topical reviews of the Standards *and* the original Handbook. Don't ignore GIPS® and corporate governance.

Save the sample exams for last. Save the Practice Exam Book and the sample exams in Book 6 of the Study Notes for the weeks just before the real exam. A good strategy is to take one exam in each of the three weeks leading up to the test. Do your best to mimic actual exam conditions (e.g., time yourself, have someone turn the heat down and up so that you go from freezing to boiling, hire a construction crew to do some blasting outside your window). Remember, no matter how challenging we make our sample exams, the actual exam will be different. Also, due mainly to exam-day stress, your perception will be that the actual exam was much more difficult than any sample exam or old exam questions you have ever seen.

Read the Candidate Bulletin. Candidate Bulletins are published in April, October, and December. They contain important information regarding changes and corrections to the exam process.

Use other Schweser study products. The Schweser Videos, SchweserPro™ QBank, Flashcards, Audio CDs, Secret Sauce™ Book and our Live Seminars will maximize your study time and effort (salesmanship aside, these really are great study tools). You can find many of these study tools in our packages: Premium, Essential, Basic, or SchweserOnline™. Everyone studies differently—use the study tools that maximize your ability to keep the material in front of you as much as possible.

If You Are Starting Late

If you are starting to study in April (or October), your strategy will differ from the one outlined. Unless you have 40 hours a week to devote to your studies, you will not be able to give each reading the full attention that it deserves. Therefore, your task is to decide which topics you should devote the bulk of your time to. *Don't forget about ethics.*

Based on your existing skill set, you will have to decide which readings to study and which readings can be skipped or glossed over. If you start your studies late and decide to read every reading in detail, you will drive yourself crazy and will not enter the exam with the self-confidence that is required to pass. This is where our seminars become invaluable. During the course of the seminar, we discuss the relative importance of each topic area and reading assignment. We also demonstrate how the material could be covered within the framework of the exam.

We have significantly enhanced our sample exams. If you're starting late—focus on the sample exams in Book 6 while you are studying Books 1 through 5.

On Exam Day (June 3, 2006 and June 4, 2006 in Eastern Asia and Oceania)
(December 2, 2006 and December 3, 2006 in Eastern Asia and Oceania)

Don't be surprised. On exam day get to the exam site early—there will be a crowd! It is CFA Institute policy to **close the doors** to the testing room **30 minutes before the exam starts.** Once you know the exam center location, decide on your travel route and where you are going to park your car. The Saturday before the exam, actually go and find the exam room. Stand in it, look it over, and get comfortable.

Know the rules and follow them. Standard VII(A) reads, "Members and candidates must not engage in any conduct that compromises the reputation or integrity of the CFA Institute or CFA designation or the integrity, validity, or security of the examinations." You cannot cheat, disobey the proctor, or do anything to disrupt or compromise the exam. If you take the exam booklet out of the room, your exam will not be graded, and you will be subject to disciplinary action.

Know what to expect. The CFA exam is a sampling process. All study sessions will be covered on the exam, but not all individual LOSs can or will be covered. The guideline topic area weights, as developed by CFA Institute, are as follows.

Topic Area Weights	
Ethical and Professional Standards	15%
Economics	10%
Quantitative Analysis	12%
Accounting and Corporate Finance	28%
Asset Valuation	30%
Portfolio Management	5%

As it has been for many years now, the 2006 Level 1 CFA examination will be 100 percent multiple choice (240 questions). Keep in mind that CFA Institute will do its best to develop tricky distractors. Distractors are potential selections for multiple-choice questions that are wrong and are intended to pull your attention away from the correct answer. So, read each question thoroughly, make your selection carefully, and move on.

Be ready to do some number crunching. Due to the quantitative nature of many of the questions, you need to be ready to calculate correct answers and provide interpretations for those answers. Be proficient in the use of your calculator and make sure P/Y is set equal to one. Keep track of the formulas, especially in the asset valuation section.

Guess if you must. Attempt to answer every question, even if it means guessing! No points are deducted for incorrect answers. Even if you don't have a clue, you have a 25 percent chance of getting a correct answer if you guess, and a 100 percent chance of getting the question wrong if you leave it blank.

Take the test! Even if you don't feel fully prepared, take the test anyway. You paid for the right to take the exam, so you might as well give it a shot. The act of actually taking the test under exam-day conditions is a learning experience in and of itself. If you don't take the test, you will be studying for the next Level 1 exam anyway. If you take the test, the experience will guide your study program next year. Nothing focuses your attention on what you don't know like an exam.

Pay attention to the fine print. The CFA Institute study guide is full of exam rules and regulations. Here are a few of the most important ones.

- You must present both your ticket and a picture ID (government-issued; cannot be expired) to be admitted to the test center.
- Bring a bunch of *sharpened #2 pencils* with *attached erasers*.
- There are only two types of calculators that are permitted in the exam—the TI BAII Plus® (including the BAII Plus Professional®) and the HP 12C® (including the HP 12C Platinum®). Decide on *one* of these models and then buy *two* of the same calculator (in case one blows up during the exam) or bring extra

batteries—CFA Institute will allow you to change batteries during the exam. If you've never used a 12C before, I'm going to save you a lot of headaches—buy the TI BAII Plus. Just remember that the TI comes preset from the factory with its payments per year (P/Y) set to 12. P/Y should equal 1.

- During the exam the only articles you may have at your desk are the exam Question Book, answer scan sheet, calculator(s), government-issue photo ID (CFA Institute prefers passports), exam ticket, and sharpened pencils. No food, loose erasers, water bottles, hats, ball caps, highlighters, correction tape/fluid, backpacks, purses, handbags, lunch bags, study materials, scratch paper, wireless communication devices, or "lucky items." You may not bring cellular phones or pagers into the exam room. Please check the candidate bulletin for other announcements on what you can and can't bring into the examination room.
- Bring a watch or timer but remember audible timers are not allowed.
- If you do not turn in your exam booklet and scan sheet before you leave the room, your exam will not be graded. Make sure your candidate number is on the exam scan sheet.

Waiting for the Results

When will I find out whether I passed? CFA Institute will release the Level 1 results in July of 2006 (January 2007).

What do I need to pass? The CFA Institute Board of Governors, the body responsible for establishing the CFA examination, sets the minimum passing score. Your goal is 70 percent! If you feel your exam was scored incorrectly you can have it manually retabulated (*not re-graded*) by submitting a written request to CFA Institute along with a $100 retabulation fee.

I'd like to take this opportunity to thank all of my colleagues at Schweser for their incredible work ethic and commitment to quality. In particular, I'd like to thank the following members of our content and editing team:

Many people would end this introduction with a "Good luck on the exam." To me, that suggests you need to be lucky to pass Level 1. With your hard work and our study products, luck will have nothing to do with it. Instead, I'll simply say: "See you next year at Level 2."

Best Regards,

Robert D. Van Eaton

Dr. Doug Van Eaton, CFA
Vice President and Level 1 Manager
Schweser Study Program

READINGS AND
LEARNING OUTCOME STATEMENTS

READINGS

The following material is a review of the Ethics and Quantitative Methods principles designed to address the learning outcome statements set forth by CFA Institute.

STUDY SESSION 1

Reading Assignments

STUDY SESSION 2

Reading Assignments
Quantitative Methods for Investment Analysis, 2nd edition, Richard A. DeFusco, Dennis W. McLeavey, Jerald E. Pinto, and David E. Runkle (CFA Institute, 2004)

STUDY SESSION 3

Reading Assignments
Quantitative Methods for Investment Analysis, 2nd edition, Richard A. DeFusco, Dennis W. McLeavey, Jerald E. Pinto, and David E. Runkle (CFA Institute, 2004)

LEARNING OUTCOME STATEMENTS (LOS)

STUDY SESSION 1

The topical coverage corresponds with the following CFA Institute assigned reading:
1. **"Code of Ethics and Standards of Professional Conduct"**
The Code of Ethics establishes the framework for ethical decision making in the investment profession. The candidate should be able to state the six components of the Code of Ethics.

The Standards of Professional Conduct are organized into seven standards:
I.   Professionalism
II.   Integrity of Capital Markets
III. Duties to Clients and Prospective Clients
IV. Duties to Employers
V.   Investment Analysis, Recommendations, and Action
VI. Conflicts of Interest
VII. Responsibilities as a CFA Institute Member or CFA Candidate

The topical coverage corresponds with the following CFA Institute assigned reading:
2. **"Guidance" for Standards I – VII**
The candidate should be able to
   a.   demonstrate a thorough knowledge of the Standards of Professional Conduct by recognizing and applying the standards to specific situations. (page 5)
   b.   distinguish between conduct that conforms to the Code and Standards and conduct that violates the Code and the Standards. (page 5)

The topical coverage corresponds with the following CFA Institute assigned reading:
3. **"Introduction to the Global Investment Performance Standards (GIPS®)"**
4. **"Global Investment Performance Standards"**
The candidate should be able to
   a.   explain why the GIPS standards were created. (page 61)
   b.   explain what parties the GIPS standards apply to and whom the standards serve. (page 61)
   c.   characterize "composites". (page 61)
   d.   explain the purpose of verification. (page 61)
   e.   explain why a global standard is needed and how it is being implemented. (page 62)
   f.   state the "vision" of the GIPS standards. (page 62)
   g.   state the objectives and key characteristics of the GIPS standards. (page 62)
   h.   state the appropriate disclosure when the GIPS standards and local regulations are in conflict. (page 63)
   i.   explain the scope of the GIPS standards with respect to definition of the firm, historical performance record, and compliance. (page 63)
   j.   name and characterize the eight major sections of the GIPS standards. (page 64)
   k.   explain the fundamentals of compliance with the GIPS standards. (page 64)

The topical coverage corresponds with the following CFA Institute assigned reading:
5. **"The Corporate Governance of Listed Companies: A Manual for Investors"**
The candidate should be able to
   a.   identify the factors in evaluating the quality of corporate governance and the relative strength of shareowner rights. (page 69)
   b.   define corporate governance and identify practices that constitute good corporate governance. (page 70)

    c.   define independence as used to describe corporate board members, and explain the role of independent board members in corporate governance. (page 70)

    d.   list and explain the major factors that enable a board to exercise its duty to act in the best long-term interests of shareowners. (page 70)

    e.   identify characteristics of a board that contribute to the board's independence, and state why each characteristic is important for shareowners' interests. (page 71)

    f.   identify factors that indicate a board and its members possess the experience required to govern the company for the benefit of its shareowners. (page 71)

    g.   explain the importance to shareowners of a board's ability to hire external consultants. (page 72)

    h.   identify advantages and disadvantages of annual board elections compared to less frequent elections. (page 72)

    i.   explain the implications of a weak corporate code of ethics with regard to related-party transactions and personal use of company assets. (page 72)

    j.   critique characteristics and practices of board committees, and determine whether they are supportive of shareowner protection. (page 73)

    k.   identify the information needed for evaluating the alignment of a company's executive compensation structure and practices with shareowner interests. (page 74)

    l.   state the provisions that should be included in a strong corporate code of ethics. (page 74)

    m.   identify components of a company's executive compensation program that positively or negatively affect shareowners' interests. (page 75)

    n.   explain the implications for shareowners of a company's proxy voting rules and practices. (page 75)

    o.   state whether a company's rules governing shareowner-sponsored board nominations, resolutions, and proposals are supportive of shareowner rights. (page 76)

    p.   explain the implications of different classes of common equity for shareowner rights. (page 77)

    q.   determine the probable effects of takeover defenses on share value. (page 78)

STUDY SESSION 2

The topical coverage corresponds with the following CFA Institute assigned reading:

6.   **"The Time Value of Money"**

The candidate should be able to

    a.   explain an interest rate as the sum of a real risk-free rate, expected inflation, and premiums that compensate investors for distinct types of risk. (page 81)

    b.   calculate and interpret the effective annual rate, given the stated annual interest rate and the frequency of compounding. (page 82)

    c.   solve time value of money problems when compounding periods are other than annual. (page 83)

    d.   calculate the PV of a perpetuity. (page 85)

    e.   calculate and interpret the FV and PV of a single sum of money, ordinary annuity, annuity due, or a series of uneven cash flows. (page 86)

    f.   draw a time line, specify a time index, and solve problems involving the time value of money as applied, for example, to mortgages and savings for college tuition or retirement. (page 95)

    g.   show and explain the connection between present values, future values, and series of cash flows. (page 100)

The topical coverage corresponds with the following CFA Institute assigned reading:

7.   **"Discounted Cash Flow Applications"**

The candidate should be able to

    a.   calculate and interpret the net present value (NPV) and the internal rate of return (IRR) of an investment. (page 111)

    b.   contrast the NPV rule to the IRR rule. (page 114)

    c.   discuss problems associated with the IRR method. (page 114)

d.   calculate, interpret, and distinguish between the money-weighted and time-weighted rates of return of a portfolio and appraise the performance of portfolios based on these measures. (page 115)

e.   calculate and interpret the bank discount yield, holding period yield, effective annual yield, and money market yield for a U.S. Treasury bill. (page 118)

f.   convert and interpret among holding period yields, money market yields, and effective annual yields. (page 120)

g.   calculate and interpret the bond equivalent yield. (page 121)

The topical coverage corresponds with the following CFA Institute assigned reading:

8.   **"Statistical Concepts and Market Returns"**

The candidate should be able to

a.   describe the nature of statistics and differentiate between descriptive statistics and inferential statistics and between a population and a sample. (page 129)

b.   explain the concepts of a parameter and a sample statistic. (page 129)

c.   explain the differences among the types of measurement scales. (page 130)

d.   define and interpret a frequency distribution. (page 131)

e.   define, calculate, and interpret a holding period return (total return). (page 130)

f.   calculate and interpret relative frequencies and cumulative relative frequencies, given a frequency distribution. (page 132)

g.   describe the properties of data presented as a histogram or a frequency polygon. (page 133)

h.   define, calculate, and interpret measures of central tendency, including the population mean, sample mean, arithmetic mean, weighted average or mean (including a portfolio return viewed as a weighted mean), geometric mean, harmonic mean, median, and mode. (page 136)

i.   describe and interpret quartiles, quintiles, deciles, and percentiles. (page 135)

j.   define, calculate, and interpret 1) a range and mean absolute deviation, and 2) a sample and a population variance and standard deviation. (page 140)

k.   contrast variance with semivariance and target semivariance. (page 143)

l.   calculate and interpret the proportion of observations falling within a specified number of standard deviations of the mean, using Chebyshev's inequality. (page 144)

m.   define, calculate, and interpret the coefficient of variation and the Sharpe ratio. (page 144)

n.   define and interpret skew, explain the meaning of a positively or negatively skewed return distribution, and describe the relative locations of the mean, median, and mode for a nonsymmetrical distribution. (page 146)

o.   define and interpret kurtosis, and measures of population and sample skew and kurtosis. (page 147)

The topical coverage corresponds with the following CFA Institute assigned reading:

9.   **"Probability Concepts"**

The candidate should be able to

a.   define a random variable, an outcome, an event, mutually exclusive events, and exhaustive events. (page 163)

b.   explain the two defining properties of probability. (page 163)

c.   distinguish among empirical, subjective, and a priori probabilities. (page 164)

d.   state the probability of an event in terms of odds for or against the event. (page 164)

e.   describe the investment consequences of probabilities that are mutually inconsistent. (page 164)

f.   distinguish between unconditional and conditional probabilities. (page 165)

g.   define a joint probability and calculate and interpret the joint probability of two events. (page 165)

h.   calculate the probability that at least one of two events will occur, given the probability of each and the joint probability of the two events. (page 166)

i.   distinguish between dependent and independent events. (page 167)

j.   calculate a joint probability of any number of independent events. (page 168)

k.   calculate, using the total probability rule, an unconditional probability. (page 169)

The topical coverage corresponds with the following CFA Institute assigned reading:

13. **"Correlation and Regression"**

The candidate should be able to

a. define and interpret a scatter plot. (page 277)

b. calculate and interpret a sample covariance and a sample correlation coefficient. (page 278)

c. formulate a test of the hypothesis that the population correlation coefficient equals zero and determine whether the hypothesis is rejected at a given level of significance. (page 281)

d. differentiate between the dependent and independent variables in a linear regression and explain the assumptions underlying linear regression. (page 283)

e. define, calculate, and interpret the standard error of estimate and the coefficient of determination. (page 286)

f. calculate a confidence interval for a regression coefficient. (page 290)

g. formulate a null and an alternative hypothesis about a population value of a regression coefficient, select the appropriate test statistic, and determine whether the null hypothesis is rejected at a given level of significance. (page 291)

h. interpret a regression coefficient. (page 291)

i. describe the use of analysis of variance (ANOVA) in regression analysis and interpret ANOVA results. (page 294)

j. calculate and interpret a predicted value and a confidence interval for the predicted value for the dependent variable given an estimated regression model and a value for the independent variable. (page 292)

k. discuss the limitations of regression analysis and identify problems with a particular regression analysis or its associated results and any conclusions drawn from them. (page 295)

CFA INSTITUTE CODE OF ETHICS AND STANDARDS OF PROFESSIONAL CONDUCT

EXAM FOCUS

You should note that for the 2006 exams, a new 9th edition of the *Standards of Practice Handbook* will be in effect[1]. There are significant revisions, restructurings, additions, and deletions in this edition that we will point out in this topic review.

In addition to reading this review of the ethics material, we strongly recommend that all candidates for the CFA® examination *purchase* their own copy of the *Standards of Practice Handbook 9th Edition* (2005) and read it multiple times. As a registered candidate, it is your responsibility to own an original copy of the *Code and Standards* and to comply with the *Code and Standards*.

BRIEF SUMMARY OF CHANGES

* The Standards have been reorganized to eliminate duplication and improve clarity.
* Some Standards have been revised and expanded to better address current issues in the investment profession. Examples include misrepresentation, duty to employer, suitability, duty of loyalty to clients, disclosure of conflicts, and the use of material non-public information.
* A new Standard has been added to address market manipulation and record retention.
* There is no longer a requirement to inform employers of the Code and Standards.
* Previously the Code and Standards were centered around U.S. laws. Now certain Standards are less U.S.-centric. Examples include the use of material non-public information and fiduciary duty.
* It has been more clearly stated that all Standards apply to both Members and Candidates.

CFA INSTITUTE CODE OF ETHICS

LOS 1: "Code of Ethics and Standards of Professional Conduct"

The Code of Ethics establishes the framework for ethical decision making in the investment profession. The candidate should be able to state the six components of the Code of Ethics.

The Standards of Professional Conduct are organized into seven standards:

I: Professionalism
II: Integrity of Capital Markets
III: Duties to Clients
IV: Duties to Employers
V: Investment Analysis, Recommendations, and Action
VI: Conflicts of Interest
VII: Responsibilities as a CFA Institute Member or CFA Candidate

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1. The new Code and Standards are effective as of January 1, 2006.

Each Standard contains multiple provisions for which the candidate is responsible. The candidate should be able to identify the ethical responsibilities required by the Code and Standards.

CODE OF ETHICS

Members of CFA Institute [including Chartered Financial Analyst® (CFA®) charterholders] and candidates for the CFA designation ("Members and Candidates") must:[2]

Professor's Note: Major changes are in italics.

- Act with integrity, competence, *diligence, respect,* and in an ethical manner with the public, clients, prospective clients, employers, employees, *colleagues in the investment profession, and other participants in the global capital markets.*

- *Place the integrity of the investment profession and the interests of clients above their own personal interests.*

- Use reasonable care and exercise independent professional judgment *when conducting investment analysis, making investment recommendations, taking investment actions, and engaging in other professional activities.*

- Practice and encourage others to practice in a professional and ethical manner that will reflect credit on themselves and the profession.

- *Promote the integrity of, and uphold the rules governing, capital markets.*

- Maintain and improve their professional competence and strive to maintain and improve the competence of other investment professionals.

STANDARDS OF PROFESSIONAL CONDUCT[3]

Professor's Note: Major changes are in italics.

I. **PROFESSIONALISM**

   A. **Knowledge of the Law.** Members and Candidates must understand and comply with all applicable laws, rules, and regulations (including the CFA Institute *Code of Ethics* and *Standards of Professional Conduct*) of any government, regulatory organization, licensing agency, or professional association governing their professional activities. In the event of conflict, Members and Candidates must comply with the more strict law, rule, or regulation. Members and Candidates must not knowingly participate or assist in any violation of laws, rules, or regulations and *must disassociate themselves from any such violation.*

   B. **Independence and Objectivity.** Members and Candidates must use reasonable care and judgment to achieve and maintain independence and objectivity in their professional activities. Members and Candidates must not offer, solicit, or accept any gift, benefit, compensation, or consideration that reasonably could be expected to compromise their own or another's independence and objectivity.

   C. **Misrepresentation.** Members and Candidates must not knowingly make any misrepresentations relating to investment analysis, recommendations, actions, or other professional activities.

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3. Copyright 2005, CFA Institute. Reproduced and republished from "Standards of Professional Conduct," from *Standards of Practice Handbook, 9th Ed., 2005,* with permission from CFA Institute. All rights reserved.

D. **Misconduct.** *Members and Candidates must not engage in any professional conduct involving dishonesty, fraud, or deceit or commit any act that reflects adversely on their professional reputation, integrity, or competence.*

II. INTEGRITY OF CAPITAL MARKETS

A. **Material Nonpublic Information.** Members and Candidates who possess material nonpublic information that could affect the value of an investment must not act or cause others to act on the information.

B. **Market Manipulation.** *Members and Candidates must not engage in practices that distort prices or artificially inflate trading volume with the intent to mislead market participants.*

III. DUTIES TO CLIENTS

A. **Loyalty, Prudence, and Care.** Members and Candidates have a duty of loyalty to their clients and must act with reasonable care and exercise prudent judgment. Members and Candidates must act for the benefit of their clients and place their clients' interests before their employer's or their own interests. In relationships with clients, Members and Candidates must determine applicable fiduciary duty and must comply with such duty to persons and interests to whom it is owed.

B. **Fair Dealing.** Members and Candidates must deal fairly and objectively with all clients when providing investment analysis, making investment recommendations, taking investment action, or engaging in other professional activities.

C. **Suitability.**

1. When Members and Candidates are in an advisory relationship with a client, they must:

   a. Make a reasonable inquiry into a client's or prospective clients' investment experience, risk and return objectives, and financial constraints prior to making any investment recommendation or taking investment action and must reassess and update this information regularly.

   b. Determine that an investment is suitable to the client's financial situation and consistent with the client's written objectives, mandates, and constraints before making an investment recommendation or taking investment action.

   c. Judge the suitability of investments in the context of the client's total portfolio.

2. When Members and Candidates are responsible for managing a portfolio to a specific mandate, strategy, or style, they must make only investment recommendations or take investment actions that are consistent with the stated objectives and constraints of the portfolio.

D. **Performance Presentation.** When communicating investment performance information, Members or Candidates must make reasonable efforts to ensure that it is fair, accurate, and complete.

E. **Preservation of Confidentiality.** Members and Candidates must keep information about current, former, and prospective clients confidential unless:

1. The information concerns illegal activities on the part of the client or prospective client,

2. Disclosure is required by law, or

3. The client or prospective client permits disclosure of the information.

IV. DUTIES TO EMPLOYERS

A. Loyalty. In matters related to their employment, Members and Candidates must act for the benefit of their employer and *not deprive their employer of the advantage of their skills and abilities, divulge confidential information*, or otherwise cause harm to their employer.

B. Additional Compensation Arrangements. Members and Candidates *must not accept gifts, benefits, compensation, or consideration that competes with, or might reasonably be expected to create a conflict of interest with, their employer's interest unless they obtain written consent from all parties involved.*

C. Responsibilities of Supervisors. Members and Candidates must make reasonable efforts to detect and prevent violations of applicable laws, rules, regulations, and the Code and Standards by anyone subject to their supervision or authority.

V. INVESTMENT ANALYSIS, RECOMMENDATIONS, AND ACTION

A. Diligence and Reasonable Basis. Members and Candidates must:

1. Exercise diligence, independence, and thoroughness in analyzing investments, making investment recommendations, and taking investment actions.

2. Have a reasonable and adequate basis, supported by appropriate research and investigation, for any investment analysis, recommendation, or action.

B. Communication with Clients and Prospective Clients. Members and Candidates must:

1. Disclose to clients and prospective clients the basic format and general principles of the investment processes used to analyze investments, select securities, and construct portfolios and must promptly disclose any changes that might materially affect those processes.

2. Use reasonable judgment in identifying which factors are important to their investment analyses, recommendations, or actions and include those factors in communications with clients and prospective clients.

3. Distinguish between fact and opinion in the presentation of investment analysis and recommendations.

C. Record Retention. Members and Candidates must develop and maintain appropriate records to support their investment analysis, recommendations, actions, and other investment-related communications with clients and prospective clients.

VI. CONFLICTS OF INTEREST

A. Disclosure of Conflicts. Members and Candidates must make full and fair disclosure of all matters that could reasonably be expected to impair their independence and objectivity or interfere with respective duties to their clients, prospective clients, and employer. Members and Candidates must ensure that such disclosures are prominent, are delivered in plain language, and communicate the relevant information effectively.

B. Priority of Transactions. Investment transactions for clients and employers must have priority over investment transactions in which a Member or Candidate is the beneficial owner.

C. **Referral Fees.** Members and Candidates must disclose to their employer, clients, and prospective clients, as appropriate, any compensation, consideration, or benefit received by, or paid to, others for the recommendation of products or services.

VII. RESPONSIBILITIES AS A CFA INSTITUTE MEMBER OR CFA CANDIDATE

A. **Conduct as Members and Candidates in the CFA Program.** *Members and Candidates must not engage in any conduct that compromises the reputation or integrity of CFA Institute or the CFA designation or the integrity, validity, or security of the CFA examinations.*

B. **Reference to CFA Institute, the CFA designation, and the CFA Program.** When referring to CFA Institute, CFA Institute membership, the CFA designation, or candidacy in the CFA Program, Members and Candidates must not misrepresent or exaggerate the meaning or implications of membership in CFA Institute, holding the CFA designation, or candidacy in the CFA Program.

STANDARDS OF PROFESSIONAL CONDUCT: GUIDANCE, COMPLIANCE, AND EXAMPLES

LOS 2: Guidance for Standards I–VII.

The guidance in the *Standards of Practice Handbook* addresses the application of the Standards of Professional Conduct. For each standard, the *Handbook* offers guidance for the standard, presents recommended procedures for compliance, and provides examples of the standard in practice. The candidate should be able to:

LOS 2.a: Demonstrate a thorough knowledge of the Standards of Professional Conduct by recognizing and applying the standards to specific situations.

LOS 2.b: Distinguish between conduct that conforms to the Code and Standards and conduct that violates the Code and the Standards.

I   Professionalism

Professor's Note: While we use the term "members" in the following, note that all of the standards apply to candidates as well.

I(A)   Knowledge of the Law. Members must understand and comply with laws, rules, regulations, and Code and Standards of any authority governing their activities. In the event of a conflict, follow the more strict law, rule, or regulation. Do not knowingly participate or assist in violations, and *dissociate from any known violation.*

Guidance—Code and Standards vs. Local Law

Members must know the laws and regulations relating to their professional activities in all countries in which they conduct business. Members must comply with applicable laws and regulations relating to their professional activity. Do not violate Code or Standards even if the activity is otherwise legal. Always adhere to the most strict rules and requirements (law or CFA Institute Standards) that apply.

Guidance—Participation or Association with Violations by Others

Members should dissociate, or separate themselves, from any ongoing client or employee activity that is illegal or unethical, even if it involves leaving an employer (an extreme case). While a member may confront the involved individual first, he must approach his supervisor or compliance department. Inaction with continued association may be construed as knowing participation.

Recommended Procedures for Compliance—Members

- Members should have procedures to keep up with changes in applicable laws, rules, and regulations.
- Compliance procedures should be reviewed on an ongoing basis to assure that they address current law, CFAI Standards, and regulations.
- Members should maintain current reference materials for employees to access in order to keep up to date on laws, rules, and regulations.
- Members should seek advice of counsel or their compliance department when in doubt.
- Members should document any violations when they disassociate themselves from prohibited activity and encourage their employers to bring an end to such activity.
- There is no requirement under the Standards to report violations to governmental authorities, but this may be advisable in some circumstances and required by law in others.

Recommended Procedures for Compliance—Firms

Members should encourage their firms to:

- Develop and/or adopt a code of ethics.
- Make available to employees information that highlights applicable laws and regulations.
- Establish written procedures for reporting suspected violation of laws, regulations, or company policies.

Application of Standard I(A) Knowledge of the Law[4]

Example 1:

Michael Allen works for a brokerage firm and is responsible for an underwriting of securities. A company official gives Allen information indicating that the financial statements Allen filed with the regulator overstate the issuer's earnings. Allen seeks the advice of the brokerage firm's general counsel, who states that it would be difficult for the regulator to prove that Allen has been involved in any wrongdoing.

Comment:

Although it is recommended that members and candidates seek the advice of legal counsel, the reliance on such advice does not absolve a member or candidate from the requirement to comply with the law or regulation. Allen should report this situation to his supervisor, seek an independent legal opinion, and determine whether the regulator should be notified of the error.

Example 2:

Kamisha Washington's firm advertises its past performance record by showing the 10-year return of a composite of its client accounts. However, Washington discovers that the composite omits the performance of accounts that have left the firm during the 10-year period and that this omission has led to an inflated performance figure. Washington is asked to use promotional material that includes the erroneous performance number when soliciting business for the firm.

Comment:

Misrepresenting performance is a violation of the Code and Standards. Although she did not calculate the performance herself, Washington would be assisting in violating this standard if she were to use the inflated performance number when soliciting clients. She must dissociate herself from the activity. She can bring the misleading number to the attention of the person responsible for calculating performance, her supervisor, or

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the compliance department at her firm. If her firm is unwilling to recalculate performance, she must refrain from using the misleading promotional material and should notify the firm of her reasons. If the firm insists that she use the material, she should consider whether her obligation to dissociate from the activity would require her to seek other employment.

I(B) Independence and Objectivity. Use reasonable care to exercise independence and objectivity in professional activities. Members and Candidates are not to offer, solicit, or accept any gift, benefit, compensation, or consideration that would compromise either their own or someone else's independence and objectivity.

Professor's Note: It is made clearer than it was under the old Standards that gifts, benefits, and other consideration are prohibited if given in an attempt to influence Members or Candidates.

Guidance

Do not let the investment process be influenced by any external sources. Modest gifts are permitted. Allocation of shares in oversubscribed IPOs to personal accounts is NOT permitted. Distinguish between gifts from clients and gifts from entities seeking influence to the detriment of the client. Gifts must be disclosed to the member's employer in any case.

Guidance—Investment-Banking Relationships

Do not be pressured by sell-side firms to issue favorable research on current or prospective investment-banking clients. It is appropriate to have analysts work with investment bankers in "road shows" only when the conflicts are adequately and effectively managed and disclosed. Be sure there are effective "firewalls" between research/investment management and investment banking activities.

Guidance—Public Companies

Analysts should not be pressured to issue favorable research by the companies they follow. Do not confine research to discussions with company management, but rather use a variety of sources, including suppliers, customers, and competitors.

Guidance—Buy-Side Clients

Buy-side clients may try to pressure sell-side analysts. Portfolio managers may have large positions in a particular security, and a rating downgrade may have an effect on the portfolio performance. As a portfolio manager, there is a responsibility to respect and foster intellectual honesty of sell-side research.

Guidance—Issuer-Paid Research

Remember that this type of research is fraught with potential conflicts. Analysts' compensation for preparing such research should be limited, and the preference is for a flat fee, without regard to conclusions or the report's recommendations.

Recommended Procedures for Compliance

- Protect the integrity of opinions—make sure they are unbiased.
- Create a restricted list and distribute only factual information about companies on the list.
- Restrict special cost arrangements—pay for one's own commercial transportation and hotel; limit use of corporate aircraft to cases in which commercial transportation is not available.
- Limit gifts—token items only. Customary, business-related entertainment is okay as long as its purpose is **not** to influence a member's professional independence or objectivity.
- Restrict employee investments in equity IPOs and private placements.

- Review procedures—have effective supervisory and review procedures.
- Firms should have formal written policies on independence and objectivity of research.

Application of Standard I(B) Independence and Objectivity

Example 1:

Steven Taylor, a mining analyst with Bronson Brokers, is invited by Precision Metals to join a group of his peers in a tour of mining facilities in several western U.S. states. The company arranges for chartered group flights from site to site and for accommodations in Spartan Motels, the only chain with accommodations near the mines, for three nights. Taylor allows Precision Metals to pick up his tab, as do the other analysts, with one exception—John Adams, an employee of a large trust company who insists on following his company's policy and paying for his hotel room himself.

Comment:

The policy of Adam's company complies closely with Standard I(B) by avoiding even the appearance of a conflict of interest, but Taylor and the other analysts were not necessarily violating Standard I(B). In general, when allowing companies to pay for travel and/or accommodations under these circumstances, members and candidates must use their judgment, keeping in mind that such arrangements must not impinge on a member or candidate's independence and objectivity. In this example, the trip was strictly for business and Taylor was not accepting irrelevant or lavish hospitality. The itinerary required chartered flights, for which analysts were not expected to pay. The accommodations were modest. These arrangements are not unusual and did not violate Standard I(B) so long as Taylor's independence and objectivity were not compromised. In the final analysis, members and candidates should consider both whether they can remain objective and whether their integrity might be perceived by their clients to have been compromised.

Example 2:

Walter Fritz is an equity analyst with Hilton Brokerage who covers the mining industry. He has concluded that the stock of Metals & Mining is overpriced at its current level, but he is concerned that a negative research report will hurt the good relationship between Metals & Mining and the investment-banking division of his firm. In fact, a senior manager of Hilton Brokerage has just sent him a copy of a proposal his firm has made to Metals & Mining to underwrite a debt offering. Fritz needs to produce a report right away and is concerned about issuing a less-than-favorable rating.

Comment:

Fritz's analysis of Metals & Mining must be objective and based solely on consideration of company fundamentals. Any pressure from other divisions of his firm is inappropriate. This conflict could have been eliminated if, in anticipation of the offering, Hilton Brokerage had placed Metals & Mining on a restricted list for its sales force.

Example 3:

Tom Wayne is the investment manager of the Franklin City Employees Pension Plan. He recently completed a successful search for firms to manage the foreign equity allocation of the plan's diversified portfolio. He followed the plan's standard procedure of seeking presentations from a number of qualified firms and recommended that his board select Penguin Advisors because of its experience, well-defined investment strategy, and performance record, which was compiled and verified in accordance with the CFA Institute Global Investment Performance Standards. Following the plan selection of Penguin, a reporter from the Franklin City Record called to ask if there was any connection between the action and the fact that Penguin was one of the sponsors of an "investment fact-finding trip to Asia" that Wayne made earlier in the year. The trip was one of several conducted by the Pension Investment Academy, which had arranged the itinerary of

meetings with economic, government, and corporate officials in major cities in several Asian countries. The Pension Investment Academy obtains support for the cost of these trips from a number of investment managers including Penguin Advisors; the Academy then pays the travel expenses of the various pension plan managers on the trip and provides all meals and accommodations. The president of Penguin Advisors was one of the travelers on the trip.

Comment:

Although Wayne can probably put to good use the knowledge he gained from the trip in selecting portfolio managers and in other areas of managing the pension plan, his recommendation of Penguin Advisors may be tainted by the possible conflict incurred when he participated in a trip paid partly for by Penguin Advisors and when he was in the daily company of the president of Penguin Advisors. To avoid violating Standard I(B), Wayne's basic expenses for travel and accommodations should have been paid by his employer or the pension plan; contact with the president of Penguin Advisors should have been limited to informational or educational events only; and the trip, the organizer, and the sponsor should have been made a matter of public record. Even if his actions were not in violation of Standard I(B), Wayne should have been sensitive to the public perception of the trip when reported in the newspaper and the extent to which the subjective elements of his decision might have been affected by the familiarity that the daily contact of such a trip would encourage. This advantage would probably not be shared by competing firms.

I(C) Misrepresentation. Do not misrepresent facts regarding investment analysis, recommendations, actions, or other professional activities.

Professor's Note: There is stronger language concerning misrepresentation—prohibition of false and misleading statements in all aspects of Members' and Candidates' professional activities and a prohibition against plagiarism.

Guidance

Trust is a foundation in the investment profession. Do not make any misrepresentations or give false impressions. This includes oral and electronic communications. Misrepresentations include guaranteeing investment performance and plagiarism. Plagiarism encompasses using someone else's work (reports, forecasts, charts, graphs, and spreadsheet models) without giving them credit.

Recommended Procedures for Compliance

A good way to avoid misrepresentation is for firms to provide employees who deal with clients or prospects a written list of the firm's available services and a description of the firm's qualifications. Employee qualifications should be accurately presented as well. To avoid plagiarism, maintain records of all materials used to generate reports or other firm products and properly cite sources (quotes and summaries) in work products. Information from recognized financial and statistical reporting services need not be cited.

Application of Standard I(C) Misrepresentations

Example 1:

Allison Rogers is a partner in the firm of Rogers and Black, a small firm offering investment advisory services. She assures a prospective client who has just inherited $1 million that "we can perform all the financial and investment services you need." Rogers and Black is well equipped to provide investment advice but, in fact, cannot provide asset allocation assistance or a full array of financial and investment services.

Comment:

Rogers has violated Standard I(C) by orally misrepresenting the services her firm can perform for the prospective client. She must limit herself to describing the range of investment advisory services Rogers and

Black can provide and offer to help the client obtain elsewhere the financial and investment services that her firm cannot provide.

Example 2:

Anthony McGuire is an issuer-paid analyst hired by publicly traded companies to electronically promote their stocks. McGuire creates a website that promotes his research efforts as a seemingly independent analyst. McGuire posts a profile and a strong buy recommendation for each company on the website indicating that the stock is expected to increase in value. He does not disclose the contractual relationships with the companies he covers on his website, in the research reports he issues, or in the statements he makes about the companies on Internet chat rooms.

Comment:

McGuire has violated Standard I(C) because the Internet site and e-mails are misleading to potential investors. Even if the recommendations are valid and supported with thorough research, his omissions regarding the true relationship between himself and the companies he covers constitute a misrepresentation. McGuire has also violated Standard VI(C) by not disclosing the existence of an arrangement with the companies through which he receives compensation in exchange for his services.

Example 3:

Claude Browning, a quantitative analyst for Double Alpha, Inc., returns in great excitement from a seminar. In that seminar, Jack Jorrely, a well-publicized quantitative analyst at a national brokerage firm, discussed one of his new models in great detail, and Browning is intrigued by the new concepts. He proceeds to test this model, making some minor mechanical changes but retaining the concept, until he produces some very positive results. Browning quickly announces to his supervisors at Double Alpha that he has discovered a new model and that clients and prospective clients alike should be informed of this positive finding as ongoing proof of Double Alpha's continuing innovation and ability to add value.

Comment:

Although Browning tested Jorrely's model on his own and even slightly modified it, he must still acknowledge the original source of the idea. Browning can certainly take credit for the final, practical results; he can also support his conclusions with his own test. The credit for the innovative thinking, however, must be awarded to Jorrely.

Example 4:

Gary Ostrowski runs a small, two-person investment management firm. Ostrowski's firm subscribes to a service from a large investment research firm that provides research reports that can be repackaged as in-house research from smaller firms. Ostrowski's firm distributes these reports to clients as its own work.

Comment:

Gary Ostrowski can rely on third-party research that has a reasonable and adequate basis, but he cannot imply that he is the author of the report. Otherwise, Ostrowski would misrepresent the extent of his work in a way that would mislead the firm's clients or prospective clients.

I(D) Misconduct. Do not engage in any professional conduct which involves dishonesty, fraud, or deceit. Do not do anything that reflects poorly on your integrity, good reputation, trustworthiness, or professional competence.

Professor's Note: There is a subtle change here versus the old Standard. The new focus is on professional rather than personal conduct. There is no longer an attempt to overreach or regulate one's personal behavior.

Guidance

CFA Institute discourages unethical behavior in all aspects of members' and candidates' lives. Do not abuse CFA Institute's Professional Conduct Program by seeking enforcement of this Standard to settle personal, political, or other disputes that are not related to professional ethics.

Recommended Procedures for Compliance

Firms are encouraged to adopt these policies and procedures:

- Develop and adopt a code of ethics and make clear that unethical behavior will not be tolerated.
- Give employees a list of potential violations and sanctions, including dismissal.
- Check references of potential employees.

Application of Standard I(D) Misconduct

Example 1:

Simon Sasserman is a trust investment officer at a bank in a small affluent town. He enjoys lunching every day with friends at the country club, where his clients have observed him having numerous drinks. Back at work after lunch, he clearly is intoxicated while making investment decisions. His colleagues make a point of handling any business with Sasserman in the morning because they distrust his judgment after lunch.

Comment:

Sasserman's excessive drinking at lunch and subsequent intoxication at work constitute a violation of Standard I(D) because this conduct has raised questions about his professionalism and competence. His behavior thus reflects poorly on him, his employer, and the investment industry.

Example 2:

Carmen Garcia manages a mutual fund dedicated to socially responsible investing. She is also an environmental activist. As the result of her participation at nonviolent protests, Garcia has been arrested on numerous occasions for trespassing on the property of a large petrochemical plant that is accused of damaging the environment.

Comment:

Generally, Standard I(D) is not meant to cover legal transgressions resulting from acts of civil disobedience in support of personal beliefs because such conduct does not reflect poorly on the member or candidate's professional reputation, integrity, or competence.

II Integrity of Capital Markets

II(A) Material Nonpublic Information. Members and Candidates in possession of nonpublic information that could affect an investment's value must not act or induce someone else to act on the information.

Professor's Note: This Standard attempts to prohibit any conduct that will damage the integrity of the markets. The new Standard is more straightforward—it states that Members and Candidates must not act or cause others to act on material nonpublic information until that same information is made public. It no longer matters whether the information is obtained in breach of a duty, is misappropriated, or relates to a tender offer.

Guidance

Information is "material" if its disclosure would impact the price of a security or if reasonable investors would want the information before making an investment decision. Ambiguous information, as far as its likely effect on price, may not be considered material. Information is "non-public" until it has been made available to the marketplace. An analyst conference call is not public disclosure. Selectively disclosing information by corporations creates the potential for insider-trading violations.

Guidance—Mosaic Theory

There is no violation when a perceptive analyst reaches an investment conclusion about a corporate action or event through an analysis of public information together with items of non-material non-public information.

Recommended Procedures for Compliance

Make reasonable efforts to achieve public dissemination of the information. Encourage firms to adopt procedures to prevent misuse of material nonpublic information. Use a "firewall" within the firm, with elements including:

- Substantial control of relevant interdepartmental communications, through a clearance area such as the compliance or legal department.
- Review employee trades—maintain "watch," "restricted," and "rumor" lists.
- Monitor and restrict proprietary trading while a firm is in possession of material nonpublic information.

Prohibition of all proprietary trading while a firm is in possession of material nonpublic information may be inappropriate because it may send a signal to the market. In these cases, firms should take the contra side of only unsolicited customer trades.

Application of Standard II(A) Material Nonpublic Information

Example 1:

Josephine Walsh is riding an elevator up to her office when she overhears the chief financial officer (CFO) for the Swan Furniture Company tell the president of Swan that he has just calculated the company's earnings for the past quarter and they have unexpectedly and significantly dropped. The CFO adds that this drop will not be released to the public until next week. Walsh immediately calls her broker and tells him to sell her Swan stock.

Comment:

Walsh has sufficient information to determine that the information is both material and nonpublic. By trading on the inside information, she has violated Standard II(A).

Example 2:

Samuel Peter, an analyst with Scotland and Pierce Incorporated, is assisting his firm with a secondary offering for Bright Ideas Lamp Company. Peter participates, via telephone conference call, in a meeting with Scotland and Pierce investment-banking employees and Bright Ideas' CEO. Peter is advised that the company's earnings projections for the next year have significantly dropped. Throughout the telephone conference call, several Scotland and Pierce salespeople and portfolio managers walk in and out of Peter's office, where the telephone call is taking place. As a result, they are aware of the drop in projected earnings for Bright Ideas. Before the conference call is concluded, the salespeople trade the stock of the company on behalf of the firm's clients and other firm personnel trade the stock in a firm proprietary account and in employee personal accounts.

Comment:

Peter violated Standard II(A) because he failed to prevent the transfer and misuse of material nonpublic information to others in his firm. Peter's firm should have adopted information barriers to prevent the communication of nonpublic information between departments of the firm. The salespeople and portfolio managers who traded on the information have also violated Standard II(A) by trading on inside information.

Example 3:

Elizabeth Levenson is based in Taipei and covers the Taiwanese market for her firm, which is based in Singapore. She is invited to meet the finance director of a manufacturing company along with the other 10 largest shareholders of the company. During the meeting, the finance director states that the company expects its workforce to strike next Friday, which will cripple productivity and distribution. Can Levenson use this information as a basis to change her rating on the company from "buy" to "sell"?

Comment:

Levenson must first determine whether the material information is public. If the company has not made this information public (a small-group forum does not qualify as a method of public dissemination), she cannot use the information according to Standard II(A).

Example 4:

Jagdish Teja is a buy-side analyst covering the furniture industry. Looking for an attractive company to recommend as a buy, he analyzed several furniture makers by studying their financial reports and visiting their operations. He also talked to some designers and retailers to find out which furniture styles are trendy and popular. Although none of the companies that he analyzed turned out to be a clear buy, he discovered that one of them, Swan Furniture Company (SFC), might be in trouble. Swan's extravagant new designs were introduced at substantial costs. Even though these designs initially attracted attention, in the long run, the public is buying more conservative furniture from other makers. Based on that and on P&L analysis, Teja believes that Swan's next-quarter earnings will drop substantially. He then issues a sell recommendation for SFC. Immediately after receiving that recommendation, investment managers start reducing the stock in their portfolios.

Comment:

Information on quarterly earnings figures is material and nonpublic. However, Teja arrived at his conclusion about the earnings drop based on public information and on pieces of nonmaterial nonpublic information (such as opinions of designers and retailers). Therefore, trading based on Teja's correct conclusion is not prohibited by Standard II(A).

II(B) Market Manipulation. Do not engage in any practices intended to mislead market participants through distorted prices or artificially inflated trading volume.

Professor's Note: This new Standard requires Members and Candidates to uphold market integrity by banning practices that distort security prices or trading volume with the intent to deceive.

Guidance

This Standard applies to transactions that deceive the market by distorting the price-setting mechanism of financial instruments or by securing a controlling position to manipulate the price of a related derivative and/or the asset itself. Spreading false rumors is also prohibited.

Application of Standard II(B) Market Manipulation

Example 1:

Matthew Murphy is an analyst at Divisadero Securities & Co., which has a significant number of hedge funds among its most important brokerage clients. Two trading days before the publication of the quarter-end report, Murphy alerts his sales force that he is about to issue a research report on Wirewolf Semiconductor, which will include his opinion that

- quarterly revenues are likely to fall short of management's guidance,
- earnings will be as much as 5 cents per share (or more than 10 percent) below consensus, and
- Wirewolf's highly respected chief financial officer may be about to join another company.

Knowing that Wirewolf had already entered its declared quarter-end "quiet period" before reporting earnings (and thus would be reluctant to respond to rumors, etc.), Murphy times the release of his research report specifically to sensationalize the negative aspects of the message to create significant downward pressure on Wirewolf's stock to the distinct advantage of Divisadero's hedge fund clients. The report's conclusions are based on speculation, not on fact. The next day, the research report is broadcast to all of Divisadero's clients and to the usual newswire services.

Before Wirewolf's investor relations department can assess its damage on the final trading day of the quarter and refute Murphy's report, its stock opens trading sharply lower, allowing Divisadero's clients to cover their short positions at substantial gains.

Comment:

Murphy violated Standard II(B) by trying to create artificial price volatility designed to have material impact on the price of an issuer's stock. Moreover, by lacking an adequate basis for the recommendation, Murphy also violated Standard V(A).

Example 2:

Sergei Gonchar is the chairman of the ACME Futures Exchange, which seeks to launch a new bond futures contract. In order to convince investors, traders, arbitragers, hedgers, and so on, to use its contract, the exchange attempts to demonstrate that it has the best liquidity. To do so, it enters into agreements with members so that they commit to a substantial minimum trading volume on the new contract over a specific period in exchange for substantial reductions on their regular commissions.

Comment:

Formal liquidity on a market is determined by the obligations set on market makers, but the actual liquidity of a market is better estimated by the actual trading volume and bid-ask spreads. Attempts to mislead participants on the actual liquidity of the market constitute a violation of Standard II(B). In this example, investors have been intentionally misled to believe they chose the most liquid instrument for some specific purpose and could eventually see the actual liquidity of the contract dry up suddenly after the term of the agreement if the "pump-priming" strategy fails. If ACME fully discloses its agreement with members to boost transactions over some initial launch period, it does not violate Standard II(B). ACME's intent is not to harm investors but on the contrary to give them a better service. For that purpose, it may engage in a liquidity-pumping strategy, but it must be disclosed.

III   Duties to Clients and Prospective Clients

III(A) Loyalty, Prudence, and Care. Members must always act for the benefit of clients and place clients' interests before their employer's or their own interests. Members must be loyal to clients, use reasonable care, exercise prudent judgment, and determine and comply with their applicable fiduciary duty to clients.

Professor's Note: This Standard continues to require that members and candidates understand and comply with their actual fiduciary duty; however, there is now a minimum level of conduct—reasonable care and prudent judgment must be exercised in all circumstances.

Guidance

Client interests always come first.

- Exercise the prudence, care, skill, and diligence under the circumstances that a person acting in a like capacity and familiar with such matters would use.
- Manage pools of client assets in accordance with the terms of the governing documents, such as trust documents or investment management agreements.
- Make investment decisions in the context of the total portfolio.
- Vote proxies in an informed and responsible manner. Due to cost benefit considerations, it may not be necessary to vote all proxies.
- Client brokerage, or "soft dollars" or "soft commissions" must be used to benefit the client.

Recommended Procedures of Compliance

Submit to clients, at least quarterly, itemized statements showing all securities in custody and all debits, credits, and transactions.

Encourage firms to address these topics when drafting policies and procedures regarding fiduciary duty:

- Follow applicable rules and laws.
- Establish investment objectives of client. Consider suitability of portfolio relative to client's needs and circumstances, the investment's basic characteristics, or the basic characteristics of the total portfolio.
- Diversify.
- Deal fairly with all clients in regards to investment actions.
- Disclose conflicts.
- Disclose compensation arrangements.
- Vote proxies in the best interest of clients and ultimate beneficiaries.
- Maintain confidentiality.
- Seek best execution.
- Place client interests first.

Application of Standard III(A) Loyalty, Prudence, and Care

Example 1:

First Country Bank serves as trustee for the Miller Company's pension plan. Miller is the target of a hostile takeover attempt by Newton, Inc. In attempting to ward off Newton, Miller's managers persuade Julian Wiley, an investment manager at First Country Bank, to purchase Miller common stock in the open market for the employee pension plan. Miller's officials indicate that such action would be favorably received and would probably result in other accounts being placed with the bank. Although Wiley believes the stock to be overvalued and would not ordinarily buy it, he purchases the stock to support Miller's managers, to maintain the company's good favor, and to realize additional new business. The heavy stock purchases cause Miller's market price to rise to such a level that Newton retracts its takeover bid.

Comment:

Standard III(A) requires that a member or candidate, in evaluating a takeover bid, act prudently and solely in the interests of plan participants and beneficiaries. To meet this requirement, a member or candidate must carefully evaluate the long-term prospects of the company against the short-term prospects presented by the takeover offer and by the ability to invest elsewhere. In this instance, Wiley, acting on behalf of his employer, the trustee, clearly violated Standard III(A) by using the profit-sharing plan to perpetuate existing management, perhaps to the detriment of plan participants and the company's shareholders, and to benefit himself. Wiley's responsibilities to the plan participants and beneficiaries should take precedence over any ties to corporate managers and self-interest. A duty exists to examine such a takeover offer on its own merits and to make an independent decision. The guiding principle is the appropriateness of the investment decision to the pension plan, not whether the decision benefits Wiley or the company that hired him.

Example 2:

Emilie Rome is a trust officer for Paget Trust Company. Rome's supervisor is responsible for reviewing Rome's trust account transactions and her monthly reports of personal stock transactions. Rome has been using Nathan Gray, a broker, almost exclusively for trust account brokerage transactions. Where Gray makes a market in stocks, he has been giving Rome a lower price for personal purchases and a higher price for sales than he gives to Rome's trust accounts and other investors.

Comment:

Rome is violating her duty of loyalty to the bank's trust accounts by using Gray for brokerage transactions simply because Gray trades Rome's personal account on favorable terms.

III(B) Fair Dealing. Members must deal fairly and objectively with all clients and prospects when providing investment analysis, making investment recommendations, taking investment action, or in other professional activities.

Professor's Note: This Standard is largely unchanged but is slightly broadened.

Guidance

Do not discriminate against any clients when disseminating recommendations or taking investment action. Fairly does not mean equally. In the normal course of business, there will be differences in the time emails, faxes, etc. are received by different clients. Different service levels are okay, but they must not negatively affect or disadvantage any clients. Disclose the different service levels to all clients and prospects, and make premium levels of service available to all who wish to pay for them.

Guidance—Investment Recommendations

Give all clients a fair opportunity to act upon every recommendation. Clients who are unaware of a change in a recommendation should be advised before the order is accepted.

Guidance—Investment Actions

Treat clients fairly in light of their investment objectives and circumstances. Treat both individual and institutional clients in a fair and impartial manner. Members and Candidates should not take advantage of their position in the industry to disadvantage clients (e.g., in the context of IPOs).

Recommended Procedures for Compliance

Encourage firms to establish compliance procedures requiring proper dissemination of investment recommendations and fair treatment of all customers and clients. Consider these points when establishing fair dealing compliance procedures:

- Limit the number of people who are aware that a change in recommendation will be made.
- Shorten the time frame between decision and dissemination.
- Publish personnel guidelines for pre-dissemination—have in place guidelines prohibiting personnel who have prior knowledge of a recommendation from discussing it or taking action on the pending recommendation.
- Simultaneous dissemination.
- Maintain list of clients and holdings—use to ensure that all holders are treated fairly.
- Develop written trade allocation procedures—ensure fairness to clients, timely and efficient order execution, and accuracy of client positions.
- Disclose trade allocation procedures.
- Establish systematic account review—to ensure that no client is given preferred treatment and that investment actions are consistent with the account's objectives.
- Disclose available levels of service.

Application of Standard III(B) Fair Dealing

Example 1:

Bradley Ames, a well-known and respected analyst, follows the computer industry. In the course of his research, he finds that a small, relatively unknown company whose shares are traded over the counter has just signed significant contracts with some of the companies he follows. After a considerable amount of investigation, Ames decides to write a research report on the company and recommend purchase. While the report is being reviewed by the company for factual accuracy, Ames schedules a luncheon with several of his best clients to discuss the company. At the luncheon, he mentions the purchase recommendation scheduled to be sent early the following week to all the firm's clients.

Comment:

Ames violated Standard III(B) by disseminating the purchase recommendation to the clients with whom he had lunch a week before the recommendation was sent to all clients.

Example 2:

Spencer Rivers, president of XYZ Corporation, moves his company's growth-oriented pension fund to a particular bank primarily because of the excellent investment performance achieved by the bank's commingled fund for the prior five-year period. A few years later, Rivers compares the results of his pension fund with those of the bank's commingled fund. He is startled to learn that, even though the two accounts have the same investment objectives and similar portfolios, his company's pension fund has significantly underperformed the bank's commingled fund. Questioning this result at his next meeting with the pension fund's manger, Rivers is told that, as a matter of policy, when a new security is placed on the recommended list, Morgan Jackson, the pension fund manger, first purchases the security for the commingled account and then purchases it on a pro rata basis for all other pension fund accounts. Similarly, when a sale is recommended, the security is sold first from the commingled account and then sold on a pro rata basis from all other accounts. Rivers also learns that if the bank cannot get enough shares (especially the hot issues) to be meaningful to all the accounts, its policy is to place the new issues only in the commingled account.

Seeing that Rivers is neither satisfied nor pleased by the explanation, Jackson quickly adds that nondiscretionary pension accounts and personal trust accounts have a lower priority on purchase and sale

recommendations than discretionary pension fund accounts. Furthermore, Jackson states, the company's pension fund had the opportunity to invest up to 5 percent in the commingled fund.

Comment:

The bank's policy did not treat all customers fairly, and Jackson violated her duty to her clients by giving priority to the growth-oriented commingled fund over all other funds and to discretionary accounts over nondiscretionary accounts. Jackson must execute orders on a systematic basis that is fair to all clients. In addition, trade allocation procedures should be disclosed to all clients from the beginning. Of course, in this case, disclosure of the bank's policy would not change the fact that the policy is unfair.

III(C) Suitability

1. When in an advisory relationship with client or prospect, Members and Candidates must:

    a. Make reasonable inquiry into clients' investment experience, risk and return objectives, and constraints prior to making any recommendations or taking investment action. Reassess information and update regularly.

    b. Be sure investments are suitable to a client's financial situation and consistent with client objectives before making recommendation or taking investment action.

    c. Make sure investments are suitable in the context of a client's total portfolio.

2. When managing a portfolio, investment recommendations and actions must be consistent with stated portfolio objectives and constraints.

Professor's Note: "Regular updates" to client information should be done at least annually. Suitability is based on a total-portfolio perspective.

Guidance

In advisory relationships, be sure to gather client information at the beginning of the relationship, in the form of an investment policy statement (IPS). Consider client's needs and circumstances and thus the risk tolerance. Consider whether or not the use of leverage is suitable for the client.

If a member is responsible for managing a fund to an index or other stated mandate, be sure investments are consistent with the stated mandate.

Recommended Procedures for Compliance

Members should:

- Put the needs and circumstances of each client and the client's investment objectives into a written IPS for each client.
- Consider the type of client and whether there are separate beneficiaries, investor objectives (return and risk), investor constraints (liquidity needs, expected cash flows, time, tax, and regulatory and legal circumstances), and performance measurement benchmarks.
- Review investor's objectives and constraints periodically to reflect any changes in client circumstances.

Application of Standard III(C) Suitability

Example 1:

Ann Walters, an investment advisor, suggests to Brian Crosby, a risk-averse client, that covered call options be used in his equity portfolio. The purpose would be to enhance Crosby's income and partially offset any untimely depreciation in value should the stock market or other circumstances affect his holdings unfavorably. Walters educates Crosby about all possible outcomes, including the risk of incurring an added tax liability if a stock rises in price and is called away and, conversely, the risk of his holdings losing protection on the downside if prices drop sharply.

Comment:

When determining suitability of an investment, the primary focus should be on the characteristics of the client's entire portfolio, not on an issue-by-issue analysis. The basic characteristics of the entire portfolio will largely determine whether the investment recommendations are taking client factors into account. Therefore, the most important aspects of a particular investment will be those that will affect the characteristics of the total portfolio. In this case, Walters properly considered the investment in the context of the entire portfolio and thoroughly explained the investment to the client.

Example 2:

Max Gubler, CIO of a property/casualty insurance subsidiary of a large financial conglomerate, wants to better diversify the company's investment portfolio and increase its returns. The company's investment policy statement (IPS) provides for highly liquid investments, such as large caps, governments, and supra-nationals, as well as corporate bonds with a minimum credit rating of AA—and maturity of no more than five years. In a recent presentation, a venture capital group offered very attractive prospective returns on some of their private equity funds providing seed capital. An exit strategy is already contemplated but investors will first have to observe a minimum three-year lock-up period, with a subsequent laddered exit option for a maximum of one third of shares per year. Gubler does not want to miss this opportunity and after an extensive analysis and optimization of this asset class with the company's current portfolio, he invests 4 percent in this seed fund, leaving the portfolio's total equity exposure still well below its upper limit.

Comment:

Gubler violates Standards III(A) and III(C). His new investment locks up part of the company's assets for at least three and for up to as many as five years and possibly beyond. Since the IPS requires investments in highly liquid investments and describes accepted asset classes, private equity investments with a lock-up period certainly do not qualify. Even without such lock-up periods an asset class with only an occasional, and thus implicitly illiquid, market may not be suitable. Although an IPS typically describes objectives and constraints in great detail, the manger must make every effort to understand the client's business and circumstances. Doing so should also enable the manager to recognize, understand, and discuss with the client other factors that may be or may become material in the investment management process.

III(D) Performance Presentation. Presentations of investment performance information must be fair, accurate, and complete.

Guidance

Members must avoid misstating performance or misleading clients/prospects about investment performance of themselves or their firms, should not misrepresent past performance or reasonably expected performance, and should not state or imply the ability to achieve a rate of return similar to that achieved in the past.

Recommended Procedures for Compliance

Encourage firms to adhere to Global Investment Performance Standards. Obligations under this Standard may also be met by:

- Considering the sophistication of the audience to whom a performance presentation is addressed.
- Presenting performance of weighted composite of similar portfolios rather than a single account.
- Including terminated accounts as part of historical performance.
- Including all appropriate disclosures to fully explain results (e.g., model results included, gross or net of fees, etc.).
- Maintaining data and records used to calculate the performance being presented.

Application of Standard III(D) Performance Presentation

Example 1:

Kyle Taylor of Taylor Trust Company, noting the performance of Taylor's common trust fund for the past two years, states in the brochure sent to his potential clients that "You can expect steady 25 percent annual compound growth of the value of your investments over the year." Taylor Trust's common trust fund did increase at the rate of 25 percent per annum for the past year which mirrored the increase of the entire market. The fund, however, never averaged that growth for more than one year, and the average rate of growth of all of its trust accounts for five years was 5 percent per annum.

Comment:

Taylor's brochure is in violation of Standard III(D). Taylor should have disclosed that the 25 percent growth occurred in only one year. Additionally, Taylor did not include client accounts other than those in the firm's common trust fund. A general claim of firm performance should take into account the performance of all categories of accounts. Finally, by stating that clients can expect a steady 25 percent annual compound growth rate, Taylor also violated Standard I(C), which prohibits statements of assurances or guarantees regarding an investment.

Example 2:

Aaron McCoy is vice president and managing partner of the equity investment group of Mastermind Financial Advisors, a new business. Mastermind recruited McCoy because he had a proven six-year track record with G&P Financial. In developing Mastermind's advertising and marketing campaign, McCoy prepared an advertisement that included the equity investment performance he achieved at G&P Financial. The advertisement for Mastermind did not identify the equity performance as being earned while at G&P. The advertisement was distributed to existing clients and prospective clients of Mastermind.

Comment:

McCoy violated Standard III(D) by distributing an advertisement that contained material misrepresentations regarding the historical performance of Mastermind. Standard III(D) requires that members and candidates make every reasonable effort to ensure that performance information is a fair, accurate, and complete representation of an individual or firm's performance. As a general matter, this standard does not prohibit showing past performance of funds managed at a prior firm as part of a performance track record so long as it is accompanied by appropriate disclosures detailing where the performance comes from and the person's specific role in achieving that performance. If McCoy chooses to use his past performance from G&P in Mastermind's advertising, he should make full disclosure as to the source of the historical performance.

III(E) Preservation of Confidentiality. All information about current and former clients and prospects must be kept confidential unless it pertains to illegal activities, disclosure is required by law, or the client or prospect gives permission for the information to be disclosed.

Professor's Note: This Standard is somewhat broader—it covers all client information, not just information "concerning matters within the scope of the relationship." Also note that the language specifically includes not only prospects but former clients. Confidentiality regarding employer information is now covered in Standard IV.

Guidance

If illegal activities by a client are involved, members may have an obligation to report the activities to authorities. The confidentiality Standard extends to former clients as well.

The requirements of this Standard are not intended to prevent Members and Candidates from cooperating with a CFA Institute Professional Conduct Program (PCP) investigation.

Recommended Procedures for Compliance

Members should avoid disclosing information received from a client except to authorized co-workers who are also working for the client.

Application of Standard III(E) Preservation of Confidentiality

Example 1:

Sarah Connor, a financial analyst employed by Johnson Investment Counselors, Inc., provides investment advice to the trustees of City Medical Center. The trustees have given her a number of internal reports concerning City Medical's needs for physical plant renovation and expansion. They have asked Connor to recommend investments that would generate capital appreciation in endowment funds to meet projected capital expenditures. Connor is approached by a local business man, Thomas Kasey, who is considering a substantial contribution either to City Medical Center or to another local hospital. Kasey wants to find out the building plans of both institutions before making a decision, but he does not want to speak to the trustees.

Comment:

The trustees gave Connor the internal reports so she could advise them on how to manage their endowment funds. Because the information in the reports is clearly both confidential and within the scope of the confidential relationship, Standard III(E) requires that Connor refuse to divulge information to Kasey.

Example 2:

David Bradford manages money for a family-owned real estate development corporation. He also manages the individual portfolios of several of the family members and officers of the corporation, including the chief financial officer (CFO). Based on the financial records from the corporation, as well as some questionable practices of the CFO that he has observed, Bradford believes that the CFO is embezzling money from the corporation and putting it into his personal investment account.

Comment:

Bradford should check with his firm's compliance department as well as outside counsel to determine whether applicable securities regulations require reporting the CFO's financial records.

IV Duties to Employers

IV(A) Loyalty. Members and Candidates must place their employer's interest before their own and must not *deprive their employer of their skills and abilities, divulge confidential information*, or otherwise harm their employer.

Professor's Note: Always act in the employer's best interests and do not deprive the employer of any of Member's/Candidate's skills or abilities. Also protect confidential information. There is now a phrase "in matters related to employment," which means that Members/Candidates are not required to subordinate important personal and family obligations to their job.

Guidance

Members must not engage in any activities which would injure the firm, deprive it of profit, or deprive it of the advantage of employees' skills and abilities. Always place client interests above interests of employer. There is no requirement that the employee put employer interests ahead of family and other personal obligations; it is expected that employers and employees will discuss such matters and balance these obligations with work obligations.

Guidance—Independent Practice

Independent practice for compensation is allowed if a notification is provided to the employer fully describing all aspects of the services, including compensation, duration, and the nature of the activities *and* if the employer consents to all terms of the proposed independent practice before it begins.

Guidance—Leaving an Employer

Members must continue to act in their employer's best interests until resignation is effective. Activities which may constitute a violation include:

- Misappropriation of trade secrets.
- Misuse of confidential information.
- Soliciting employer's clients prior to leaving.
- Self-dealing.
- Misappropriation of client lists.

Once an employee has left a firm, simple knowledge of names and existence of former clients is generally not confidential. Also there is no prohibition on the use of experience or knowledge gained while with a former employer.

Guidance—Whistleblowing

There may be isolated cases where a duty to one's employer may be violated in order to protect clients or the integrity of the market, and not for personal gain.

Guidance—Nature of Employment

The applicability of this Standard is based on the nature of the employment—employee versus independent contractor. If Members and Candidates are independent contractors, they still have a duty to abide by the terms of the agreement.

Application of Standard IV(A) Loyalty

Example 1:

James Hightower has been employed by Jason Investment Management Corporation for 15 years. He began as an analyst but assumed increasing responsibilities and is now a senior portfolio manager and a member of the firm's investment policy committee. Hightower has decided to leave Jason Investment and start his own investment management business. He has been careful not to tell any of Jason's clients that he is leaving, because he does not want to be accused of breaching his duty to Jason by soliciting Jason's clients before his departure. Hightower is planning to copy and take with him the following documents and information he developed or worked on while at Jason: (1) the client list, with addresses, telephone numbers, and other pertinent client information; (2) client account statements; (3) sample marketing presentations to prospective clients containing Jason's performance record; (4) Jason's recommended list of securities; (5) computer models to determine asset allocations for accounts with different objectives; (6) computer models for stock selection; and (7) personal computer spreadsheets for Hightower's major corporate recommendations which he developed when he was an analyst.

Comment:

Except with the consent of their employer, departing employees may not take employer property, which includes books, records, reports, and other materials, and may not interfere with their employer's business opportunities. Taking any employer records, even those the member or candidate prepared, violates Standard IV(A).

Example 2:

Dennis Elliot has hired Sam Chisolm who previously worked for a competing firm. Chisolm left his former firm after 18 years of employment. When Chisolm begins working for Elliot, he wants to contact his former clients because he knows them well and is certain that many will follow him to his new employer. Is Chisolm in violation of the Standard IV(A) if he contacts his former clients?

Comment:

Because client records are the property of the firm, contacting former clients for any reason through the use of client lists or other information taken from a former employer without permission would be a violation of Standard IV(A). In addition, the nature and extent of the contact with former clients may be governed by the terms of any non-compete agreement signed by the employee and the former employer that covers contact with former clients after employment.

But, simple knowledge of the name and existence of former clients is not confidential information, just as skills or experience that an employee obtains while employed is not "confidential" or "privileged" information. The Code and Standards do not impose a prohibition on the use of experience or knowledge gained at one employer from being used at another employer. The Code and Standards also do not prohibit former employees from contacting clients of their previous firm, absent a non-compete agreement. Members and candidates are free to use public information about their former firm after departing to contact former clients without violating Standard IV(A).

In the absence of a non-compete agreement, as long as Chisolm maintains his duty of loyalty to his employer before joining Elliot's firm, does not take steps to solicit clients until he has left his former firm, and does not make use of material from his former employer without its permission after he has left, he would not be in violation of the Code and Standards.

Example 3:

Several employees are planning to depart their current employer within a few weeks and have been careful to not engage in any activities that would conflict with their duty to their current employer. They have just learned that one of their employer's clients has undertaken a request for proposal (RFP) to review and possibly hire a new investment consultant. The RFP has been sent to the employer and all of its competitors. The group believes that the new entity to be formed would be qualified to respond to the RFP and eligible for the business. The RFP submission period is likely to conclude before the employees' resignations are effective. Is it permissible for the group of departing employees to respond to the RFP under their anticipated new firm?

Comment:

A group of employees responding to an RFP that their employer is also responding to would lead to direct competition between the employees and the employer. Such conduct would violate Standard IV(A) unless the group of employees received permission from their employer as well as the entity sending out the RFP.

IV(B) Additional Compensation Arrangements. No gifts, benefits, compensation or consideration are to be accepted which may create a conflict of interest with the employer's interest unless written consent is received from all parties.

Professor's Note: The new language broadens "compensation" to include "gifts, benefits, compensation, or consideration."

Guidance

Compensation includes direct and indirect compensation from a client and other benefits received from third parties. Written consent from a member's employer includes email communication.

Recommended Procedures for Compliance

Make an immediate written report to employer detailing proposed compensation and services, if additional to that provided by employer.

Application of Standard IV(B) Additional Compensation Arrangements

Example 1:

Geoff Whitman, a portfolio analyst for Adams Trust Company, manages the account of Carol Cochran, a client. Whitman is paid a salary by his employer, and Cochran pays the trust company a standard fee based on the market value of assets in her portfolio. Cochran proposes to Whitman that "any year that my portfolio achieves at least a 15 percent return before taxes, you and your wife can fly to Monaco at my expense and use my condominium during the third week of January. Whitman does not inform his employer of the arrangement and vacations in Monaco the following January as Cochran's guest.

Comment:

Whitman violated Standard IV(B) by failing to inform his employer in writing of this supplemental, contingent compensation arrangement. The nature of the arrangement could have resulted in partiality to Cochran's account, which could have detracted from Whitman's performance with respect to other accounts he handles for Adams Trust. Whitman must obtain the consent of his employer to accept such a supplemental benefit.

IV(C) Responsibilities of Supervisors. All Members and Candidates must make reasonable efforts to detect and prevent violations of laws, rules, regulations, and the Code and Standards by any person under their supervision or authority.

Professor's Note: The focus is on establishing and implementing reasonable compliance procedures in order to meet this Standard.

Guidance

Members must take steps to prevent employees from violating laws, rules, regulations, or the Code and Standards and make reasonable efforts to detect violations.

Guidance—Compliance Procedures

Understand that an adequate compliance system must meet industry standards, regulatory requirements, and the requirements of the Code and Standards. Members with supervisory responsibilities have an obligation to bring an inadequate compliance system to the attention of firm's management and recommend corrective action. While investigating a possible breach of compliance procedures, it is appropriate to limit the suspected employee's activities.

Recommended Procedures for Compliance

A member should recommend that his employer adopt a code of ethics. Employers should not commingle compliance procedures with the firm's code of ethics—this can dilute the goal of reinforcing one's ethical obligations. Members should encourage employers to provide their code of ethics to clients.

Adequate compliance procedures should:

- Be clearly written.
- Be easy to understand.
- Designate a compliance officer with authority clearly defined.
- Have a system of checks and balances.
- Outline the scope of procedures.
- Outline what conduct is permitted.
- Contain procedures for reporting violations and sanctions.

Once the compliance program is instituted, the supervisor should:

- Distribute it to the proper personnel.
- Update it as needed.
- Continually educate staff regarding procedures.
- Issue reminders as necessary.
- Require professional conduct evaluations.
- Review employee actions to monitor compliance and identify violations.
- Enforce procedures once a violation occurs.

If there is a violation, respond promptly and conduct a thorough investigation while placing limitations on the wrongdoer's activities.

Application of Standard IV(C) Responsibilities of Supervisors

Example 1:

Jane Mattock, senior vice president and head of the research department of H&V, Inc., a regional brokerage firm, has decided to change her recommendation for Timber Products from buy to sell. In line with H&V's procedures, she orally advises certain other H&V executives of her proposed actions before the report is prepared for publication. As a result of his conversation with Mattock, Dieter Frampton, one of the executives of H&V accountable to Mattock, immediately sells Timber's stock from his own account and from certain discretionary client accounts. In addition, other personnel inform certain institutional customers of the changed recommendation before it is printed and disseminated to all H&V customers who have received previous Timber reports.

Comment:

Mattock failed to supervise reasonably and adequately the actions of those accountable to her. She did not prevent or establish reasonable procedures designed to prevent dissemination of or trading on the information by those who knew of her changed recommendation. She must ensure that her firm has procedures for reviewing or recording trading in the stock of any corporation that has been the subject of an unpublished change in recommendation. Adequate procedures would have informed the subordinates of their duties and detected sales by Frampton and selected customers.

Example 2:

Deion Miller is the research director for Jamestown Investment Programs. The portfolio managers have become critical of Miller and his staff because the Jamestown portfolios do not include any stock that has been the subject of a merger or tender offer. Georgia Ginn, a member of Miller's staff, tells Miller that she has been studying a local company, Excelsior, Inc., and recommends its purchase. Ginn adds that the company has been widely rumored to be the subject of a merger study by a well-known conglomerate and discussions between them are under way. At Miller's request, Ginn prepares a memo recommending the stock. Miller passes along Ginn's memo to the portfolio managers prior to leaving for vacation, noting that he has not reviewed the memo. As a result of the memo, the portfolio managers buy Excelsior stock immediately. The day Miller returns to the office, Miller learns that Ginn's only sources for the report were her brother, who is an acquisitions analyst with Acme Industries and the "well-known conglomerate" and that the merger discussions were planned but not held.

Comment:

Miller violated Standard IV(C) by not exercising reasonable supervision when he disseminated the memo without checking to ensure that Ginn had a reasonable and adequate basis for her recommendations and that Ginn was not relying on material nonpublic information.

V   Investment Analysis, Recommendations, and Action

V(A) Diligence and Reasonable Basis

1. When analyzing investments, making recommendations, and taking investment actions use diligence, independence, and thoroughness.

2. Investment analysis, recommendations, and actions should have a reasonable and adequate basis, supported by research and investigation.

Professor's Note: There is no real change to the old Standard—it is now just more clearly stated that Members and Candidates must act with diligence, independence, and thoroughness when performing investment analysis and making a recommendation or taking investment action.

Guidance

The application of this Standard depends on the investment philosophy adhered to, members' and candidates' roles in the investment decision-making process, and the resources and support provided by employers. These factors dictate the degree of diligence, thoroughness of research, and the proper level of investigation required.

Guidance—Using Secondary or Third-Party Research

See that the research is sound. Examples of criteria to use to evaluate:

- Review assumptions used.
- How rigorous was the analysis?
- How timely is the research?
- Evaluate objectivity and independence of the recommendations.

Guidance—Group Research and Decision Making

Even if a member does not agree with the independent and objective view of the group, he does not necessarily have to decline to be identified with the report, as long as there is a reasonable and adequate basis.

Recommended Procedures for Compliance

Members should encourage their firms to consider these policies and procedures supporting this Standard:

- Have a policy requiring that research reports and recommendations have a basis that can be substantiated as reasonable and adequate.
- Have detailed, written guidance for proper research and due diligence.
- Have measurable criteria for judging the quality of research.

Application of Standard V(A) Diligence and Reasonable Basis

Example 1:

Helen Hawke manages the corporate finance department of Sarkozi Securities, Ltd. The firm is anticipating that the government will soon close a tax loophole that currently allows oil and gas exploration companies to pass on drilling expenses to holders of a certain class of shares. Because market demand for this tax-advantaged class of stock is currently high, Sarkozi convinces several companies to undertake new equity financings at once before the loophole closes. Time is of the essence, but Sarkozi lacks sufficient resources to conduct adequate research on all the prospective issuing companies. Hawke decides to estimate the IPO prices based on the relative size of each company and to justify the pricing later when her staff has time.

Comment:

Sarkozi should have taken on only the work that it could adequately handle. By categorizing the issuers as to general size, Hawke has bypassed researching all the other relevant aspects that should be considered when pricing new issues and thus has not performed sufficient due diligence. Such an omission can result in investors purchasing shares at prices that have no actual basis. Hawke has violated Standard V(A).

Example 2:

Evelyn Mastakis is a junior analyst asked by her firm to write a research report predicting the expected interest rate for residential mortgages over the next six months. Mastakis submits her report to the fixed-income investment committee of her firm for review, as required by firm procedures. Although some committee members support Mastakis's conclusion, the majority of the committee disagrees with her conclusion and the report is significantly changed to indicate that interest rates are likely to increase more than originally predicted by Mastakis.

Comment:

The results of research are not always clear, and different people may have different opinions based on the same factual evidence. In this case, the majority of the committee may have valid reasons for issuing a report that differs from the analyst's original research. The firm can issue a report different from the original report of the analyst as long as there is a reasonable or adequate basis for its conclusions. Generally, analysts must write research reports that reflect their own opinion and can ask the firm not to put their name on reports that ultimately differ from that opinion. When the work is a group effort, however, not all members of the team may agree with all aspects of the report. Ultimately, members and candidates can ask to have their names removed from the report, but if they are satisfied that the process has produced results or conclusions that have a reasonable or adequate basis, members or candidates do not have to dissociate from the report even when they do not agree with its contents. The member or candidate should document the difference of opinion and any request to remove his or her name from the report.

V(B) Communication With Clients and Prospective Clients

1. Disclose to clients and prospects basic format and general principles of investment processes used to analyze and select securities and construct portfolios. Promptly disclose any process changes.

2. Use reasonable judgment in identifying relevant factors important to investment analyses, recommendations, or actions, and include factors when communicating with clients and prospects.

3. Investment analyses and recommendations should clearly differentiate facts from opinions.

Professor's Note: This combines two of the prior Standards. There is no longer a need to distinguish between types of communication, research report versus a recommendation. This Standard covers communication in any form.

Guidance

Proper communication with clients is critical to provide quality financial services. Members must distinguish between opinions and facts and always include the basic characteristics of the security being analyzed in a research report.

Members must illustrate to clients and prospects the investment decision-making process utilized. The suitability of each investment is important in the context of the entire portfolio.

All means of communication are included here, not just research reports.

Recommended Procedures for Compliance

Selection of relevant factors in a report can be a judgment call, so be sure to maintain records indicating the nature of the research, and be able to supply additional information if it is requested by the client or other users of the report.

Application of Standard V(B) Communication with Clients and Prospective Clients

Example 1:

Sarah Williamson, director of marketing for Country Technicians, Inc., is convinced that she has found the perfect formula for increasing Country Technician's income and diversifying its product base. Williamson plans to build on Country Technician's reputation as a leading money manager by marketing an exclusive and expensive investment advice letter to high-net-worth individuals. One hitch in the plan is the complexity of Country Technician's investment system—a combination of technical trading rules (based on historical price and volume fluctuations) and portfolio-construction rules designed to minimize risk. To simplify the newsletter, she decides to include only each week's top-five buy and sell recommendations and to leave out details of the valuation models and the portfolio-structuring scheme.

Comment:

Williamson's plans for the newsletter violate Standard V(B) because she does not intend to include all the relevant factors behind the investment advice. Williamson need not describe the investment system in detail in order to implement the advice effectively, clients must be informed of Country Technician's basic process and logic. Without understanding the basis for a recommendation, clients cannot possibly understand its limitations or its inherent risks.

Example 2:

Richard Dox is a mining analyst for East Bank Securities. He has just finished his report on Boisy Bay Minerals. Included in his report is his own assessment of the geological extent of mineral reserves likely to be found on the company's land. Dox completed this calculation based on the core samples from the company's latest drilling. According to Dox's calculations, the company has in excess of 500,000 ounces of gold on the property. Dox concludes his research report as follows: "Based on the fact that the company has 500,000 ounces of gold to be mined, I recommend a strong BUY."

Comment:

If Dox issues the report as written, he will violate Standard V(B). His calculation of the total gold reserves for the property is an opinion, not a fact. Opinion must be distinguished from fact in research reports.

Example 3:

May & Associates is an aggressive growth manager that has represented itself since its inception as a specialist at investing in small-capitalization domestic stocks. One of May's selection criteria is a maximum capitalization of $250 million for any given company. After a string of successful years of superior relative performance, May expanded its client base significantly, to the point at which assets under management now exceed $3 billion. For liquidity purposes, May's chief investment officer (CIO) decides to lift the maximum permissible market-cap ceiling to $500 million and change the firm's sales and marketing literature accordingly to inform prospective clients and third-party consultants.

Comment:

Although May's CIO is correct about informing potentially interested parties as to the change in investment process, he must also notify May's existing clients. Among the latter group might be a number of clients who not only retained May as a small-cap manager but also retained mid-cap and large-cap specialists in a multiple-manager approach. Such clients could regard May's change of criteria as a style change that could distort their overall asset allocations.

Example 4:

Rather than lifting the ceiling for its universe from $250 million to $500 million, May & Associates extends its small-cap universe to include a number of non-U.S. companies.

Comment:

Standard V(B) requires that May's CIO advise May's clients of this change because the firm may have been retained by some clients specifically for its prowess at investing in domestic small-cap stocks. Other variations requiring client notification include introducing derivatives to emulate a certain market sector or relaxing various other constraints, such as portfolio beta. In all such cases, members and candidates must disclose changes to all interested parties.

V(C) Record Retention. Maintain all records supporting analysis, recommendations, actions, and all other investment-related communications with clients and prospects.

Professor's Note: The issue of record retention is now explicitly broken out into a new Standard, emphasizing its importance.

Guidance

Members must maintain research records that support the reasons for the analyst's conclusions and any investment actions taken. Such records are the property of the firm. If no other regulatory standards are in place, CFA Institute recommends at least a 7-year holding period.

Recommended Procedures for Compliance

This record-keeping requirement generally is the firm's responsibility.

Application of Standard V(C) Record Retention

Example 1:

One of Nikolas Lindstrom's clients is upset by the negative investment returns in his equity portfolio. The investment policy statement for the client requires that the portfolio manager follow a benchmark-oriented approach. The benchmark for the client included a 35 percent investment allocation in the technology sector, which the client acknowledged was appropriate. Over the past three years, the portion put into the segment of technology stocks suffered severe losses. The client complains to the investment manager that so much money was allocated to this sector.

Comment:

For Lindstrom, it is important to have appropriate records to show that over the past three years the percentage of technology stocks in the benchmark index was 35 percent. Therefore, the amount of money invested in the technology sector was appropriate according to the investment policy statement. Lindstrom should also have the investment policy statement for the client stating that the benchmark was appropriate for the client's investment objectives. He should also have records indicating that the investment had been explained appropriately to the client and that the investment policy statement was updated on a regular basis.

VI Conflicts of Interest

VI(A) Disclosure of Conflicts. Members and Candidates must make full and fair disclosure of all matters which may impair their independence or objectivity or interfere with their duties to employer, clients and prospects. Disclosures must be prominent, in plain language, and effectively communicate the information.

Professor's Note: Emphasis in the new Standard is on meaningful disclosure—prominent and in plain language.

Guidance

Members must fully disclose to clients, prospects, and their employers all actual and potential conflicts of interest in order to protect investors and employers. These disclosures must be clearly stated.

Guidance—Disclosure to Clients

The requirement that all potential areas of conflict be disclosed allows clients and prospects to judge motives and potential biases for themselves. Disclosure of broker/dealer market-making activities would be included here. Board service is another area of potential conflict.

The most common conflict which requires disclosure is actual ownership of stock in companies that the member recommends or that clients hold.

Guidance—Disclosure of Conflicts to Employers

Members must give the employer enough information to judge the impact of the conflict. Take reasonable steps to avoid conflicts, and report them promptly if they occur.

Recommended Procedures of Compliance

Any special compensation arrangements, bonus programs, commissions, and incentives should be disclosed.

Application of Standard VI(A) Disclosure of Conflicts

Example 1:

Hunter Weiss is a research analyst with Farmington Company, a broker and investment banking firm. Farmington's merger and acquisition department has represented Vimco, a conglomerate, in all of its acquisitions for 20 years. From time to time, Farmington officers sit on the boards of directors of various Vimco subsidiaries. Weiss is writing a research report on Vimco.

Comment:

Weiss must disclose in his research report Farmington's special relationship with Vimco. Broker/dealer management of and participation in public offerings must be disclosed in research reports. Because the position of underwriter to a company presents a special past and potential future relationship with a company that is the subject of investment advice, it threatens the independence and objectivity of the report and must be disclosed.

Example 2:

Samantha Dyson, a portfolio manager for Thomas Investment Counsel, Inc., specializes in managing defined-benefit pension plan accounts, all of which are in the accumulative phase and have long-term investment objectives. A year ago, Dyson's employer, in an attempt to motivate and retain key investment professionals, introduced a bonus compensation system that rewards portfolio managers on the basis of quarterly performance relative to their peers and certain benchmark indexes. Dyson changes her investment strategy and purchases several high-beta stocks for client portfolios in an attempt to improve short-term performance. These purchases are seemingly contrary to the client investment policy statement. Now, an officer of Griffin Corporation, one of Dyson's pension fund clients, asks why Griffin Corporation's portfolio seems to be dominated by high-beta stocks of companies that often appear among the most actively traded issues. No change in objective or strategy has been recommended by Dyson during the year.

Comment:

Dyson violated Standard VI(A) by failing to inform her clients of the changes in her compensation arrangement with her employer that created a conflict of interest. Firms may pay employees on the basis of performance, but pressure by Thomas Investment Counsel to achieve short-term performance goals is in basic conflict with the objectives of Dyson's accounts.

Example 3:

Bruce Smith covers East European equities for Marlborough investments, an investment management firm with a strong presence in emerging markets. While on a business trip to Russia, Smith learns that investing in Russian equity directly is difficult but that equity-linked notes that replicate the performance of the underlying Russian equity can be purchased from a New York-based investment bank. Believing that his firm would not be interested in such a security, Smith purchases a note linked to a Russian telecommunications company for his own account without informing Marlborough. A month later, Smith decides that the firm should consider investing in Russian equities using equity-linked notes, and he prepares a write-up on the market that concludes with a recommendation to purchase several of the notes. One note recommended is linked to the same Russian telecom company that Smith holds in his personal account.

Comment:

Smith violated Standard VI(A) by failing to disclose his ownership of the note linked to the Russian telecom company. Smith is required by the standard to disclose the investment opportunity to his employer and look to his company's policies on personal trading to determine whether it was proper for him to purchase the note for his own account. By purchasing the note, Smith may or may not have impaired his ability to make an unbiased and objective assessment of the appropriateness of the derivative instrument for his firm, but Smith's failure to disclose the purchase to his employer impaired his employer's ability to render an opinion regarding whether the ownership of a security constituted a conflict of interest that might have affected future recommendations. Once he recommended the notes to his firm, Smith compounded his problems by not disclosing that he owned the notes in his personal account—a clear conflict of interest.

VI(B) Priority of Transactions. Investment transactions for clients and employers must have priority over those in which a Member or Candidate is a beneficial owner.

Professor's Note: Language has been simplified in the new Standard—transactions for clients and employers always have priority over personal transactions.

Guidance

Client transactions take priority over personal transactions and over transactions made on behalf of the member's firm. Personal transactions include situations where the member is a "beneficial owner." Personal transactions may be undertaken only after clients and the member's employer have had an adequate opportunity to act on a recommendation. Note that family-member accounts that are client accounts should be treated just like any client account; they should not be disadvantaged.

Recommended Procedures for Compliance

All firms should have in place basic procedures that address conflicts created by personal investing. The following areas should be included:

- Limited participation in equity IPOs. Members can avoid these conflicts by not participating in IPOs.
- Restrictions on private placements. Strict limits should be placed on employee acquisition of these securities and proper supervisory procedures should be in place. Participation in these investments raises conflict of interest issues, similar to IPOs.

- Establish blackout/restricted periods. Employees involved in investment decision-making should have blackout periods prior to trading for clients—no "front running" (i.e., purchase or sale of securities in advance of anticipated client or employer purchases and sales). The size of the firm and the type of security should help dictate how severe the blackout requirement should be.
- Reporting requirements. Supervisors should establish reporting procedures, including duplicate trade confirmations, disclosure of personal holdings/beneficial ownership positions, and pre-clearance procedures.
- Disclosure of policies. When requested, members must fully disclose to investors their firm's personal trading policies.

Application of Standard VI(B) Priority of Transactions

Example 1:

Erin Toffler, a portfolio manager at Esposito Investments, manages the retirement account established with the firm by her parents. Whenever IPOs become available, she first allocates shares to all her other clients for whom the investment is appropriate; only then does she place any remaining portion in her parents' account, if the issue is appropriate for them. She has adopted this procedure so that no one can accuse her of favoring her parents.

Comment:

Toffler has breached her duty to her parents by treating them differently from her other accounts simply because of the family relationship. As fee-paying clients of Esposito Investments, Toffler's parents are entitled to the same treatment as any other client of the firm. If Toffler has beneficial ownership in the account, however, and Esposito Investments has preclearance and reporting requirements for personal transactions, she may have to preclear the trades and report the transactions to Esposito.

Example 2:

A brokerage's insurance analyst, Denise Wilson, makes a closed-circuit report to her firm's branches around the country. During the broadcast, she includes negative comments about a major company within the industry. The following day, Wilson's report is printed and distributed to the sales force and public customers. The report recommends that both short-term traders and intermediate investors take profits by selling that company's stocks. Several minutes after the broadcast, Ellen Riley, head of the firm's trading department, closes out a long call position in the stock. Shortly thereafter, Riley establishes a sizable "put" position in the stock. Riley claims she took this action to facilitate anticipated sales by institutional clients.

Comment:

Riley expected that both the stock and option markets would respond to the "sell" recommendation, but she did not give customers an opportunity to buy or sell in the options market before the firm itself did. By taking action before the report was disseminated, Riley's firm could have depressed the price of the "calls" and increased the price of the "puts." The firm could have avoided a conflict of interest if it had waited to trade for its own account until its clients had an opportunity to receive and assimilate Wilson's recommendations. As it is, Riley's actions violated Standard VI(B).

VI(C) Referral Fees. Members and Candidates must disclose to their employers, clients, and prospects any compensation consideration or benefit received by, or paid to, others for recommendations of products and services.

Guidance

Members must inform employers, clients, and prospects of any benefit received for referrals of customers and clients, allowing them to evaluate the full cost of the service as well as any potential impartiality. All types of consideration must be disclosed.

Application of Standard VI(C) Referral Fees

Example 1:

Brady Securities, Inc., a broker/dealer, has established a referral arrangement with Lewis Brothers, Ltd., an investment counseling firm. Under this arrangement, Brady Securities refers all prospective tax-exempt accounts, including pension, profit-sharing, and endowment accounts, to Lewis Brothers. In return, Lewis Brothers makes available to Brady Securities on a regular basis the security recommendations and reports of its research staff, which registered representatives of Brady Securities use in serving customers. In addition, Lewis Brothers conducts monthly economic and market reviews for Brady Securities personnel and directs all stock commission business generated by referral account to Brady Securities. Willard White, a partner in Lewis Brothers, calculates that the incremental costs involved in functioning as the research department of Brady Securities amount to $20,000 annually. Referrals from Brady Securities last year resulted in fee income of $200,000, and directing all stock trades through Brady Securities resulted in additional costs to Lewis Brothers' clients of $10,000.

Diane Branch, the chief financial officer of Maxwell Inc., contacts White and says that she is seeking an investment manager for Maxwell's profit-sharing plan. She adds, "My friend Harold Hill at Brady Securities recommended your firm without qualification, and that's good enough for me. Do we have a deal?" White accepts the new account but does not disclose his firm's referral arrangement with Brady Securities.

Comment:

White violated Standard VI(C) by failing to inform the prospective customer of the referral fee payable in services and commissions for an indefinite period to Brady Securities. Such disclosure could have caused Branch to reassess Hill's recommendation and make a more critical evaluation of Lewis Brothers' services.

Example 2:

James Handley works for the Trust Department of Central Trust Bank. He receives compensation for each referral he makes to Central Trust's brokerage and personal financial management department that results in a sale. He refers several of his clients to the personal financial management department but does not disclose the arrangement within Central trust to his clients.

Comment:

Handley has violated Standard V(C) by not disclosing the referral arrangement at Central Trust Bank to his clients. The Standard does not distinguish between referral fees paid by a third party for referring clients to the third party and internal compensation arrangements paid within the firm to attract new business to a subsidiary. Members and candidates must disclose all such referral fees. Therefore, Handley would be required to disclose, at the time of referral, any referral fee agreement in place between Central Trust Bank's departments. The disclosure should include the nature and the value of the benefit and should be made in writing.

Example 3:

Yeshao Wen is a portfolio manager for a bank. He receives additional monetary compensation from his employer when he is successful in assisting in the sales process and generation of assets under management.

The assets in question will be invested in proprietary product offerings such as affiliate company mutual funds.

Comment:

Standard VI(C) is meant to address instances where the investment advice provided by a member or candidate appears to be objective and independent but in fact is influenced by an unseen referral arrangement. It is not meant to cover compensation by employers to employees for generating new business when it would be obvious to potential clients that the employees are "referring" potential clients to the services of their employers.

If Wen is selling the bank's investment management services in general, he does not need to disclose to potential clients that he will receive a bonus for finding new clients and acquiring new assets under management for the bank. Potential clients are likely aware that it would be financially beneficial both to the portfolio manager and the manager's firm for the portfolio manager to sell the services of the firm and attract new clients. Therefore, sales efforts attempting to attract new investment management clients need not disclose this fact.

However, in this example, the assets will be managed in "proprietary product offerings" of the manager's company (for example, an in-house mutual fund) and Wen will receive additional compensation for selling firm products. Some sophisticated investors may realize that it would be financially beneficial to the portfolio manager and the manager's firm if the investor buys the product offerings of the firm.

Best practice, however, dictates that the portfolio manager must disclose to clients that they are compensated for referring clients to firm products. Such discloser will meet the purpose of Standard VI(C), which is to allow investors to determine whether there is any partiality on the part of the portfolio manager when making investment advice.

VII   Responsibilities as a CFA Institute Member or CFA Candidate

VII(A) Conduct as Members and Candidates in the CFA Program. Members and Candidates must not engage in conduct that compromises the reputation or integrity of CFA Institute or the CFA designation or the integrity, validity, or security of the CFA exams.

Professor's Note: The Standard is intended to cover conduct such as cheating on the CFA exam or otherwise violating rules of CFA Institute or the CFA program. It is not intended to prevent anyone from expressing any opinions or beliefs concerning CFA Institute or the CFA program.

Members must not engage in any activity that undermines the integrity of the CFA charter. This Standard applies to conduct which includes:

- Cheating on the CFA exam or any exam.
- Not following rules and policies of the CFA program.
- Giving confidential information on the CFA program to Candidates or the public.
- Improperly using the designation to further personal and professional goals.
- Misrepresenting information on the Professional Conduct Statement (PCS) or the CFA Institute Professional Development Program.

Members and candidates are not precluded from expressing their opinions regarding the exam program or CFA Institute.

Application of Standard VII(A) Conduct as Members and Candidates in the CFA Program

Example 1:

Ashlie Hocking is writing Level II of the CFA examination in London. After completing the exam, she immediately attempts to contact her friend in Sydney, Australia, to tip him off to specific questions on the exam.

Comment:

Hocking has violated Standard VII(A) by attempting to give her friend an unfair advantage, thereby compromising the integrity of the CFA examination process.

Example 2:

Jose Ramirez is an investment-relations consultant for several small companies that are seeking greater exposure to investors. He is also the program chair for the CFA Institute society in the city where he works. To the exclusion of other companies, Ramirez only schedules companies that are his clients to make presentations to the society.

Comment:

Ramirez, by using his volunteer position at CFA Institute to benefit himself and his clients, compromises the reputation and integrity of CFA Institute, and, thus, violates Standard VII(A).

VII(B) Reference to CFA Institute, the CFA designation, and the CFA Program. Members and Candidates must not misrepresent or exaggerate the meaning or implications of membership in CFA Institute, holding the CFA designation, or candidacy in the program.

Professor's Note: Replacing the vague language "dignified and judicious..." this new Standard is clearer as it prohibits Candidates from engaging in any conduct that may "misrepresent or exaggerate the meaning or implications of membership in CFA Institute, holding the CFA designation, or candidacy in the CFA program." The requirement that Candidates still not reference any "partial" designation remains since this also misrepresents or exaggerates credentials.

Guidance

Members must not make promotional promises or guarantees tied to the CFA designation. Do not:

- Over-promise individual competence.
- Over-promise investment results in the future (i.e., higher performance, less risk, etc.).

Guidance—CFA Institute Membership

Members must satisfy these requirements to maintain membership:

- Sign PCS annually.
- Pay CFA Institute membership dues annually.

If they fail to do this, they are no longer active members.

Guidance—Using the CFA Designation

Do not misrepresent or exaggerate the meaning of the designation.

Guidance—Referencing Candidacy in the CFA Program

There is no partial designation. It is acceptable to state that a Candidate successfully completed the program in three years, if in fact they did, but claiming superior ability because of this is not permitted.

Guidance—Proper Usage of the CFA Marks

The Chartered Financial Analyst and CFA marks must always be used either after a charterholder's name or as adjectives, but not as nouns, in written and oral communications.

Recommended Procedures for Compliance

Make sure that members' and candidates' firms are aware of the proper references to a member's CFA designation or candidacy, as this is a common error.

Application of Standard VII(B) Reference to CFA Institute, the CFA Designation, and the CFA Program

Example 1:

An advertisement for AZ Investment Advisors states that all the firm's principals are CFA charterholders and all passed the three examinations on their first attempt. The advertisement prominently links this fact to the notion that AZ's mutual funds have achieved superior performance.

Comment:

AZ may state that all principals passed the three examinations on the first try as long as this statement is true and is not linked to performance or does not imply superior ability. Implying that (1) CFA charterholders achieve better investment results and (2) those who pass the exams on the first try may be more successful than those who do not violates Standard VII(B).

Example 2:

Five years after receiving his CFA charter, Louis Vasseur resigns his position as an investment analyst and spends the next two years traveling abroad. Because he is not actively engaged in the investment profession, he does not file a completed Professional Conduct Statement with CFA Institute and does not pay his CFA Institute membership dues. At the conclusion of his travels, Vasseur becomes a self-employed analyst, accepting assignments as an independent contractor. Without reinstating his CFA Institute membership by filing his Professional Conduct Statement and paying his dues, he prints business cards that display "CFA" after his name.

Comment:

Vasseur has violated Standard VII(B) because Vasseur's right to use the CFA designation was suspended when he failed to file his Professional Conduct Statement and stopped paying dues. Therefore, he no longer is able to state or imply that he is an active CFA charterholder. When Vasseur files his Professional Conduct Statement and resumes paying CFA Institute dues to activate his membership, he will be eligible to use the CFA designation upon satisfactory completion of CFA Institute reinstatement procedures.

EXAM FLASHBACKS

Required CFA Institute disclaimer: Due to CFA curriculum changes from year to year, published sample exam questions and guideline answers prior to the current year may not reflect the current curriculum.

Exam Flashback # 1
Source: Question #1 from '99–'03 sample exams.

The CFA Institute Code of Ethics specifically addresses all of the following **EXCEPT**:
A. competence.
B. integrity and dignity.
C. independent judgment.
D. importance of contractual obligations.

Exam Flashback # 2
Source: Question #4 from '99–'03 sample exams.

A CFA Institute member resides in Country A, where securities laws are *more* strict than the CFA Institute Standards of Professional Conduct, and does all of his business in Country B, where securities laws are *less* strict than the CFA Institute Standards. The laws of Country A state that professional conduct is governed by the laws of the locality in which business is conducted. According to the CFA Institute *Standards of Practice Handbook*, the member has a duty to adhere to:
A. the laws of Country A.
B. the laws of Country B.
C. CFA Institute Standards of Professional Conduct.
D. a basic standard of competence and diligence.

Exam Flashback # 3
Source: Question #38 from '91 actual exam.

SSB, an investment banking firm, is the principal underwriter for Hitech's convertible debentures. Betsy Anderson, an analyst with SSB, discovers from talking with retailers that all retailers have returned Hitech's only product because it is defective. Anderson found that this information is not in the prospectus. The preliminary red herring prospectus has already been distributed. Anderson's best course of action is to:
A. report her findings to the appropriate supervisory persons in SSB.
B. report her findings to the Division of Corporation Finance of the SEC.
C. notify potential investors of the omission on a fair and equitable basis.
D. not act on the information because it is material nonpublic information.

Exam Flashback # 4
Source: Question #13 from '94 and '96 actual exams.

Judy Albert and Bob Tye, who recently started their own investment advisory business, plan to take the Level III CFA examination next year. Albert's business card reads, "Judy Albert, CFA Candidate." Tye has not put anything about the CFA designation on his business card. However, the firm's promotional material describes the CFA requirements and indicates that Tye participates in the CFA program and has completed Levels I and II. According to CFA Institute's Standards of Professional Conduct:
A. Albert has violated the Standards, but Tye has not.
B. Tye has violated the Standards, but Albert has not.
C. both Albert and Tye have violated the Standards.
D. neither Albert nor Tye has violated the Standards.

Exam Flashback # 5

Source: Question #64 from '90 actual exam.
In the new 9th Edition of the Standards of Practice Handbook, professional misconduct is covered under Standard I(D) and knowledge of the law is covered under Standard I(A).

Standard II(B) (Professional Misconduct) differs from Standard IA (Fundamental Responsibilities: Members shall maintain knowledge of and comply with all applicable laws, etc.) in that:
A. Standard II(B) encompasses the broad anti-fraud prohibitions of the Securities Act of 1933 and the Securities Exchange Act of 1934.
B. Standard II(B) extends beyond compliance with the laws and regulations to cover behavior that reflects adversely on the entire profession.
C. violation of Standard II(B) is dependent upon conviction by a court of law.
D. Standard IA includes the responsibility to exercise reasonable supervision over subordinates by establishing supervisory procedures and by ensuring that those procedures are followed.

Exam Flashback # 6

Source: Question #3 from '96 actual exam, and '97 and '98 sample exams.

Clifford Brown is a partner in a money management firm. Brown recently attended a seminar at which he learned about a quantitative model presented by Art Dixon. Upon returning to his office, Brown began testing the model and making a few minor alterations to it. He showed his model to his partners, who were impressed and decided to promote the model as proof of the money management firm's continuing innovation and value added. In the next newsletter mailed by the firm, Brown included a discussion of the model, the results of tests of the model, and financial data on several stocks selected by the model. These factual corporate financial data were taken from a Standard & Poor's publication. According to CFA Institute Standards of Professional Conduct, which of the following actions is Brown required to take?
A. Brown must credit both Dixon for having developed the original model and Standard & Poor's as the source of the data.
B. Brown must credit Dixon for having developed the original model, but need not credit Standard & Poor's as the source of the data.
C. Brown must credit Standard & Poor's as the source of the data, but need not credit Dixon for having developed the original model.
D. Brown need not credit either Dixon for having developed the original model or Standard & Poor's as the source of the data.

Exam Flashback # 7

Source: Question #9 from '94 actual exam.

Alan Johnson, CFA, is a portfolio manager with an investment firm based in New York. One of his firm's clients told Johnson that she would compensate him beyond that provided by his firm, based on the capital appreciation of the portfolio each year. Johnson should:
A. turn down the additional compensation because it will result in conflicts with the interests of the other client accounts.
B. receive permission from CFA Institute for the compensation arrangement.
C. inform his employer, customers, and clients if he accepts the compensation arrangement.
D. turn down the additional compensation because it would create undue pressure on him to achieve strong short-term performance results.

Exam Flashback # 8

Source: Question #65 from '90 actual exam.

Gen Ral, CFA, supervises a group of five research analysts, none of whom are CFAs or CFA candidates. She has attempted to get her firm to adopt an adequate compliance system to assure that applicable laws and regulations are met. However, the firm's principals have refused to adopt her recommendations. Therefore, Ral should:
A. take no action, because she has discharged her obligations under the CFA Institute Standards by bringing the inadequate compliance system to the attention of the principals.
B. take no action, because she has no supervisory obligations under the CFA Institute Standards for employees who are not CFAs or CFA candidates.
C. decline in writing to accept supervisory responsibility until reasonable compliance procedures are adopted.
D. resign, because she has no other alternative that will keep her in compliance with CFA Institute Standards.

Exam Flashback # 9
Source: Question #3 from the '99–'03 sample exams.

George Moses, CFA, analyzes Technicorp for a brokerage company. Extensive study has led Moses to rate Technicorp as a "hold," largely because of increasing competition in the industry. At a recent CFA Institute Society meeting, Moses discussed Technicorp's prospects with two other analysts. Although the other analysts did not give a reason, both said that Technicorp was about to experience rapid earnings growth. Immediately following the meeting, Moses issued a "buy" recommendation for Technicorp. According to the CFA Institute Standards of Practice Handbook, did Moses violate the CFA Institute Standards of Professional Conduct?
A. No.
B. Yes, because he copied the opinions of others.
C. Yes, because he did not seek approval of the change from his supervisor.
D. Yes, because he did not have a reasonable and adequate basis for his recommendation.

Exam Flashback # 10
Source: Question #9 from '92 actual exam, but has been modified from its original form.

Tim Gaines, a CFA candidate, works for a regional brokerage firm. He estimates that Walkton Industries will increase its dividend by $1.50 per share during the next year. This increase is upon pending legislation that would, if enacted, give Walkton a substantial tax break. The Congressman in Walkton's home district has told Gaines that, although he is lobbying hard for the bill and prospects for passage look good, concern over the federal deficit could cause the tax break bill to be voted down. The company has not made any statements regarding a change in dividend policy. Gaines writes in his research report, "We expect Walkton's stock price to rise by at least $8.00 per share by the end of the year. The stock price gain will be fueled, in large part, by a dividend increase of $1.50 per share. Investors buying the stock at the current time should expect to realize a total return of at least 15 percent on the stock." Which of the following is **TRUE**?
A. Gaines violated CFA Institute Standards of Professional Conduct because he used material inside information.
B. Gaines violated CFA Institute Standards of Professional Conduct because he has a material misrepresentation in his report.
C. Gaines violated CFA Institute Standards of Professional Conduct because he has divulged confidential information.
D. Gaines' actions did not violate CFA Institute Standards of Professional Conduct.

Exam Flashback # 11
Source: Question #2 from '93 actual exam, but has been modified from its original form.

West is a CFA charterholder and trust officer for REO Trust Company. Soon after beginning work for REO, West finds that REO has been conducting all its securities transactions through her brother who is a registered representative. West's brother charges the REO commissions that are equal to the lowest available from any other broker. West's brother tells her that if she continues doing business with him, he will give her a substantial discount on all personal transactions she conducts through him. West:
A. does not need to inform her employer of the arrangement because the commissions her brother charges the firm are the lowest possible.
B. must inform her employer of the arrangement because she is doing business with a member of her immediate family.
C. does not need to inform her employer of the arrangement because REO had been doing business with her brother before she was hired.
D. must reject the arrangement because it provides her with additional compensation.

Exam Flashback # 12
Source: Question #15 from '96 sample exam.

Don Cheney, CFA, manages an employee profit-sharing plan for Dugal Corp. John Dugal, the company president, recently asked Cheney to vote the shares in the firm's profit-sharing plan in favor of the company-nominated slate of directors and against the directors proposed by a dissident group. Cheney does not want to lose this client because he directs all the client's trades to a brokerage firm that provides Cheney with useful information about tax-free investments. Although this information is not of value in managing the Dugal account, it does help in managing several other accounts. Although the transactions conducted by this brokerage firm are of only marginal quality, its prices are reasonable and the quality of the tax-free investment information is high. Cheney investigates the proxy-fight issue, concludes that management's slate of directors is only slightly worse than those recommended by the dissident group, and votes in favor of the management-backed directors. According to the CFA Institute Standards of Professional Conduct, Cheney:
A. did not violate the Standards when he voted the shares in the manner requested by Dugal or in directing trades to the brokerage firm.
B. violated the Standards when he voted the shares in the manner requested by Dugal, but not in directing trades to the brokerage firm.
C. violated the Standards when he directed trades to the brokerage firm, but not in voting the shares in as requested by Dugal.
D. violated the Standards when he voted the shares in the manner requested by Dugal and in directing trades to the brokerage firm.

Exam Flashback # 13
Source: Question #8 from '94 and '96 sample exam.

Bill Elliott, CFA, manages the discretionary account of the Jones Corporation employees' profit-sharing plan. Diane Jones, the company president, recently asked Elliott to vote the shares in the firm's profit-sharing plan in favor of the company-nominated slate of directors and against the directors sponsored by a dissident stockholder group. Elliott does not want to lose Jones as a client, because he directs all the client's trades to a brokerage firm that provides Elliott with useful information about tax-free investments. Although this information is not of value in managing the Jones Corporation account, it does help in managing several other accounts. The brokerage firm providing this information also offers the lowest prices for trades. Elliott investigates the proxy-fight issue, concludes that management's slate of directors is better for the long-run performance of the firm than those recommended by the dissident group, and votes accordingly. According to the CFA Institute Standards of Professional Conduct, Elliott:
A. violated the Standards by voting the shares in the manner requested by Jones, *but not* by directing trades to the brokerage firm.
B. *did not* violate the Standards by voting the shares in the manner requested by Jones or by directing trades to the brokerage firm.
C. violated the Standards by directing trades to the brokerage firm, *but not* by voting the shares as requested by Jones.
D. violated the Standards by voting the shares in the manner requested by Jones *and* by directing trades to the brokerage firm.

Exam Flashback # 14
Source: Question #9 from '93 and '96 actual exams, and '97 and '98 sample exams.

Susan Wright, CFA, works for an investment counseling firm. James Johnson, a new client of the firm, is meeting with Wright for the first time. Johnson had used another counseling firm for financial advice for years, but he switched his account to Wright's firm. At the beginning of their meeting, Wright explains to Johnson that she has discovered a highly undervalued stock that offers large potential gains. Wright recommends that Johnson buy the stock for his account. Wright's actions violated the CFA Institute Standards of Professional Conduct. Which of the following statements best describes the action Wright should have taken? Wright should have:
A. asked Johnson why he changed counseling firms. If the discovery process indicated that he had been treated unfairly at the other firm, Wright would have a responsibility to notify CFA Institute of any violation.
B. determined Johnson's needs, objectives, and tolerance for risk before making a recommendation for any type of security.
C. explained the characteristics of the company to Johnson, including the characteristics of the industry in which the company operates.
D. explained her qualifications, including her education, training, experience, and the meaning of the CFA designation.

Exam Flashback # 15
Source: Question #7 from '92, '93 and '96 actual exams, and '97 and '98 sample exams.

Jim Orlando, CFA, is a research analyst with a brokerage firm. He decided to change his recommendation on the common stock of Alpha, Inc. from a "buy" to a "sell." He faxed this change in investment advice to all his current customers. The next day, a new customer calls with a "buy" order for 500 shares of Alpha. According to CFA Institute Standards of Professional Conduct, Orlando:
A. may accept the order because he has complied with the standard on fair dealing with customers.
B. should advise the customer of the change in recommendation before accepting the order.
C. should delay executing the order until five days have elapsed after the communication of the change in recommendation.
D. may accept the order when the customer specifies in writing that he was notified of the change in the recommendation.

Exam Flashback # 16
Source: Question #35 from '91 actual exam.
In the new 9th Edition of the Standards of Practice Handbook, priority of transactions is covered under Standard VI(B).

The basic idea behind Standard IV(B.4), Priority of Transactions, of the CFA Institute Standards of Professional Conduct is that employees should **NOT**:
A. attempt to notify their best customers about critical investment information sooner than their other customers.
B. place their own personal interests above those of their customers or employer.
C. place a buy or sell order in non-discretionary accounts without the prior consent of the client.
D. give preferential treatment to one client over another in security transactions.

Exam Flashback # 17
Source: Question #9 from '93 and '96 actual exams, and '97 and '98 sample exams.

Don Smith, a CFA candidate employed by Hunter Investment Counselors, Inc., provides investment advice to the board of trustees of a private university endowment fund. The trustees have given Smith the fund's financial information, including planned expenditures. Smith received a phone call on Friday afternoon from Mark Murdock, a prominent alumnus, requesting that Smith fax to him comprehensive financial information about the fund. According to Murdock, he has a potential contributor but needs the information today to close the deal and cannot contact any of the trustees. Based on CFA Institute Standards of Professional Conduct, Smith may:
A. send Murdock the information provided he promptly notifies the trustees.
B. not send Murdock the information until after the trustees have been notified.
C. send Murdock the information because such disclosure would benefit the client.
D. not send Murdock the information because it is material nonpublic information.

Exam Flashback # 18
Source: Question #63 from '90 actual exam and '98 sample exam.

Janzen tells a prospective client, "I may not have a long-term track record yet, but I am sure that you will be very pleased with my recommendations and service. In the three years that I have been in the business, my equity-oriented clients have averaged a total return of more than 26 percent a year." The statement is true; but Janzen only has a few clients, and one of his clients took a large position in a penny stock (against Janzen's advice) and realized a huge gain. This large gain caused the average of all of Janzen's clients to exceed 26 percent per year. Without this one investment, the average gain would have been 8 percent per year. Has Janzen violated the Standards of Professional Conduct?
A. Yes, because the statement misrepresents Janzen's track record.
B. Yes, because the statement on return ignores the risk preferences of his clients.
C. No, because the statement is a true and accurate description of Janzen's track record.
D. No, because Janzen is not promising that he can earn a 26 percent return in the future.

Exam Flashback # 19
Source: Question #19 from '96 actual exam.

Bob Little, CFA, is the chemical industry research analyst at a brokerage firm. Just after buying 500 shares of Kern Electronics for his personal account, Little learns that his firm is preparing an investment report with a buy recommendation on the Kern stock. According to CFA Institute Standards of Professional Conduct, Little must:
A. take no action to disclose his beneficial ownership in the Kern stock.
B. request that his firm disclose his beneficial ownership in the investment report.
C. sell his stock immediately and disclose this action to his firm.
D. sell his stock within 30 days and disclose this action to his firm.

Exam Flashback # 20
Source: Question #77 from '89 actual exam, but has been modified from its original form.

All of the following statements concerning Standard IV(B.8), Disclosure of Referral Fees, are true EXCEPT:
A. one purpose of the standard is to help the client evaluate the full cost of the services.
B. one purpose of the standard is to help the client evaluate any possible partiality shown in the recommendation of services.
C. one purpose of the standard is to help the client evaluate potential conflicts of interest as a result of "immediate family" participation in transactions.
D. consideration includes all fees, whether paid in cash, in soft dollars, or in kind.

Exam Flashback # 21
Source: Question #16 from '99–'03 sample exams.

Louis Stark, CFA, is employed in the merger and acquisitions department of an investment firm. His friend, Elizabeth Mackie, CFA, is a portfolio manager in the investment management department of the same firm. Stark is helping a client acquire Gamma Corporation. According to the CFA Institute Standards of Practice Handbook, which of the following is the most appropriate action to take involving communication between the two departments?
A. Stark may tell Mackie about the pending merger if Mackie promises not to release the information to the public.
B. The investment firm should build a Fire Wall between the merger and acquisitions department and the investment management department.
C. The investment firm must add Gamma Corporation to its list of stocks that cannot be added to portfolios managed by employees of the investment firm.
D. Stark may tell Mackie about the pending merger if Mackie promises in advance not to use this information to help make an investment decision about Gamma Corporation.

CONCEPT CHECKERS: STANDARDS OF PRACTICE HANDBOOK

1. Jamie Hutchins, CFA, is a portfolio manager for CNV Investments Inc. Over the years, Hutchins has made several poor personal investments that have led to financial distress and personal bankruptcy. Hutchins feels that her business partner, John Smith, is mostly to blame for her situation since "he did not invest enough money in her investment opportunities and caused them to fail." Hutchins reports Smith to CFA Institute claiming Smith violated the Code and Standards relating to misconduct. Which of the following statements is correct?
    A. Neither Hutchins nor Smith violated the Code and Standards.
    B. By reporting Smith to CFA Institute, Hutchins has misused the Professional Conduct Program, thus violating the Code and Standards.
    C. Hutchins' bankruptcy reflects poorly on her professional reputation and thus violates the Code and Standards.
    D. Smith's lack of investment in Hutchin's opportunities violated the priority of transactions, and he was appropriately reported to CFA Institute.

2. While working on a new underwriting project, Jean Brayman, CFA, has just received information from her client that leads her to believe that the firm's financial statements in the registration statement overstate the firm's financial position. Brayman should:
    A. report her finding to the appropriate governmental regulatory authority.
    B. immediately dissociate herself from the underwriting in writing to the client.
    C. seek advice from her firm's compliance department as to the appropriate action to take.
    D. inform the client of the problem and issue a press release correcting the statements.

3. Karen Jones, CFA, is an outside director for Valley Manufacturing. At a director's meeting, Jones finds out that Mfg. Corp. has made several contributions to foreign politicians that she suspects were illegal. Jones checks with her firm's legal counsel and determines that the contributions were indeed illegal. At the next board meeting Jones urges the board to disclose the contributions. The board, however, votes not to make a disclosure. Jones should:
    A. protest the board's actions in writing to the executive officer of Valley.
    B. resign from the board and seek legal counsel as to her legal disclosure requirements.
    C. inform her supervisor of their discovery and cease attending meetings until the matter is resolved.
    D. resign form the board, sell any stock she owns in the firm, and issue a press release explaining her actions.

4. Carrie Carlson, CFA, is a citizen of Emerging Market Country (EMC) with no securities laws governing the use of inside information. Carlson has clients in Emerging Market Country and in Neighboring Country (NC), which has a few poorly defined laws governing the use of inside information. Should Carlson have inside information on a publicly traded security, she:
    A. can inform her clients in EMC, but not NC.
    B. can only trade for her own account when she has inside information.
    C. can use the information for her NC clients to the extent permitted by the laws of NC.
    D. cannot use the information to trade in either EMC or NC.

5. In order to dispel the myth that emerging market stocks are illiquid investments, Green Brothers, a "long only" emerging market fund manager, has two of its subsidiaries simultaneously buy and sell emerging market stocks. In its marketing literature, Green Brothers cites the overall emerging market volume as evidence of the market's liquidity. As a result of its actions, more investors participate in the emerging markets fund. Which of the following is **TRUE**? Green Brothers:
    A. did not violate the Code and Standards.
    B. violated the Code and Standards by failing to consider the suitability of emerging market investments.
    C. violated the Code and Standards by manipulating the volume in the emerging securities markets.
    D. would not have violated the Code and Standards if the subsidiaries only traded stocks not included in the fund.

6. Over the past two days, Lorraine Quigley, CFA, manager of a hedge fund, has been purchasing large quantities of Craeger Industrial Products' common stock while at the same time shorting put options on the same stock. Quigley did not notify her clients of the trades although they are aware of the fund's general strategy to generate returns. Which of the following statements is **TRUE**? Quigley:
    A. did not violate the Code and Standards.
    B. violated the Code and Standards by manipulating the prices of publicly traded securities.
    C. violated the Code and Standards by failing to disclose the transactions to clients before they occurred.
    D. violated the Code and Standards by failing to establish a reasonable and adequate basis before making the trades.

7. Which of the following statements is **FALSE**? A member or candidate:
    A. can participate or assist in a violation simply by having knowledge of the violation and not taking action to stop it.
    B. is held responsible for participating in illegal acts in instances where violation of the law is evident to those who know or should know the law.
    C. when confronted with potentially illegal activities, should consult with her supervisor and her employer's counsel.
    D. must report evidence of legal violations to the appropriate governmental or regulatory organization.

8. Paula Osgood, CFA is promoting her new money management firm by issuing an advertisement. Which of these items is **NOT** *in conflict* with the professional designation Standard? The advertisement states that:
    A. she passed three exams covering ethics, financial statement analysis, asset valuation, and portfolio management and that she is a member of the local society. Osgood signs the advertisement followed by the letters CFA in oversized and bold strike letters.
    B. she passed three exams totaling over 18 hours over the minimum period of one and a half years. Knowledge tested included ethics, financial statement analysis, asset valuation, and portfolio management. In addition, she is a member of the local society.
    C. because of her extensive CFA training she will be able to achieve better investment results than non-CFA managers since she is one of very few professionals to have been awarded this designation.
    D. she is one of very few professionals to have been awarded this designation and that she is a member of the local society. She signs the advertisement followed by the letters CFA in oversized and bold strike letters.

9. Melvin Byrne, CFA, manages a portfolio for James Martin, a very wealthy client. Martin's portfolio is well diversified with a slight tilt toward capital appreciation. Martin requires very little income from the portfolio. Recently Martin's brother, Cliff, has become a client of Byrne. Byrne proceeds to invest Cliff's portfolio in a similar manner to James' portfolio based on the fact that both brothers have a similar lifestyle and are only two years apart in age. Which of the following statements is **TRUE**? Byrne:
    A. violated the Code and Standards by deviating from his investment mandate related to James' portfolio.
    B. violated the Code and Standards by knowingly creating a conflict of interest between James' and Cliff's portfolio.
    C. violated the Code and Standards by failing to determine Cliff's objectives and constraints prior to investing his portfolio.
    D. did not violate the Code and Standards.

10. In which of the following has the analyst **NOT** committed plagiarism?
    A. Julie Long takes performance projections and charts from a company she is researching, combines them with her own analysis, and publishes them under her own name.
    B. Bill Cooper finds a statistical table in the Federal Reserve Bulletin that supports the work he has done in his industry analysis and has his secretary include the table as part of his report without citing the source.
    C. Jan Niedfeldt gets a call from one of her fellow analysts stating that the analyst's research shows that XYZ Company is a buy. Niedfeldt calls up her major clients and tells them that her research shows XYZ is a buy.
    D. To speed up an acquisition project, Jim Zijacek's boss gives him a report from another firm also working on the project and tells Zijacek to print the report on company letterhead, sign it, and mail it out to the stockholders.

11. Jessica Ellis, CFA, manages an international stock portfolio for a group of wealthy investors with similar investment objectives. According to the investment policy statement, the portfolio is to pursue an aggressive growth strategy while maintaining sufficient international diversification. The fund is prohibited from using leverage. Ellis has just received a request from all of the group of investors to purchase a large position in German bonds which they believe to be significantly undervalued. Which of the following actions should Ellis take to avoid violating the Code and Standards?
    A. Purchase the bonds since it was requested by the clients to whom Ellis has a fiduciary duty.
    B. Inform the investors that she is unable to make the purchase since it is inconsistent with the international stock portfolio's investment mandate.
    C. Purchase the bonds only after receiving a written consent statement signed by each portfolio investor that states they are aware that the investment is not suitable for the portfolio.
    D. Change the investment policy statement to reflect an investment strategy that would allow the bond purchase and then purchase the undervalued bonds for the portfolio.

12. In a marketing brochure, DNR Asset Managers presents the performance of several composite portfolios managed according to similar investment strategies. In constructing composites, the firm excludes individual portfolios with less than $1 million in assets, excludes terminated portfolios, and includes simulated results. DNR includes the following disclosure in the brochure: "Past performance is no guarantee of future results. Composites exclude portfolios under $1 million in assets and include results from simulated model portfolios with similar strategies." DNR's brochure:
    A. does not violate the Code and Standards.
    B. violates the Code and Standards by failing to include terminated portfolios in the performance presentation.
    C. violates the Code and Standards by excluding portfolios under $1 million from the composite performance presentations.
    D. violates the Code and Standards by including simulated results of model portfolios even with a disclosure in the presentation.

13. Connie Fletcher, CFA, works for a small money management firm that specializes in pension accounts. Recently, a friend asked her to act as an unpaid volunteer manager for the city's street sweep pension fund. As part of the position, the city would grant Fletcher a free parking space in front of her downtown office. Fletcher is considering the offer. Before she accepts she should:
    A. do nothing since this is a volunteer position.
    B. inform her current clients in writing and discuss the offer with her employer.
    C. inform her current clients in writing, get their permission, and discuss the offer with her employer.
    D. disclose the details of the volunteer position to her employer and obtain written permission from her employer.

14. Which of the following statements about an investment supervisor's responsibilities is **FALSE**? A supervisor:
    A. is expected to know what constitutes an adequate compliance system.
    B. should bring an inadequate compliance system to the attention of management and recommend corrective action.
    C. is responsible for instructing those to whom he has delegated authority about methods to detect and prevent violations of the law and standards.
    D. need only report employee violations of the Code and Standards to upper management and provide a written warning to the employee to cease such activities.

15. Robert Blair, CFA, Director of Research, has had an ongoing battle with management about the adequacy of the firm's compliance system. Recently, it has come to Blair's attention that the firm's compliance procedures are inadequate in that they are not being monitored and not carefully followed. What should Blair do?
    A. Resign from the firm unless the compliance system is strengthened and followed.
    B. Send his superior a memo outlining the problem. This will discharge his obligation under the Code.
    C. Take no action since his job is supervision and not policy making.
    D. Decline in writing to continue to accept supervisory responsibility until reasonable compliance procedures are adopted.

16. Ahmed Jamal, CFA, head of research for Valley Brokers, decided it was time to change his recommendation on D&R Company from buy to sell. He orally announced his decision during the Monday staff meeting and said his written report would be finished and disseminated to Valley's customers by the middle of next week. As a result of this announcement, Doris Smith, one of Jamal's subordinates, immediately sold her personal shares in D&R, and Martin Temple told his largest institutional customers of the change the following day. Which Standards have been *violated*?
    A. Jamal violated Standard IV(C), Responsibilities of Supervisors; and Smith violated Standard II(A), Material Nonpublic Information.
    B. Jamal violated Standard IV(C), Responsibilities of Supervisors; Smith violated Standard II(A), Material Nonpublic Information; and Temple violated Standard VI(B), Priority of Transactions.
    C. Jamal violated Standard IV(C), Responsibilities of Supervisors; Smith violated Standard VI(B), Priority Transactions; and Temple violated Standard III(B), Fair Dealing.
    D. Smith violated Standard VI(B), Priority of Transactions; and Temple violated Standard III(B), Fair Dealing.

17. Jack Schleifer, CFA, is an analyst for Brown Investment Managers (BIM). Schleifer has recently accepted an invitation to visit the facilities of ChemCo, a producer of chemical compounds used in a variety of industries. ChemCo offers to pay for Schleifer's accommodations in a penthouse suite at a luxury hotel and allow Schleifer to use the firm's private jet to travel to its three facilities located in New York, Hong Kong, and London. In addition, ChemCo offers two tickets to a formal high-society dinner in New York and a small desk clock with the ChemCo logo. Schleifer declines to use ChemCo's corporate jet and allow for the firm to pay for his accommodations but accepts the clock and the tickets to the dinner (which he discloses to his employer) since he will be able to market his firm's mutual funds to other guests at the dinner. Has Schleifer violated any CFA Institute Standards of Professional Conduct?
    A. Yes.
    B. No, since he is using the gifts accepted to benefit his employer's interests.
    C. No, since the gifts he accepted were fully disclosed in writing to his employer.
    D. No, since the gifts that he accepted were of nominal value and he declined to accept the hotel accommodations and the use of ChemCo's jet.

18. Based on the Standards of Professional Conduct, a financial analyst is required to do all the following **EXCEPT**:
    A. disclose the fact that his firm is the underwriter for securities issued by a company he covers.
    B. report to his employer the receipt of gifts and additional compensation from clients.
    C. pay for chartered transportation and lodging while visiting a company's remotely located facilities.
    D. pay for commercial transportation and lodging while visiting a company's headquarters.

19. Beth Anderson, CFA, is a portfolio manager for several wealthy clients including Reuben Carlyle. Anderson manages Carlyle's personal portfolio of stock and bond investments. Carlyle recently told Anderson that he is under investigation by the IRS for tax evasion related to his business, Carlyle Concrete (CC). After learning about the investigation, Anderson proceeds to inform a friend at a local investment bank so that they may withdraw their proposal to take CC public. Which of the following is **TRUE**? Anderson:
    A. violated the Code and Standards by failing to immediately terminate the client relationship with Carlyle.
    B. violated the Code and Standards by failing to maintain the confidentiality of her client's information.
    C. violated the Code and Standards by failing to detect and report the tax evasion to the proper authorities.
    D. did not violate the Code and Standards since the information she conveyed pertained to illegal activities on the part of her client.

20. Gail Stefano, CFA, an analyst for a U.S. brokerage firm that serves U.S. investors, researches public utilities in South American emerging markets. Stefano makes the following statement in a recent report: "Based on the fact that the South American utilities sector has seen rapid growth in new service orders, we expect that most companies in the sector will be able to convert the revenue increases into significant profits. We also believe the trend will continue for the next three to five years." The report goes on to describe the major risks of investing in this market, in particular the political and exchange rate instability associated with South American countries. Stefano's report:
    A. has not violated the Code and Standards.
    B. violated the Code and Standards by failing to properly distinguish factual information from opinions.
    C. violated the Code and Standards by recommending an investment which would not be suitable for all of its clients.
    D. violated the Code and Standards by failing to properly identify details related to the operations of South American utilities.

21. All of the following violate Standard III(B), Fair Dealing, **EXCEPT**:
    A. before disseminating a change in the analyst's buy recommendation, the analyst calls his best clients and tells them about the change.
    B. a firm makes investment recommendations and also manages a mutual fund. The firm routinely begins trading for the fund's account ten minutes before announcing recommendation changes to client accounts.
    C. after releasing the general recommendation to all clients, an analyst calls the firm's largest institutional clients to discuss the recommendation in more detail.
    D. a portfolio manager allocates IPO shares to her brother's fee-based retirement account only after allocating shares to all other accounts.

22. Which of the following does **NOT** constitute a violation of Standard VI(B), Priority of Transactions?
    A. Failure by an analyst to make or change a recommendation until he trades for his own account.
    B. An analyst trades for her son's trust account on the same day her firm changes its buy/sell recommendation.
    C. An analyst takes a position in a stock she recommended one week after the recommendation was made public.
    D. An analyst trades for the firm's account before handling client trades.

23. Jamie Olson, CFA, has just started work as a trainee with Neuvo Management Corp., a small regional money management firm started six months ago. She has been told to make a few cold calls and round up some new clients. In which of the following statements has Olson **NOT** violated the Standards of Practice?
    A. "Sure, we can perform all the financial and investment services you need. We've consistently outperformed the market indexes and will continue to do so under our current management."
    B. "Sure, we can assist you with all the financial and investment services you need. If we don't provide the service in-house, we have arrangements with other full-service firms that I would be happy to tell you about."
    C. "Believe me, I've been at this game long enough to know what I'm talking about. I personally guarantee this investment. It's a sure winner."
    D. "Our firm has a long history of successful performance for our clients. While we can't guarantee future results, we do believe we will continue to benefit our clients."

24. Mary Herbst, CFA, a pension fund manager at GBH Investments, is reviewing some of FreeTime, Inc.'s pension fund activities over the past years. Which of the following actions related to FreeTime, Inc.'s pension fund is *least likely* to be a breach of her fiduciary duties?
    A. Paying higher-than-average brokerage fees to obtain research materials used in the management of other funds by the investment group.
    B. Trading with selected brokers so that the brokers will recommend GBH's managers to potential clients.
    C. Substantially increasing the risk of the fund in order to minimize FreeTime, Inc.'s future contributions.
    D. Selectively choosing brokers for the quality of research provided for managing FreeTime's pension.

25. Kevin Minter is an investment analyst at Bradley & Company, a small money management firm. Minter keeps detailed records and documentation of relevant research pertaining to his investment recommendations. Because of the limited physical and electronic storage space, however, three days after issuing a recommendation, Minter shreds hard copy documentation and deletes all electronic files except for the recommendation itself. There is no regulatory requirement to maintain records for any specified time period. Which of the following statements is **TRUE**? Minter:
    A. has not violated the Code and Standards.
    B. violated the Code and Standards because he did not maintain hard copy and electronic documentation supporting all of his current recommendations.
    C. violated the Code and Standards because he did not maintain hard copy or electronic documentation supporting his recommendations for seven years.
    D. violated the Code and Standards because he did not maintain hard copy and electronic documentation supporting all of his recommendations.

26. Eugene Nieder, CFA, has just accepted a new job as a quantitative analyst for Paschal Investments, LLP. Nieder developed a complex model while working for his previous employer and plans to recreate the model for Paschal. Nieder did not make copies of the model or any supporting documents since his employer refused to grant him permission to do so. Nieder will recreate the model from memory. Which of the following statements is **TRUE**?
    A. Nieder can recreate the model without violating the Code and Standards as long as he also generates supporting documentation.
    B. Nieder can recreate the model without violating the Code and Standards as long as he obtains permission to do so from his former employer.
    C. Nieder can recreate the model without violating the Code and Standards without documentation if the model is modified from its original form.
    D. Nieder cannot recreate the model without violating the Code and Standards because it is the property of his former employer.

27. As part of an agreement with Baker Brokerage, Hern Investment Company, a money manager for individual clients, provides monthly emerging market overviews in exchange for prospective client referrals and European equity research from Baker. Clients and prospects of Hern are not made aware of the agreement, but clients unanimously rave about the high quality of the research provided by Baker. As a result of the research, many non-discretionary clients have earned substantial returns on their portfolios. Managers at Hern have also used the research to earn outstanding returns for the firm's discretionary portfolios. Which of the following statements is **TRUE**? Hern:
    A. has not violated the Code and Standards.
    B. has violated the Code and Standards by using third-party research in discretionary accounts.
    C. has violated the Code and Standards by failing to disclose the referrals made by Baker.
    D. has violated the Code and Standards by failing to communicate the basic investment characteristics to discretionary clients.

28. Frist Investments, Inc. has just hired Michael Pulin to manage institutional portfolios, most of which are pension related. Pulin has just taken the Level III CFA exam and is awaiting his results. Pulin has over 15 years of investment management experience with individual clients but has never managed an institutional portfolio. Pulin joined the CFA Institute as an affiliate member two years ago and is in good standing with the organization. Which of the following statements would be most appropriate for Frist to use in advertising Pulin as a new member of the firm? Pulin:
   A. has many years of investment experience which, along with his participation in the CFA program, will allow him to deliver superior investment performance relative to other managers.
   B. is a CFA Level III and passed the first two exams on the first attempt. He is an affiliate member of the CFA Institute. We expect him to become a regular member if he passes the Level III examination.
   C. will be a CFA once he passes the Level III CFA Exam. He has vast amounts of practical experience as well as an enhanced understanding of the investment process as a result of his participation in the CFA program.
   D. is a Level III CFA candidate and has many years of excellent performance in the investment management industry. Pulin is an affiliate member of the CFA Institute and will be eligible to become a CFA charterholder and regular member if he passes the Level III CFA Exam.

29. Before joining Mitsui Ltd. as an analyst covering the electrical equipment manufacturing industry, Pam Servais, CFA, worked for Internet Security Systems (ISS) where she had access to nonpublic information. While at ISS, Servais learned of a severe environmental problem at two firms handling boron-based components. It is common knowledge that seven firms in the industry worldwide use the same boron handling technique. The two firms for which Servais has knowledge announced the problem last week and had immediate stock price declines of 11 and 17 percent, respectively. The other five firms have not made an announcement. Servais issues a report recommending Mitsui clients sell shares of the remaining five firms. Servais' issuance of this recommendation:
   A. is not a violation of CFA Institute Standards.
   B. is a violation of CFA Institute Standards insofar as it fails to have adequate basis in fact.
   C. is a violation of CFA Institute Standards insofar as it fails to distinguish between opinion and fact.
   D. constitutes a violation of the Standard pertaining to the use of material nonpublic information.

30. Zanuatu, an island nation, does not have any regulations precluding the use of non-public information. Alfredo Romero has a friend and fellow CFA charterholder there with whom he has shared nonpublic information regarding firms outside of his industry. The information concerns several firms' internal earnings and cash flow projections. The friend may:
   A. trade on the information under the laws of Zanuatu, which govern her behavior.
   B. not trade on the information under CFA Institute Standards, which govern her behavior.
   C. not trade on the information under the laws of Zanuatu, which govern her behavior.
   D. trade on the information under CFA Institute Standards since the firms concerned are outside of Romero's industry.

31. Samantha Donovan, CFA, is an exam proctor for the Level II CFA exam. The day before the exam is to be administered, Donovan faxes a copy of one of the questions to two friends, James Smythe and Lynn Yeats, who are Level II candidates in the CFA program. Donovan, Smythe, and Yeats had planned the distribution of an exam question months in advance. Smythe used the fax to prepare for the exam. Yeats, however, had second thoughts and threw the fax away without looking at its contents. Which of the following statements is correct?
   A. Donovan violated the Code and Standards but Yeats did not.
   B. Yeats violated the Code and Standards but Smythe did not.
   C. Donovan violated the Code and Standards but Smythe did not.
   D. Donovan and Yeats both violated the Code and Standards.

32. Julia Green, CFA, has friends from her previous employer who have suggested that she agree to receive non-public information anonymously from them via an Internet chat room. In this way, she receives news about an exciting new product being developed by a firm in Singapore that has the potential to double the firm's revenue. The firm has not previously revealed any information regarding the product to the public. According to the Code and Standards, this information is:
    A. not material and may be traded upon.
    B. both material and nonpublic and may not be traded upon in Singapore, but may be traded on elsewhere.
    C. both material and nonpublic and may not be traded upon in any jurisdiction.
    D. public by virtue of its release in the chat room and may be traded upon.

33. Sally Albright, CFA, works full-time for Frank & Company, an investment management firm, as a fixed-income security analyst. Albright has been asked by a business contact at KDG Enterprises to accept some analytical work from KDG on a consulting basis. The work would entail investigating potential distressed debt investments in the small-cap market. Albright should:
    A. accept the work as long as she obtains consent to all the terms of the engagement from Frank & Company.
    B. not accept the work as it violates the Code and Standards by creating a conflict of interest.
    C. accept the work so long as she obtains written consent from KDG.
    D. not accept the work since this will likely expose her to material nonpublic information in violation of the Code and Standards.

34. William Bixby, CFA, oversees a mid-cap fund that is required to invest in a minimum of 40 and a maximum of 60 different issues. Bixby uses a quantitative approach to actively manage the assets. In promotional materials, he states that "through our complex quantitative approach, securities are selected that have similar exposures to a number of risk factors that are found in the S&P 500 Index. Thus the fund is designed to track the performance of the S&P 500 Index but will receive a return premium of between 2 and 4 percent according to our model's risk-return measures." This statement is:
    A. permissible since the assertion is supported by modern portfolio theory and estimates from the firms' model.
    B. not permissible since Bixby is misrepresenting the services that she and/or her firm are capable of performing.
    C. not permissible since Bixby is misrepresenting the investment performance she and/or her firm can reasonably expect to achieve.
    D. permissible since the statement describes the basic characteristics of the fund's risk and return objectives.

35. Josef Karloff, CFA, acts as liaison between Pinnacle Financial (an investment management firm) and Summit Inc. (an investment banking boutique specializing in penny stocks). When Summit underwrites an IPO, Karloff routinely has Pinnacle issue vague statements implying that the firm has cash flows, financial resources, and growth prospects that are far better than is the case in reality. This action is:
    A. permissible under CFA Institute Standards.
    B. a violation of the Standard concerning fair dealing.
    C. a violation of the Standard concerning responsibilities of supervisors.
    D. a violation of the Standard concerning professional misconduct.

36. Shane Matthews, CFA, is a principal at Carlson Brothers, a leading regional investment bank specializing in initial public offerings of small to mid-sized biotech firms. Just before many of the IPOs are offered to the general public, Matthews arranges for 10 percent of the shares of the firm going public to be distributed to management at 75 percent of the expected IPO price. This action is:
    A. permissible under CFA Institute Standards.
    B. a violation of the Standard concerning professionalism.
    C. a violation of the Standard concerning disclosure of conflicts of interest.
    D. a violation of the Standard concerning suitability.

37. Will Hunter, CFA, is a portfolio manager at NV Asset Managers in Baltimore, which specializes in managing labor union pension fund accounts. A friend of Hunter's who is an investment banker asks Hunter to purchase shares in their new IPOs in order to support the price long enough for insiders to liquidate their holdings. Hunter realizes that the price of the shares will almost certainly fall dramatically after his buying support ceases. NV management "strongly suggests" that Hunter "not rock the boat" and honor the investment banker's request since NV has had a long-standing relationship with the investment bank. Hunter agrees to make the purchases. Hunter has:
    A. not violated the Code and Standards.
    B violated the Code and Standards by failing to report fair, accurate, and complete data to his clients.
    C. violated the Code and Standards by attempting to distort natural market forces.
    D. violated the Code and Standards by failing to place orders in the appropriate transaction priority.

38. Neiman Investment Co. receives brokerage business from Pick Asset Management in exchange for referring prospective clients to Pick. Pick advises pension clients—in writing at the time the relationship is established—of the nature of its arrangement with Neiman. With regard to this practice, Pick has:
    A. fully complied with the Code and Standards.
    B. violated the Code and Standards by failing to preserve the confidentiality of the agreement with Neiman.
    C. violated the Code and Standards by inappropriately negotiating an agreement that creates a conflict of interest.
    D. violated the Code and Standards by inappropriately delegating its fiduciary responsibilities to Neiman.

39. Fred Johnson, CFA, a financial analyst and avid windsurfer, has begun an investment survey of the water sports leisure industry. His brother sells windsurfing gear in Tampa and tells him that Swordfish9 is the "hottest windsurfing rig on the market and will be highly profitable for Swordfish Enterprises." Johnson had never heard of Swordfish previously but after testing the board himself became very excited about the Swordfish9 and issued an investment recommendation of "buy" on Swordfish Enterprises. As a result of issuing the recommendation, Johnson has:
    A. not violated the Code and Standards.
    B. violated the Code and Standards by failing to establish a reasonable and adequate basis.
    C. violated the Code and Standards concerning professionalism by placing recreational interests ahead of his fiduciary duty to his clients.
    D. violated the Code and Standards by failing to consider the suitability of the investment for his clients.

40. Daniel Lyons, CFA, is an analyst for a French firm that sells investment research to European companies. Lyons' aunt owns 30,000 shares of French National Bank (FNB). She informs Lyons that as a part of her estate planning she has created a trust in his name into which she has placed 2,000 shares of FNB. The trust is structured so that Lyons will not receive control of the assets for two years, at which time his aunt will also gift her current home to Lyons and move into a retirement community. Lyons is due to update his research coverage of FNB next week. Lyons should:
    A. advise his superiors that he is no longer able to issue research recommendations on FNB.
    B. update the report without notification since the shares are held in trust and are beyond his direct control.
    C. disclose the situation to his employer and, if then asked to prepare a report, also disclose the situation in the report.
    D. disclose the situation to his employer and then prepare the report with no disclosure.

ANSWERS – EXAM FLASHBACKS

1. **D** The easiest way to know the components of the CFA Institute Code of Ethics is to memorize it. This is material you should look over close to the exam date because it tends to be forgotten easily.

2. **A** Members must abide by the strictest requirements.

3. **A** Note in this situation that Anderson is not a supervisor. If she were, she would be required to take actions to assure the appropriate disclosures are made in the prospectus.

4. **A** Note the material points out that in addition to everything else, you cannot say you are a candidate unless you are enrolled for the next available exam or awaiting exam results.

    Professor's Note: Candidates should refer to Exhibit 4 under Standard VII(B) in the 9th Edition of the Handbook for detailed guidance on this issue.

5. **B** Besides the law this standard covers excessive drinking, multiple misdemeanor convictions, and any activity that involves dishonesty, fraud, and misrepresentation that would reflect badly on the profession.

    Professor's Note: Under the revised Code and Standards, Standard I(D) covers professional misconduct and applies to professional activities. Multiple misdemeanors that are unrelated to business activities (e.g., acts of civil disobedience that result in misdemeanor offenses) may not constitute a violation of the Standard.

6. **B** Plagiarism includes:
    - Reporting other people's research as your own.
    - Using excerpts from reports without acknowledgments.
    - Citing specific quotes attributable to "leading analysts."
    - Presenting and citing statistical estimates prepared by others but without qualifying caveats.
    - Using charts and graphs without reference. Data from statistical reporting services can be used without acknowledgment.
    - Copying proprietary computerized spreadsheets or algorithms without authorization.

7. **C** Note the person giving the gift is an existing client and not someone trying to influence Johnson. Standard III(D) says an employee must inform his or her employer in writing of additional compensation arrangements.

    Standard IV(A.3) states that bonuses or gifts may be accepted from clients but must be disclosed to the employer if the bonus exceeds $100, but members cannot accept gifts over $100 from someone (brokers) trying to influence their judgment. Note also that Standard IV(B.7) says members shall disclose to clients and prospects all matters that reasonably could be expected to impair their ability to make objective recommendations.

    Professor's Note: In the new 9th Edition of the Standards of Practice Handbook, additional compensation arrangements are covered under Standard IV(B). Independence and objectivity are covered under Standard I(B) but the $100 gift rule has been eliminated. Standard VI(A) covers disclosure of conflicts of interest that may impair a member or candidate's independence and objectivity.

8. **C** Standard III(E) states that a member should bring an inadequate compliance system to the attention of the firm's senior managers. If the member cannot discharge his or her duties due to the inadequacies of the system the member should decline in writing to accept supervisory responsibility until the firm adopts reasonable procedures.

    Professor's Note: In the new 9th Edition of the Standards of Practice Handbook, responsibilities of supervisors are covered under Standard IV(C). Note that "CFA" cannot be used as a noun as it was here.

9. **D** Since the analysts that told Moses about Technicorp's prospects did not provide any basis for their comment, Moses did not have a reasonable basis, thus violating Standard IV(A.1).

    Professor's Note: In the new 9th Edition of the Standards of Practice Handbook, investment analysis, recommendations, and actions (and the requirement to have a reasonable and adequate basis for recommendations) are covered under Standard V(A).

10. **B** Gains violated Standard IV(A.2) because he has not used reasonable judgment regarding the exclusion of the fact that the company's dividend policy is contingent upon the outcome of the pending legislation. Furthermore, Standard IV(B.6) has been violated because it is a misrepresentation of the investment to make the statement that: "Investors buying the stock at the current time should expect to realize a total return of at least 15 percent on the stock."

Professor's Note: In the new 9th Edition of the Standards of Practice Handbook, reasonable judgment regarding key investment factors to include in investment analysis, recommendations, and actions is covered under Standard V(B). Misrepresentations of investments are covered under Standard I(C).

11. **D** Standard III(D) states that gifts from clients must be separated from gifts made to influence a member. The receipt of the discount may be perceived to affect West's independence or objectivity in making investment recommendations, which is a violation of Standard IV(A.3).

Professor's Note: In the new 9th Edition of the Standards of Practice Handbook, additional compensation from clients is covered under Standard IV(B). Gifts intended to influence a member's or candidate's independence and objectivity are prohibited under Standard I(B).

12. **D** Under Standard IV(B.1), Fiduciary Duty, and the Employee Retirement Income Security Act (ERISA), pension fund managers have a fiduciary duty to the plan's beneficiaries. Cheney violated his fiduciary duty because he voted in favor of management's slate of directors even though he determined it to be worse that than the slate proposed by the dissident group. The direction of trades to the broker is a violation because the transactions are of only marginal quality.

Professor's Note: In the new 9th Edition of the Standards of Practice Handbook, fiduciary duties must be established and complied with according to Standard III(A). The details of the ERISA act are no longer included in the guidance for the Standard but Cheney would still have violated his fiduciary duty and Standard III(A) by not acting in the best interest of his clients (the plan beneficiaries).

13. **B** Here, the pension director did not violate Standard IV(B.1) or ERISA, because he would have voted the plan's shares in favor of the Jones company-nominated slate of directors, regardless of his relationship with Jones. Directing the trades is acceptable because it offers the lowest prices and the research is used in managing other firm accounts.

Watch out. Compare this question to the last one. You can see that the questions are almost identical to each other but the answers are significantly different. On the exam read all the questions carefully. Don't just answer a question because it looks like an old exam question. Read the question carefully and all the way through, then answer it.

Professor's Note: In the new 9th Edition of the Standards of Practice Handbook, fiduciary duties must be established and complied with according to Standard III(A). The details of the ERISA act are no longer included in the guidance for the Standard.

14. **B** Standard IV(B.2.a) requires members to make a reasonable inquiry into a client's financial situation and investment objectives and constraints prior to making any type of investment recommendation.

Professor's Note: In the new 9th Edition of the Standards of Practice Handbook, members' and candidates' responsibility to make reasonable inquiry into a client's financial situation, objectives, and constraints is covered under Standard III(C).

15. **B** Standard IV(B.3) requires that members deal fairly and objectively with all clients. This means both new and existing clients. Note that Standard IV(B.3) deals with how members treat all clients in relation to each other, whereas Standard IV(B.4) covers trading the client's accounts before your own accounts.

Professor's Note: In the new 9th Edition of the Standards of Practice Handbook, equitable treatment of client transactions is covered under Standard III(B) Fair Dealing. Standard VI(B) Priority of Transactions covers members' and candidates' responsibility not to trade personal accounts ahead of client accounts.

16. **B** Standard IV(B.4) pertains to members trading ahead of their clients' accounts. Standard IV(B.3) deals with treating all clients fairly and objectively.

17. **B**   Don't fall for this one. Even though it would be to the benefit of the fund, Standard IV(B.5), Preservation of Confidentiality, prohibits members from disseminating clients' information without their approval. Standard IV(B.5), Preservation of Confidentiality, upholds that the financial analyst must preserve the confidentiality of information from a client if: (i) the financial analyst receives the information due to his or her special ability to conduct the client's business, and (ii) the information the analyst receives is relevant to the client's business that is the subject of the special or confidential relationship. The information on the fund falls under both of these criteria.

     Professor's Note: In the new 9th Edition of the Standards of Practice Handbook, preservation of client confidentiality is covered under Standard III(E).

18. **A**   It is obvious that a violation has occurred. Now, you have to select the "yes" answer that is most appropriate. Standard IV(B.6), Prohibition Against Misrepresentation, prohibits members from providing false or misleading statements about an investment.

     Professor's Note: In the new 9th Edition of the Standards of Practice Handbook, Standard I(C) prohibits members and candidates from misrepresenting their investment track record.

19. **B**   Under Standard IV(B.7), Disclosure of Conflicts to Clients and Prospects, members must disclose anything that could reasonably influence their investment recommendations. This includes beneficial ownership of securities or other investments. This means that Little must disclose his ownership of Kern Electronics.

     Professor's Note: In the new 9th Edition of the Standards of Practice Handbook, disclosure of conflicts of interest, such as ownership of a company being researched, is covered under Standard VI(A).

20. **C**   Standard IV(B.8), Disclosure of Referral Fees, is intended to assure that clients are informed about any benefit received from the referral of clients and customers.

     Professor's Note: In the new 9th Edition of the Standards of Practice Handbook, Standard VI(C) requires that members and candidates disclose all types of referral fees to their employer, clients, and prospects.

21. **B**   This situation falls under Standard V(A), Prohibition of the Use of Material Nonpublic Information. A "Fire Wall" is a common and widely used method for preventing insider trading by employees. The purpose of a fire wall is to prevent the communication of material nonpublic or sensitive information between departments. The minimum elements of a fire wall are: substantial control of interdepartmental communications, review of employee trading, and heightened review of proprietary trading while the firm is in possession of material nonpublic information.

     Professor's Note: In the new 9th Edition of the Standards of Practice Handbook, Standard II(A) prohibits any use of material nonpublic information and recommends that firms create firewalls to prevent material nonpublic information from being inappropriately shared between departments.

ANSWERS – CONCEPT CHECKERS: STANDARDS OF PRACTICE HANDBOOK

1. **B** Hutchins' personal bankruptcy may reflect poorly on her professional reputation if it resulted from fraudulent or deceitful business activities. There is no indication of this, however, and the bankruptcy is thus not a violation. Smith has not violated the Code and Standards by refusing to invest with Hutchins in what turned out to be bad investment opportunities. By reporting Smith to CFA Institute for a violation, Hutchins has misused the Professional Conduct Program to settle a dispute unrelated to professional ethics and has thus violated Standard I(D), Misconduct.

2. **C** According to Standard I(A), informing her supervisor or firm's compliance department is appropriate. Dissociating herself would be premature. She should report her suspicions to a supervisory person and attempt to remedy the situation.

3. **B** According to Standard I(A), since she has taken steps to stop the illegal activities and the board has ignored her, Jones must dissociate from the board and seek legal advice as to what other actions would be appropriate in this instance. She may need to inform legal or regulatory authorities of the illegal activities.

4. **D** According to Standard II(A), members and candidates are under no circumstances allowed to use inside information to trade securities. Carlson must abide by the Code and Standards, which is the most strict regulation in the scenario.

5. **C** The intent of Green Brothers' actions is to manipulate market liquidity in order to attract investment to its own funds. The increased trading activity was not based on market fundamentals or an actual trading strategy to benefit investors. It was merely an attempt to mislead market participants in order to increase assets under Green Brothers' management. The action violates Standard II(B), Market Manipulation.

6. **A** Quigley's trades are most likely an attempt to take advantage of an arbitrage opportunity that exists between Craeger's common stock and its put options. She is not manipulating the prices of securities in an attempt to mislead market participants, which would violate Standard II (A). She is pursuing a legitimate investment strategy. Participants in her hedge fund are aware of the fund's investment strategy, and thus Quigley did not violate the Code and Standards by not disclosing this specific set of trades in advance of trading.

7. **D** According to Standard I(A), in some instances reporting a legal violation to governmental or regulatory officials may be appropriate, but this isn't always necessary, and it isn't required under Standard I(A).

8. **B** According to Standard VII(B), any explanation of the designation in print form should be a concise description of the requirements or of CFA Institute. The other statements contains violations of Standard VII(B), in particular the presentation of the letters CFA. Also, she may not imply superior performance as a result of being a CFA charterholder.

9. **C** Standard III(C) requires that before taking investment action, members and candidates must make a reasonable inquiry into a client's or prospect's investment objectives and constraints as well as their prior investment experience. Byrne cannot assume that because the brothers have similar lifestyles and are close in age that they should have similarly managed portfolios. Byrne should have interviewed Cliff directly before investing his portfolio.

10. **B** According to Standard I(C), factual data from a recognized statistical reporting service need not be cited.

11. **B** According to Standard III(C), Ellis must consider the suitability of each new investment (as well as current holdings) in light of the portfolio mandate. Ellis must only make investments that are in accordance with the portfolio's investment policy statement. Therefore, Ellis should not purchase the unsuitable bonds as requested by her clients.

12. **B** By failing to include terminated portfolios in the performance presentation, the performance will have an inherent upward bias, making results appear better than they truly are. By excluding the terminated portfolios, DNR misleads its potential investors and thus violates Standard III(D), Performance Presentation.

13. **D** According to Standard IV(A), members and candidates are expected to act for the benefit of the employer and not deprive the employer of their skills. Fletcher is performing work similar to the services that her employer provides for a fee. Although the position is a volunteer position, Fletcher will receive compensation in the form of a free parking space. In light of the circumstances, Fletcher must disclose the details of the position and get written permission before accepting the volunteer position.

14. **D** According to Standard IV(C), reporting the violation and warning the employee to cease activities that violate the law or the Code and Standards are not enough. The supervisor must takes steps (such as limiting employee activity or increasing the level of employee monitoring) to prevent further violations while he conducts an investigation.

15. **D** According to Standard IV(C), because he is aware that the firm's compliance procedures are not being monitored and followed and because he has repeatedly tried to get company management to correct the situation, Blair should decline supervisory responsibility until adequate procedures to detect and prevent violations of laws, regulations, and the Code and Standards are adopted and followed. If he does not do so, he will be in violation of the Code and Standards.

16. **C** Jamal failed to properly supervise employees and provide adequate procedures and policies to prevent employee violations. Smith should not have traded her own account ahead of client accounts. Temple should not have disclosed the recommendation change selectively but should have informed his clients fairly and objectively. No inside information was used in the question.

17. **A** Standard I(B) requires that members and candidates reject offers of gifts or compensation that could compromise their independence or objectivity. Schleifer has appropriately rejected the offer of the hotel accommodations and the use of ChemCo's jet. He may accept the desk clock since this gift is of nominal value and is unlikely to compromise his independence and objectivity. Schleifer cannot accept the tickets to the dinner, however. Since it is a formal high-society dinner, the tickets are most likely expensive or hard to come by. Even though he has disclosed the gift to his employer and he plans to use the dinner as a marketing opportunity for his firm, the gift itself may influence Schliefer's future research in favor of ChemCo. Allowing such potential influence is a violation of Standard I(B).

18. **C** Standard I(B) requires that an analyst maintain his independence and objectivity by having his firm pay for ordinary travel expenses to visit companies that are the subject of research. However, in some cases, such as remotely located facilities, the company may pay for modest accommodations and chartered flights as long as the transportation and lodging is not lavish and is not intended to exert influence over the analyst.

19. **B** Anderson must maintain the confidentiality of client information according to Standard III(E). Confidentiality may be broken in instances involving illegal activities on the part of the client, but the client's information shall be relayed to proper authorities. Anderson did not have the right to inform the investment bank of her client's investigation.

20. **A** Historical growth can be cited as a fact since it actually happened. Stefano states that her firm expects further growth and profitability which is an opinion. She does not claim that these are facts. In addition, Stefano identifies relevant factors and highlights in particular the most significant risks of investing in South American utilities. She has fully complied with Standard V(B), Communication with Clients and Prospective Clients. Under the Standard, it is not necessary to include every detail about a potential investment in a report. Members and candidates are expected to use their judgment and identify the most important factors to include.

21. **C** This is not necessarily a violation. Firms can offer different levels of service to clients as long as this is disclosed to all clients. The largest institutional clients would likely be paying higher fees for a greater level of service. Also note that the analyst's brother's account in answer D should be treated similarly to any other client account.

22. **C** One week is likely an acceptable waiting period.

23. **B** Standard I(C)—in the other choices, Olson misrepresents the services that she or her firm are capable of performing, her qualifications, her academic or professional credentials, or the firm's credentials. The firm is small and most likely cannot perform all investment services the client may require. The firm cannot guarantee future outperformance of the market indexes. Olson hasn't been in the business for a long time as she claims and cannot guarantee the performance of any investment. The firm doesn't have a long history (only 6 months).

24. **D** Standard III(A)—Herbst is acting as a fiduciary for the pension plan beneficiaries. She may pay higher-than-average brokerage fees so long as doing so benefits the pension beneficiaries, not other clients. Trading with selected brokers solely to gain referrals is not likely to be in the pension beneficiaries' best interest since it does not take into account other important factors for selecting brokerage firms. Minimizing contributions benefits the plan sponsor, not the plan beneficiaries to whom the fiduciary duty is owed. Choosing brokers based on quality of services provided is reasonable.

25. **C** Minter is required by Standard V(C) to maintain hard copy *or* electronic documentation of the research and data supporting current investment recommendations for at least seven years (since there is no regulatory guidance on an appropriate length of time).

26. **A** Nieder must not take models or documents from his previous employer without explicit permission to do so [Standard IV(A)]. He is allowed, however, to reproduce the model from memory but must recreate the supporting documentation to maintain compliance with Standard V(C), Record Retention.

27. **C** According to Standard VI(C), Referral Fees, Hern must disclose the referral arrangement between itself and Baker so that potential clients can judge the true cost of Hern's services and assess whether there is any partiality inherent in the recommendation of services.

28. **D** Standard VII(B) governs acceptable methods of referencing the CFA Institute, CFA designation, and CFA Program. Candidates may reference their candidacy if they are enrolled for or waiting for the results of a CFA exam. Pulin may also reference his membership status with the CFA Institute as well as his remaining eligibility requirements to become a CFA charterholder.

29. **A** There is no indication that Servais has inside information pertaining to the situation at the five firms in question—only the two firms that have already gone public with the information. It is common knowledge that the other five firms follow the same boron handing procedures. She is, therefore, in compliance with Standard V(A) concerning the use of material nonpublic information in the issuance of the investment recommendation.

30. **B** Even though the laws of Zanuatu would not preclude trading on the information, as a CFA Charterholder the friend is bound by the CFA Institute Code and Standards. Standard II(A) prohibits the use of material nonpublic information, and the friend may not trade the stocks about which she has such information under any circumstances.

31. **D** In this situation, Donovan, Smythe, and Yeats all violated Standard VII(A), Conduct as Members and Candidates in the CFA Program. The Standard prohibits conduct that compromises the integrity, validity, or security of the CFA exams. Donovan clearly breached the exam security. Smythe and Yeats both compromised the integrity of the exams by planning to use the actual exam question to gain an advantage over other candidates. Even though Yeats did not ultimately use the information to study for the exam, she participated in a scheme to cheat on the CFA exam.

32. **C** The furtive release of such information to a limited circle via an internet chat room does not cause the information to be public. The information is also clearly material. Therefore Green is not allowed to trade on the information under Standard II(A).

33. **A** Albright is entitled to accept work for which she receives outside compensation as long as the appropriate consent is obtained. Under Standard IV(A), such consent must be obtained from her employer prior to beginning the work.

34. **C** It is not reasonable for Bixby to expect a 40-to-60 stock mid-cap portfolio to track the entire S&P 500 Index, which is a large-cap index. He should know that there will be periods of wide variance between the performance of the portfolio and the S&P 500 Index. There is no assurance that a premium of 2 to 4% will consistently be obtained. Bixby is in violation of Standard III(D) since he has made an implicit guarantee of the fund's expected performance.

35. **D** Since the statements are vague, we have no direct evidence that a violation of securities law has occurred. However, under Standard I(C), members and candidates are prohibited from engaging in activities involving false or misleading statements. Karloff's action is a clear attempt to deceive the investing public regarding the value of Summit IPOs.

36. **B** Members and candidates are required to maintain knowledge of and comply with the applicable securities laws governing their professional activities. This type of securities fraud would almost certainly be against the law in most jurisdictions. Matthews's actions, therefore, are in violation of Standard I(A), which require knowledge of and adherence to applicable laws. He has also violated Standard I(D), which prohibits professional misconduct involving fraud and other acts that reflect poorly on the professional's reputation.

37. **C** NV management is asking Hunter to violate Standard II(B), which prohibits taking actions that are designed to distort prices or artificially increase trading volume. The intent of Hunter's actions is to mislead market participants and allow corporate insiders to take advantage of the artificially high prices.

38. **A** There is no violation of the CFA Institute Standards regarding this matter. The referral arrangement is fully disclosed to clients before they agree to do business with Pick. Therefore clients can fully assess how the agreement will affect their account before hiring Pick as their asset manager.

39. **B** Johnson has apparently let his recreational passion cloud his judgment. This is not to say that Swordfish Enterprises is not or will not be an excellent investment. However, if he had never heard of the firm previously, issuing an investment recommendation without conducting a thorough financial investigation indicates a failure to exercise diligence and also indicates that he lacks a reasonable and adequate basis for his recommendation. He is in violation of Standard V(A).

40. **C** Even though the shares are held in trust, this could still be construed as a conflict of interest. Lyons is obligated under Standard VI(A) to inform his employer of the potential conflict. If he is then authorized to issue investment recommendations on the security in question, the existence of a potential conflict must be disclosed in the report.

GLOBAL INVESTMENT PERFORMANCE STANDARDS

EXAM FOCUS

This topic review covers the key features of the Global Investment Performance Standards (GIPS®) as adopted by CFA Institute in 1999 and subsequently updated. Compliance with GIPS is voluntary. For the Level I exam you are responsible for only the "Introduction to the Global Investment Performance Standards (GIPS®)" and the Preface, Section I, and Section II (through II.0: Fundamentals of Compliance) of the GIPS document. The GIPS document is included in the book of candidate readings for Level 1 and is also available on the CFA Institute website. There is a helpful glossary of the terms included in the document. Candidates should not underestimate the importance of this material for the exam.

LOS 4.a: Explain why the GIPS standards were created.

In the past, a variety of reporting procedures were misleading at best. Some of these misleading practices included:

- *Representative accounts*—showing a top-performing portfolio as representative of firm's results.
- *Survivorship bias*—excluding "weak performance" accounts that have been terminated.
- *Varying time periods*—showing performance for selected time periods with outstanding returns.

GIPS is a set of ethical principles based on a standardized, industry-wide approach. Investment firms can *voluntarily* follow GIPS in their presentation of historical investment results to prospective clients. These standards seek to avoid misrepresentations of performance.

LOS 4.b: Explain what parties the GIPS standards apply to and whom the standards serve.

GIPS apply to investment management firms and are intended to serve prospective and existing clients of investment firms. GIPS allow clients to more easily compare investment performance among investment firms and have more confidence in reported performance.

LOS 4.c: Characterize "composites."

A composite is a grouping of individual portfolios representing a similar investment strategy, objective, or mandate. Examples of possible composites are "Large Capitalization Equities non-taxable accounts" and "Investment Grade Domestic Bonds."

LOS 4.d: Explain the purpose of verification.

Once a firm claims to be GIPS-compliant, the firm has an option to hire an independent third party to verify the claim of compliance.

The purpose of verification is to provide assurance that compliance has been adhered to on a firm-wide basis. Verification adds credibility.

LOS 4.e: Explain why a global standard is needed and how it is being implemented.

A global standard is needed for these reasons:

- Our industry is becoming more global, accentuating the need for standardization of calculation and presentation of investment performance.
- Prospective clients and investment management firms benefit from a set standard that is recognized globally.
- Firms can compete for business on an equal footing if every firm's investment performance information is complete and presented fairly.
- Existing and prospective clients benefit from the GIPS standards because they will have a higher degree of confidence in the historical information being provided.

The most recent GIPS revisions are effective January 1, 2006. All presentations that include results for periods subsequent to December 31, 2005 must meet all the requirements of these newly revised GIPS in order to be compliant. Prior performance results which do not include this most recent data may be prepared using the 1999 version of the GIPS standards.

LOS 4.f: State the "vision" of the GIPS standards.

An internationally accepted performance standard allows firms to present results that are comparable across all geographic locations.

The GIPS standards may better enable discussions between investment managers and prospects about the more crucial issues of how the investment management firm achieved its results, and how it determines future strategies.

LOS 4.g: State the objectives and key characteristics of the GIPS standards.

Objectives:

- To obtain global acceptance of calculation and presentation standards in a fair, comparable format with full disclosure.
- To ensure consistent, accurate investment performance data in areas of reporting, records, marketing, and presentations.
- To promote fair competition among investment management firms in all markets without unnecessary entry barriers for new firms.
- To promote global "self regulation."

Key characteristics:

- To claim compliance, an investment management firm must define its "firm," and this definition should reflect the "distinct business entity" that is held out to clients and prospects as the investment firm.
- GIPS are ethical standards for performance presentation which ensure fair representation of results and full disclosure.
- Include all actual fee-paying, discretionary portfolios in composites for a minimum of five years or since firm or composite inception. After presenting five years of compliant data, the firm must add annual performance each year going forward up to a minimum of ten years.
- Firms are required to use certain calculation and presentation standards and make specific disclosures.
- Input data must be accurate.
- GIPS contain both *required* and *recommended* provisions—firms are *encouraged* to adopt the *recommended* provisions.
- Firms are encouraged to present all pertinent additional and supplemental information.
- There will be no partial compliance and only full compliance can be claimed.

- For cases in which a local or country-specific law or regulation conflicts with GIPS, follow the local law, but disclose the conflict.
- Certain "recommendations" may become "requirements" in the future.
- Supplemental "private equity" and "real estate" provisions, contained in the GIPS standards, are to be applied to those asset classes.

LOS 4.h: State the appropriate disclosure when the GIPS standards and local regulations are in conflict.

To comply with both GIPS and local laws and regulations, firms must disclose any local laws or regulations that are in conflict with the GIPS standards. The local law must be followed, but the conflict must be disclosed.

LOS 4.i: Explain the scope of the GIPS standards with respect to definition of the firm, historical performance record, and compliance.

Definition of firm:

- In order to properly conform to GIPS, the "firm" must be clearly identified—it may be an actual subsidiary or a business entity or division.
- It must be held out to clients and prospects as a distinct business entity.

Historical performance record:

- Minimum of five years of historical data are required, or since firm or composite inception.
- After the five years has been achieved, the firm must add one additional year of performance each year up to a minimum of ten years.
- Non-GIPS-compliant information may be linked to compliant history. However, no noncompliant performance can be presented after January 1, 2000. Firms must clearly identify the noncompliant portion of results.
- If a firm has previously claimed compliance with an Investment Performance Council-endorsed Country Version of GIPS (CVG), it is granted reciprocity to claim compliance with GIPS for historical periods prior to January 1, 2006.

Compliance:

- The GIPS standards are newly revised and effective January 1, 2006. All presentations for performance after this date must use the requirements of the revised GIPS.
- All GIPS requirements must be met in order for a firm to claim to be GIPS-compliant.
- The following are "future requirements" which are *presently recommendations*:
  - After January 1, 2008, real estate investments must be valued at least quarterly.
  - After January 1, 2010, firms must value portfolios on the date of all large external cash flows.
  - After January 1, 2010, portfolios must be valued as of the calendar month end or the last business day of the month.
  - After January 1, 2010, composite returns must be calculated by asset weighting individual portfolio returns, at least monthly.
  - After January 1, 2010, carve-out returns cannot be included in single asset class composite returns. There is an exception if the carve-outs are managed separately and maintain their own separate cash balances.

Professor's Note: Carve-out returns refer to returns on a segment of a composite, for example, the equities contained in a "balanced funds" composite that contains both debt and equity securities.

- Internal compliance checks to verify accurate GIPS results are encouraged.

LOS 4.j: Name and characterize the eight major sections of the GIPS standards.

0. *Fundamentals of Compliance*: These are issues for firms to consider when claiming GIPS compliance. Definition of the firm is part of this. The next LOS covers this section in greater detail.

1. *Input Data*: Input data should be consistent in order to establish full, fair, and comparable investment performance presentations.

2. *Calculation Methodology*: Certain methodologies are required for portfolio and composite return calculations. Uniformity in methods is required.

3. *Composite Construction*: Creation of meaningful, asset-weighted composites is important to achieve a fair presentation.

4. *Disclosures*: Certain information must be disclosed about the presentation and the policies adopted by the firm.

5. *Presentation and Reporting*: Investment performance must be presented according to GIPS requirements, and when appropriate, other firm-specific information should be included.

6. *Real Estate*: These provisions apply to all real estate investments (land, buildings, etc.) regardless of the level of control the firm has over management of the investment.

7. *Private Equity*: These must be valued according to the GIPS Private Equity Valuation Principles, which are contained in Appendix D, unless it is an open-end or evergreen fund (which must follow regular GIPS).

LOS 4.k: Explain the fundamentals of compliance with the GIPS standards.

Fundamentals of compliance contain both requirements and recommendations:

Definition of the Firm—Requirements:

- To apply GIPS on a firm-wide basis.
- Firm must be defined as a distinct business unit.
- Total firm assets includes total market value of discretionary and non-discretionary assets, including fee-paying and non-fee-paying accounts.
- Include asset performance of sub-advisors, as long as the firm has discretion over sub-advisor selection.
- If a firm changes its organization, historical composite results cannot be changed.

Definition of the Firm—Recommendations:

- Include the broadest definition of the firm, including all geographical offices marketed under the same brand name.

Document Policies and Procedures—Requirements:

- Document, in writing, policies and procedures the firm uses to comply with GIPS.

Claim of Compliance—Requirements:

- Once GIPS requirements have been met, the following compliance statement must be used:

    "[Insert name of firm] has prepared and presented this report in compliance with the Global Investment Performance Standards (GIPS®)."

- There is no such thing as partial compliance.
- There are to be no statements referring to calculation methodologies used in a composite presentation as being "in accordance with GIPS. . . etc."
- Similarly, there should be no such statements referring to the performance of an individual, existing client as being "calculated in accordance with GIPS. . . etc." unless a compliant firm is reporting results directly to the client.

Firm Fundamental Responsibilities—Requirements:

- Firms must provide a compliant presentation to *all* prospects (prospect must receive presentation within the previous 12 months).
- Provide a composite list and composite description to all prospects that make a request. List discontinued composites for at least five years.
- Provide, to clients requesting it, a compliant presentation and a composite description for any composite included on the firm's list.
- When jointly marketing with other firms, if one of the firms claims GIPS compliance, be sure it is clearly defined as separate from noncompliant firms.
- Firms are encouraged to comply with recommendations and must comply with all requirements. Be aware of updates, guidance statements, etc.

Verification—Recommendations:

- Firms are encouraged to pursue independent verification. Verification applies to the entire firm's performance measurement practices and methods, not a selected composite.
- Verified firms should include the following disclosure language:

"[Insert name of firm] has been verified for the periods [insert dates] by [name of verifier]. A copy of the verification report is available upon request."

CONCEPT CHECKERS: GLOBAL INVESTMENT PERFORMANCE STANDARDS

1. Which of the following statements most accurately describes the parties that the GIPS standards are intended to apply to and serve? The GIPS standards apply to:
   A. consultants and serve their existing and prospective clients.
   B. firms that issue securities and serve investment management firms.
   C. investment management firms and serve securities regulatory entities.
   D. investment management firms and they serve their existing and prospective clients.

2. At a regional conference for institutional portfolio managers, Jason Morris makes four comments in a presentation centered on explaining the reasons for the creation of the GIPS standards. Which of Morris' comments is incorrect? GIPS were created:
   A. to improve comparability of performance results among investment firms.
   B. to reduce historical performance inflation caused by excluding results of terminated portfolios.
   C. to alleviate the ambiguity caused by investment firms' manipulated forecasts of expected portfolio returns.
   D. in response to performance reporting abuses which included only reporting results over periods of exceptional returns.

3. Which of the following statements most accurately describes verification under the GIPS standards? Verification:
   A. may be performed on single composites.
   B. is required for a firm to claim GIPS compliance.
   C. requires a verification report to be issued for the entire firm.
   D. is required for all composites in existence prior to January 1, 2000.

4. Benson Asset Management Inc. is seeking to become compliant with the GIPS standards. The firm has hired an independent consultant to assist in ensuring that Benson's policies and procedures conform to the standards. All of the following recommendations made by the consultant are required under GIPS **EXCEPT**:
   A. Benson must disclose the results of its independent verification process in its composite presentations.
   B. All of Benson's accounts managed by third party advisors selected by Benson must be included in the firm's composites.
   C. Benson's policies and procedures that are instrumental to maintaining compliance with GIPS must be documented in writing.
   D. Compliant presentations for discontinued composites must be made available to any prospect requesting one up to five years after discontinuation.

5. Assume that on January 1, 2001, a 15-year-old firm with no GIPS-compliant performance history wishes to claim compliance with the GIPS standards. Which of the following statements accurately reflects appropriate actions for the firm to take to claim compliance with the standards?
   A. Report up to ten years of non-GIPS-compliant history, as long as it discloses why the performance presentation is not in compliance with GIPS.
   B. Retroactively comply with GIPS for the year beginning January 1, 2000, and report nine additional years of performance history (ten total) with a disclosure of why the earlier years are not GIPS-compliant.
   C. Retroactively comply with GIPS for the year beginning January 1, 2000, and report four additional years of performance history (five total) with a disclosure of why the earlier years are not GIPS-compliant.
   D. Retroactively comply with the GIPS standards for the 5-year period January 1, 1996, through December 31, 2000, and report five additional years of non-GIPS-compliant performance with a disclosure of why the performance in these years is not GIPS-compliant.

6. Vivian Müller, compliance director for ABC Investments, is reviewing GIPS compliance policies put in place by her employees. In the policies, the employees have included several future GIPS requirements that are currently only recommended. All of the following policies are only recommended, **EXCEPT:**
    A. Calculation of composite returns must be done based on monthly asset weighted returns of the underlying portfolios.
    B. Restructuring of the firm's organization cannot be used as a basis to change the historical performance results of a composite.
    C. All investments in land, in-process building construction, and finished buildings must be valued every three months.
    D. Performance presentation for carve-out returns is only permitted for portfolios managed separately with their own cash balances.

7. Which of the following is an inaccurate statement about the major components of the GIPS standards?
    A. GIPS cover the professional qualifications of those responsible for managing assets at a firm claiming compliance.
    B. GIPS cover the way investment firms calculate composite returns as well as the method used to create the composite itself.
    C. GIPS apply to many categories of portfolio assets including stocks, bonds, real estate, and private equity.
    D. GIPS require that firms claiming compliance disclose certain information related to performance presentations and firm policies.

8. Mason Smith is trying to decide which of the following composite definitions, submitted by his junior analysts, would be considered a viable composite according to the GIPS standards. Which composite will meet the standards? A composite that includes:
    A. all accounts that are managed directly from the firm's Hong Kong office.
    B. all actively managed portfolios but excludes passively managed portfolios.
    C. portfolios that have experienced at least a positive three percent return over the last five years.
    D. all portfolios that are managed to provide a return equal to that of the S&P 500 Index return.

9. Mack Stevens has assembled several articles written about the GIPS standards. Each article has listed at least one objective of the GIPS standards. All of the following statements collected from the articles correctly describe objectives of GIPS **EXCEPT:**
    A. GIPS attempt to gain worldwide acceptance of performance calculation and presentations standards in a fair format with full disclosure.
    B. GIPS try to provide an opportunity for large and small firms to compete on an equal footing through external rules and regulations.
    C. GIPS seek to encourage equitable competition among investment firms in all markets without stifling new market entrants in the process.
    D. GIPS strive to make sure that firms have reliable and precise reporting, record keeping, advertising, and presenting of investment performance.

10. An investment management firm, Investco, Inc., was recently audited by the United States Securities and Exchange Commission (SEC). Investco included the following statement in its performance presentation report: "This report has been verified as GIPS-compliant by Investco's Compliance Department and the United States Securities and Exchange Commission." Does this constitute acceptable verification under the GIPS standards?
    A. Yes, but only because the SEC conducted an audit.
    B. No, only one party may perform GIPS verification.
    C. No, neither party involved in the audit may perform a GIPS verification.
    D. Yes, because an audit was performed by both the SEC and the firm's internal audit team.

ANSWERS – CONCEPT CHECKERS: GLOBAL INVESTMENT PERFORMANCE STANDARDS

1. **D** The GIPS standards apply to investment management firms. They are intended to serve prospective and existing clients of investment firms and consultants who advise these clients.

2. **C** GIPS were created to reduce ambiguity of performance reporting among investment firms. Past abuses of performance reporting include representative accounts (showing only top performers), survivorship bias (deleting poor performers), and varying time period (showing only the time period with the best performance).

3. **C** A single verification report is issued in respect to the whole firm: GIPS verification cannot be carried out for a single composite.

4. **A** Verification is not currently required under GIPS. Firms that choose to undergo the verification process are encouraged, but not required, to make a specific verification disclosure in composite presentations and advertisements that reference the firm's GIPS verification.

5. **D** In order to claim GIPS compliance, a firm must present at least five years of annual investment performance that is compliant with GIPS. If a firm or composite is less than five years old, the performance since the inception of the firm or composite must be presented.

6. **B** Firms cannot alter historical performance records of composites simply because of a reorganization of the firm. This is a current requirement of the GIPS standards. All of the other answer choices are future requirements that are currently only recommended.

7. **A** There are no GIPS related directly to the qualifications of employees managing assets at an investment firm whether it claims compliance with the standards or not. The major sections of GIPS are: input data, calculation methodology, composite construction, disclosures, presentation and reporting, real estate, and private equity.

8. **D** Composites are groups of portfolios that represent a similar investment strategy, objective, or mandate. Clearly, grouping all portfolios managed to mirror the S&P 500 Index return constitutes a composite according to this definition. Organizing composites by office, a generic active management category, or by return history is not acceptable as these categories do not reflect any sort of strategy, objective, or mandate.

9. **B** GIPS seeks to promote global self regulation through voluntary acceptance and adherence to the standards. The other statements correctly state objectives of the GIPS standards.

10. **C** GIPS verification must be performed by an independent third party. The SEC audit does not constitute a GIPS verification.

THE CORPORATE GOVERNANCE OF LISTED COMPANIES: A MANUAL FOR INVESTORS

EXAM FOCUS

Due to the collapses of some major corporations and associated investor losses, corporate governance has become a hot topic in the investment community. The prominence of the issue has likely been a factor in the decision to include this new reading in the 2006 CFA curriculum. Corporate governance is the firm's set of internal controls and outlines how a firm is managed. There is a fair amount of material (17 LOS) here, but it is not particularly challenging. You need to understand well the specific issues that are covered under the heading of "corporate governance" and which practices are considered good. Several LOS refer to the independence and effectiveness of a firm's board of directors, and you should know the characteristics of an independent and effective board of directors. Much of the rest of the material has to do with shareholder interests and whether a firm's actions and procedures promote the interests of shareholders.

LOS 5.a: Identify the factors in evaluating the quality of corporate governance and the relative strength of shareowner rights.

Corporate governance refers to a firm's internal controls and procedures by which the firm is managed. It provides a framework under which management, the board, and investors all work together.

In evaluating the board of directors, investors should:

- Establish how many members are truly independent.
- Evaluate the qualifications of board members to meet challenges the firm faces.
- Assess whether the board of directors has authority to hire independent third-party consultants.
- Determine whether all board members are elected annually or if there are staggered terms (staggered terms make it more difficult for shareholders to significantly change the board of directors).
- Investigate whether the firm has outside business relationships with board members, executives, employees, or the families of these groups.
- Determine whether board members have relevant finance and accounting experience.
- Find out if the firm has a committee of independent members to set executive pay.
- Find out if the firm has a nominations committee to recruit new members.
- Inquire as to other board committees responsible for governance, mergers and acquisitions, legal matters, and risk management.

In evaluating management, investors should:

- Verify that the firm has committed to an ethical framework and adopted a code of ethics.
- See if the firm permits board members or management to use firm assets for personal reasons.
- Analyze executive compensation to assess whether it is commensurate with responsibilities and performance.
- Look into the size, purpose, means of financing, and duration of any share-repurchase programs.

In evaluating shareholder rights, investors should:

- Find out if shareholders must be present in order to vote proxies.
- Determine whether shareholders can submit votes confidentially.

- See if shareholders can cast the cumulative number of votes associated with their shares for just one or for a limited number of board nominees.
- Observe whether shareholders can approve corporate structure changes.
- Determine whether shareholders can nominate board members.
- Find out if shareholders can submit proposals for board consideration and whether the board and management are required to implement shareholder-approved proposals.
- See if the firm's ownership structure has different classes of stock that separate voting rights from the economic value of shares.
- Evaluate corporate governance codes or any local legal statutes that allow shareholders to seek legal or regulatory action to enforce and protect shareholder rights.
- Analyze any existing or proposed takeover defenses.

LOS 5.b: Define corporate governance and identify practices that constitute good corporate governance.

Corporate governance is the set of internal controls, processes, and procedures by which firms are managed. It defines the appropriate rights, roles, and responsibilities of management, the board of directors, and shareholders within an organization. It is the firm's series of checks and balances. Good corporate governance practices seek to ensure that:

- The board of directors protects shareholder interests.
- The firm acts lawfully and ethically in dealings with shareholders.
- The rights of shareholders are protected and shareholders have a voice in governance.
- The board acts independently from management.
- There are proper procedures and controls covering management's day-to-day operations.
- The firm's financial, operating, and governance activities are reported to shareholders in a fair, accurate, and timely manner.

LOS 5.c: Define independence as used to describe corporate board members, and explain the role of independent board members in corporate governance.

To be independent, a board member must not have any material relationship with:

- The firm and its subsidiaries, including former employees, executives, and their families.
- Individuals or groups, such as a shareholder(s) with a controlling interest, which can influence the firm's management.
- Executive management and their families.
- The firm's advisers, auditors, and their families.
- Any entity which has a cross-directorship with the firm.

An independent board member must work to protect shareholders' long-term interests. Board members need to have not only independence, but experience and resources. The board of directors must have autonomy to operate independently from management.

If a board is not independent, it may be more likely to make decisions that benefit either management or those who have influence over management, thus harming shareholders' long-term interests.

LOS 5.d: List and explain the major factors that enable a board to exercise its duty to act in the best long-term interests of shareowners.

In order to properly protect shareholders' long-term interests, investors should consider whether:

- A majority of the board of directors is comprised of independent members (not management).
- The board meets regularly outside the presence of management.

- The chairman of the board is also the CEO. This may impair the ability and willingness of independent board members to express opinions contrary to managements'.
- Independent board members have a primary or leading board member in cases where the chairman is *not* independent.
- Board chair is a former CEO of the firm. This could affect abilities of independent board members to act independently of management.
- There are board members who are closely aligned with a firm supplier, customer, share-option plan or pension adviser, etc. Can board members recuse themselves on any potential areas of conflict?

A non-independent board is more likely to make decisions which unfairly or improperly benefit management interests and those who have influence over management. These also may harm shareholders' long-term interests.

LOS 5.e: Identify characteristics of a board that contribute to the board's independence, and state why each characteristic is important for shareowners' interests.

An independent board will more properly protect shareowner interests. Investors should determine whether:

- The majority of the board is comprised of independent members. *This will limit undue management influence in board decisions.*
- Independent members meet outside of management's presence. *This allows free discussion without influence from executive board members.*
- The board chair is also the CEO. *Combining these positions inhibits the power of independent Board members.*
- The board chair is a former CEO of the firm. *Again, this arrangement can impair the board's ability to act independently, free from management influence.*
- A major supplier or customer is aligned with a member of management or the board. *A board member may have to recuse themselves to avoid a conflict of interest.*

LOS 5.f: Identify factors that indicate a board and its members possess the experience required to govern the company for the benefit of its shareowners.

Board members without the requisite skills and experience are more likely to defer to management when making decisions. This can be a threat to shareholder interests.

When considering the qualifications of board members, consider whether board members:

- Can make informed decisions about the firm's future.
- Can act with care and competence as a result of their experience with:
  - Technologies, products, services which the firm offers.
  - Financial operations and accounting and auditing topics.
  - Legal issues.
  - Strategies, planning.
  - Business risks the firm faces.
- Have made any public statements indicating their ethical stances.
- Have had any legal or regulatory problems as a result of working for or serving on the firm's board or the board of another firm.
- Have other board experience.
- Regularly attend meetings.
- Are committed to shareholders. Do they have significant stock positions? Have they eliminated any conflicts of interest?
- Have necessary experience and qualifications?
- Have served on board for more than 10 years. While this adds experience, these board members may be too closely allied with management.

Investors should also consider:

- How many board and committee meetings are held, and what is the attendance record?
- Was there a self-assessment of the board and its committees?
- Does the board provide adequate training for its members?

LOS 5.g: Explain the importance to shareowners of a board's ability to hire external consultants.

There is often a need for specific, specialized, independent advice on various firm issues/risks, including compensation, mergers and acquisitions, legal, regulatory, financial, and issues relating to the firm's reputation. A truly independent board will have the ability to hire external consultants without management approval.

This ability to hire external consultants ensures that the board will receive specialized advice on technical issues and provides the board with independent advice that is not influenced by management interests.

LOS 5.h: Identify advantages and disadvantages of annual board elections compared to less frequent elections.

Anything beyond an annual limit on board member tenure (i.e., two- or three-year terms) limits shareowners' ability to change the board's composition, in the event that board members fail to represent shareowners' interests fairly.

While reviewing firm policy regarding election of the board, investors should consider:

- Whether there are annual elections or staggered multiple-year terms (a *classified* board). Staggered board may serve another purpose—to act as a takeover defense.
- Whether the board filled a vacant position for a remaining term without shareholder approval.
- Whether shareholders can remove a board member.
- Whether the board is the proper size for the specific facts and circumstances of the firm.

LOS 5.i: Explain the implications of a weak corporate code of ethics with regard to related-party transactions and personal use of company assets.

To make sure that board members act independently, there should be policies to discourage the following practices:

- Receiving consulting fees for work done on the firm's behalf.
- Receiving finders' fees for any mergers, acquisitions, and sales which are brought to management's attention.

Further, procedures should be considered which would limit board members' and associates' abilities to receive consulting fees beyond the scope of their board responsibilities.

The firm should disclose all material related-party transactions or commercial relationships it has with board members or nominees. The same goes for any property which is leased, lent, or otherwise provided to the firm from board members or executive officers.

Receiving personal benefits from the firm can create conflicts of interest.

LOS 5.j: Critique characteristics and practices of board committees, and determine whether they are supportive of shareowner protection.

Audit Committee

This committee ensures that the financial information provided to shareholders is complete, accurate, reliable, relevant, and timely. Investors must determine whether:

- Proper accounting and auditing procedures have been followed (GAAP and GAAS).
- The external auditor is free from management influence.
- Any conflicts between the external auditor and the firm are resolved in a manner that favors the shareholder.
- Independent auditors have authority over the audit of all the company's affiliates and divisions.
- All board members serving on the audit committee are independent.
- Committee members are considered financial experts.
- The shareholders vote on the approval of the board's selection of the external auditor.
- The audit committee has authority to approve or reject any other proposed non-audit engagements with the external audit firm.
- The firm has provisions and procedures that specify to whom the internal auditor reports. Internal auditors must have no restrictions on their contact with the audit committee.
- There have been any discussions between the audit committee and the external auditor resulting in a change in financial reports due to questionable interpretation of accounting rules, fraud, etc.
- The audit committee controls the audit budget.

Remuneration/Compensation Committee

Investors should be sure that there is a committee of independent board members that sets executive compensation, commensurate with responsibilities and performance. The committee can further these goals by:

- Making sure all committee members are independent.
- Linking compensation to long-term firm performance/profitability.

Investors, when analyzing this committee, should determine whether:

- Executive compensation is appropriate.
- The firm has provided loans or the use of company property to board members.
- There is regular attendance by committee members.
- There are policies and procedures for this committee.
- The firm has provided details to shareholders regarding compensation in public documents.
- Terms and conditions of options granted are reasonable.
- Any obligations regarding share-based compensation are met through issuance of new shares.
- The firm and the board are required to receive approval by shareholders for any share-based remuneration plans, since these plans can affect shareholder proportional ownership and create potential dilution issues.
- Senior executives from other firms have cross-directorship links with the firm or committee members. Watch for situations where individuals may benefit directly from reciprocal decisions on remuneration.

Nominations Committee

The nominations committee handles recruiting of new (independent) board members. It is responsible for:

- Recruiting qualified board members.
- Regularly reviewing performance, independence, skills, and experience of existing board members.
- Creating nominations procedures and policies.
- Preparing an executive management succession plan.

Candidates proposed by this committee will affect whether or not the board works for the benefit of shareholders. Performance assessment of board members should be fair and appropriate.

Investors should review company reports over several years to see if this committee has properly recruited board members who have fairly protected shareholder interests. Investors should also review:

- Criteria for selecting new board members.
- Composition, background, and expertise of present board members. How do proposed new members complement the existing board?
- The process for finding new members (i.e., input from outside the firm versus management suggestions).
- Attendance records.
- Succession plans for executive management (if such plans exist).
- The committee's report, including any actions/decisions and discussion of the committee.

Other Board Committees

Additional committees can provide more insight into goals and strategies of the firm. It is more likely that these committees will fall outside of typical corporate governance codes, so they are more likely to be comprised of members of executive management. Be wary of this—independence is once again critical to maintain shareowners' best interests.

LOS 5.k: Identify the information needed for evaluating the alignment of a company's executive compensation structure and practices with shareowner interests.

Both the amount paid to key executives and the manner in which the compensation is provided should be examined closely.

Compensation should be in line with the level of responsibilities and performance, and incentives should encourage long-term growth. Investors should be able to evaluate whether they are getting a proper return on investment for the amount of compensation paid to management. When reviewing compensation disclosures, consider the following:

- Compensation strategy. Does the program reward long-term growth? Compare rewards with the competition.
- Executive compensation analysis should be performed in terms of corporate performance.
- Share-based compensation terms should also be examined. Consider how the program affects shares outstanding, dilution, and share values.
- All stock option grants should show up on the financial statements as an expense.
- Performance-based compensation should be linked to long-term profitability and share price performance.
- Option repricing. Has the firm lowered strike prices of previously issued options?
- Share ownership by management should be examined. Does management own shares other than those related to stock option grants?

Information on executive compensation may be found in the annual report and in proxy materials. Additional information may be contained on the firm's website.

LOS 5.l: State the provisions that should be included in a strong corporate code of ethics.

A code of ethics for a firm sets the standard for basic principles of integrity, trust, and honesty. It gives the staff behavioral standards and addresses conflicts of interest. Ethical breaches can lead to big problems for firms, resulting in sanctions, fines, management turnover, and unwanted negative publicity. Having an ethical code can be a mitigating factor from regulators if a breach occurs.

When analyzing ethics codes, these are items to be considered:

- Make sure the board of directors receives relevant corporate information in a timely manner.
- Ethics codes should be in compliance with the corporate governance code of the location country or with the governance requirements set forth by the local stock exchange. Firms should disclose whether they adhered to their own ethical code, including any reasons for failure.
- The ethical code should prohibit advantages to the firm's insiders that are not offered to shareowners.
- There should be a person designated to be responsible for corporate governance.
- If there are waivers from the ethics code for selected management personnel, there should be reasons given.
- Were any provisions of the ethics code waived recently? Why?
- The firm's ethics code should be periodically audited and improved.

LOS 5.m: Identify components of a company's executive compensation program that positively or negatively affect shareowners' interests.

- Compensation programs rewarding short-term performance negatively affect long-term shareholder value.
- Compensation programs should be in line with corporate performance.
- If there are share-based compensation programs, they should be reviewed and assessed for possible dilution of shareholder interest and overall share values.
- Stock-options need to be properly accounted for, and expensed.
- All performance-based compensation should be properly tied to superior performance.
- Investors should watch for "option repricing"—sometimes the company will lower the strike prices of stock options already issued to management.

LOS 5.n: Explain the implications for shareowners of a company's proxy voting rules and practices.

The ability to vote proxies is a fundamental shareholder right. If the firm makes it difficult to vote proxies, it limits the ability of shareholders to express their views and affect the firm's future direction.

Investors should consider whether the firm:

- Limits the ability to vote shares (i.e., by requiring attendance at annual meeting).
- Groups its meetings to be held the same day as other companies in the same region and also requires attendance to cast votes.
- Allows proxy voting by some remote mechanism.
- Is allowed under its governance code to use share blocking, a mechanism that prevents investors who wish to vote their shares from trading their shares during a period prior to the annual meeting.

Confidential Voting

Investors should determine if shareholders are able to cast confidential votes—this can encourage unbiased voting. In looking at this issue, investors should consider whether:

- The firm uses a third party to tabulate votes.
- The third party or the firm retains voting records.
- The tabulation is subject to audit.
- Shareholders are entitled to vote only if present.

Cumulative Voting

Shareholders may be able to cast the cumulative number of votes allotted to their shares for one or a limited number of board nominees. Be cautious in the event the firm has a considerable minority shareholder group, such as a founding family, that can serve its own interests through cumulative voting.

Information on possible cumulative voting rights will be contained in the articles of organization and by-laws, the prospectus, or Form 8-A, which must be filed with the Securities and Exchange Commission in the United States.

Voting for Other Corporate Changes

Certain other changes to corporate structure or policies can change the relationship between shareholders and the firm. Watch for changes to:

- Articles of organization.
- By-laws.
- Governance structures.
- Voting rights and procedures.
- Poison pill provisions (these are impediments to an acquisition of the firm).
- Provisions for change-in-control.

An example of a change that would adversely affect shareholder value is the adoption of an anti-takeover mechanism or provision making it too expensive for potential acquirers to consider a purchase.

Regarding issues requiring shareholder approval, consider whether shareholders:

- Must approve other corporate change proposals with supermajority votes.
- Will be able to vote on the sale of the firm, or part of it, to a third-party buyer.
- Will be able to vote on major executive compensation issues.
- Will be able to approve any anti-takeover measures.
- Will be able to periodically reconsider and re-vote on rules that require supermajority voting to revise any governance documents.
- Have the ability to vote for changes in articles of organization, by-laws, governance structures, and voting rights and procedures.
- Have the ability to use their relatively small ownership interest to force a vote on a special interest issue.

Investors should also be able to review issues such as:

- Share buy-back programs which may be used to fund share-based compensation grants.
- Amendments or other changes to a firm's charter and by-laws.
- Issuance of new capital stock.

LOS 5.o: State whether a company's rules governing shareowner-sponsored board nominations, resolutions, and proposals are supportive of shareowner rights.

Shareowner-Sponsored Board Nominations

Investors need to determine whether the firm's shareholders have the power to put forth an independent board nominee. Having such flexibility is a positive for investors as it allows them to address their concerns and protect their interests through direct board representation. Additional items to consider:

- Under what circumstances can a shareholder nominate a board member? Can shareowners vote to remove a board member?
- How does the firm handle contested board elections?

The proxy statement is a good source document for information about these issues in the United States. In many jurisdictions, articles of organization and corporate by-laws are other good sources of information on shareholder rights.

Shareowner-Sponsored Resolutions

The right to propose initiatives for consideration at the annual meeting is an important shareholder method to send a message to management.

Investors should look at whether:

- The firm requires a simple majority or a supermajority vote to pass a resolution.
- Shareholders can hold a special meeting to vote on a special initiative.
- Shareholder-proposed initiatives will benefit all shareholders, rather than just a small group.

Advisory or Binding Shareowner Proposals

Investors should find out if the board and management are required to actually implement any shareholder-approved proposals. Investors should determine whether:

- The firm has implemented or ignored such proposals in the past.
- The firm requires a supermajority of votes to approve changes to its by-laws and articles of organization.
- Any regulatory agencies have pressured firms to act on the terms of any approved shareholder initiatives.

LOS 5.p: Explain the implications of different classes of common equity for shareowner rights.

Shareowner Rights Issues—Ownership Structure

There may be different classes of common equity within a firm that separate the voting rights of those shares from their economic value.

Firms with dual classes of common equity could encourage prospective acquirers to only deal directly with shareholders with the supermajority rights. Firms that separate voting rights from economic rights have historically had more trouble raising equity capital for capital investment and product development than firms that combine those rights.

When looking at a firm's ownership structure, examine whether:

- There are safeguards in the by-laws and articles of organization to protect shareholders who have inferior voting rights.
- The firm was recently privatized by a government entity and whether the selling entity retained voting rights—this may prevent shareholders from receiving full value for their shares.
- Any super-voting rights kept by certain classes of shareholders impair the firm's ability to raise equity capital. If a firm has to turn to debt financing, the increase in leverage can harm the firm.

Information on these issues can be found in the proxy, website, prospectus, or notes to the financial statements.

Shareowner Rights Issues—Shareowner Legal Rights

Examine whether the investor has the legal right under the corporate governance code and other legal statutes of the jurisdiction in which the firm is headquartered to seek legal redress or regulatory action to enforce and protect shareholder rights.

Investors should determine whether:

- Legal statutes allow shareholders to take legal actions to enforce ownership rights.
- The local market regulator, in similar situations, has taken action to enforce shareholder rights.

- Shareholders are allowed to take legal or regulatory action against firm's management or board in the case of fraud.
- Shareholders have "dissenters' rights," which require the firm to repurchase their shares at fair market value in the event of a problem.

LOS 5.q: Determine the probable effects of takeover defenses on share value.

Examples of takeover defenses include golden parachutes, poison pills, and greenmail (use of corporate funds to buy back the shares of a hostile acquirer at a premium to their market value). All of these defenses may be used to counter a hostile bid, and the probable effect is to decrease share value.

When reviewing the firm's takeover defenses, investors should:

- Ask whether the firm requires shareholder approval to implement such takeover measures.
- Ask whether firm has received any acquisition interest in the past.
- Consider that the firm may use its cash to "pay off" a hostile bidder. Investors should take steps to discourage this activity.
- Consider whether any change of control issues would invoke interest of a national or local government and as a result pressure the seller to change the terms of the acquisition or merger.

THE TIME VALUE OF MONEY

EXAM FOCUS

This topic review covers time value of money (TVM) concepts and applications. Procedures are presented for the calculation of the future value and present value of single cash flows, annuities, and uneven cash flows. The impact of different compounding periods is examined, along with the procedures for solving for other variables in TVM problems. Your main objective in this chapter is to master TVM mechanics (i.e., learn how to crunch the numbers). There will be a lot of TVM problems and applications on the exam, so be prepared to deal with them. Work all the questions and problems found at the end of this review. Make sure you know how to grind out all the TVM problems on your calculator. The more rapidly you can do them (correctly), the more time you will have for the less predictable parts of the exam.

WARM-UP: TIME VALUE OF MONEY CONCEPTS AND APPLICATIONS

In order for money to have time value, it must be possible to invest it at a positive rate of return. The rate of return (interest rate) that provides money with time value is composed of three components:

- *Risk-free rate*. This is the rate that is earned on a riskless investment, and it represents the compensation that investors require to defer current consumption. The rate on short-term U.S. Treasury securities is typically used to represent the risk-free rate.
- *Inflation premium*. This is the return that investors require to compensate them for the change of purchasing power over an investment horizon.
- *Risk premium*. This is the compensation that investors require for being exposed to various types of investment risk.

The concept of *compound interest* and *interest on interest* is deeply embedded in time value of money (TVM) procedures. When an investment is subjected to compound interest, the growth in the value of the investment from period to period reflects not only the interest earned on the original principal amount but also on the interest earned on the previous period's interest earnings—the interest on interest. TVM applications frequently call for the determination of the future value (FV) of an investment's cash flows as a result of the effects of compound interest. The process of computing FV involves projecting the cash flows forward, on the basis of an appropriate compound interest rate, to the end of the investment's life. The computation of the present value (PV) works in the opposite direction—it brings the cash flows from an investment back to the beginning of the investment's life on the basis of an appropriate compound rate of return. The usefulness of being able to measure the PV and/or FV of an investment's cash flows comes into play when comparing investment alternatives because the value of the investment's cash flows must be measured at some common point in time, either at the end of the investment horizon (FV) or at the beginning of the investment horizon (PV).

Using a Financial Calculator

It is imperative that you are able to use a financial calculator when working TVM problems because the exam is constructed under the assumption that candidates have the ability to do so. There is simply no other way that you will have time to solve TVM problems. **CFA Institute allows only two types of calculators to be used for the exam—the TI BAII Plus® (including the BAII Plus Professional) and the HP 12C® (including the HP 12C**

Platinum). This topic review is written primarily with the TI BAII Plus in mind. If you don't already own a calculator, go out and buy a **TI BAII Plus!** However, if you already own the HP 12C and are comfortable with it, by all means continue to use it.

The TI BAII Plus comes preloaded from the factory with the periods per year function (P/Y) set to 12. This automatically converts the annual interest rate (I/Y) into monthly rates. While appropriate for many loan-type problems, this feature is not suitable for the vast majority of the TVM applications we will be studying. So prior to using our Study Notes, please set your P/Y key to "1" using the following sequence of keystrokes:

[2nd] [P/Y] "1" [ENTER] [2nd] [QUIT]

As long as you do not change the P/Y setting, it will remain set at one period per year until the battery from your calculator is removed (it does not change when you turn the calculator on and off). If you want to check this setting at any time, press [2nd] [P/Y]. The display should read P/Y = 1.0. If it does, press [2nd] [QUIT] to get out of the "programming" mode. If it doesn't, repeat the procedure previously described to set the P/Y key. With P/Y set to equal 1, it is now possible to think of I/Y as the interest rate per compounding period and N as the number of compounding periods under analysis. Thinking of these keys in this way should help you keep things straight as we work through TVM problems.

Before we begin working with financial calculators, you should familiarize yourself with your TI by locating the TVM keys noted below. These are the only keys you need to know to work virtually all TVM problems.

- N = Number of compounding periods.
- I/Y = Interest rate per compounding period.
- PV = Present value.
- FV = Future value.
- PMT = Annuity payments, or constant periodic cash flow.
- CPT = Compute.

Professor's Note: There is a free downloadable calculator tutorial available at our web site (www.schweser.com) for both the TI and HP calculators. This file contains detailed information—complete with illustrations—on setting up your calculator, handling multiple payment periods, various types of TVM calculations, and more. If you are even the least bit unsure about how to use your calculator, you'd be well advised to access and review this tutorial!

Time Lines

It is often a good idea to draw a time line before you start to solve a TVM problem. A *time line* is simply a diagram of the cash flows associated with a TVM problem. A cash flow that occurs in the present (today) is put at time 0. Cash outflows (payments) are given a negative sign, and cash inflows (receipts) are given a positive sign. Once the cash flows are assigned to a time line, they may be moved to the beginning of the investment period to calculate the PV through a process called *discounting* or to the end of the period to calculate the FV using a process called *compounding*.

Figure 1 illustrates a time line for an investment that costs $1,000 today (outflow) and will return a stream of cash payments (inflows) of $300 per year at the end of each of the next five years.

Figure 1: Time Line
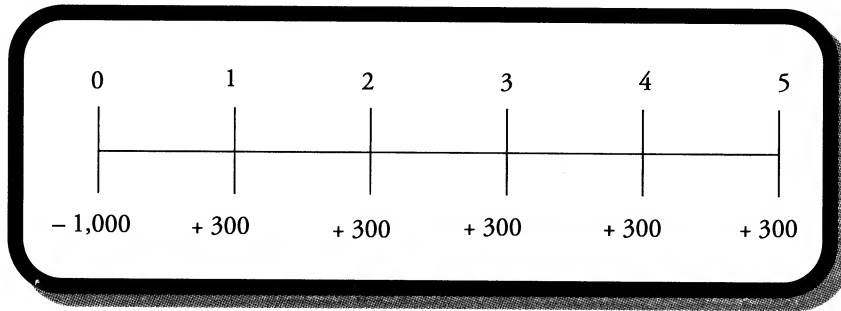
Please recognize that the cash flows occur at the end of the period depicted on the time line. Furthermore, note that the end of one period is the same as the beginning of the next period. For example, the end of t = 2 is the same as the beginning of t = 3, but the beginning of year 3 cash flow appears at time t = 2 on the time line. Keeping this convention in mind will help you keep things straight when you are setting up TVM problems.

Professor's Note: Throughout the problems in this review, rounding differences may occur between the use of different calculators or techniques presented in this document. So don't panic if you are a few cents off in your calculations.

LOS 6.a: Explain an interest rate as the sum of a real risk-free rate, expected inflation, and premiums that compensate investors for distinct types of risk.

The **real risk-free rate** of interest is a theoretical rate on a single period loan that has no expectation of inflation in it. When we speak of a real rate of return, we are referring to an investor's increase in purchasing power (after adjusting for inflation). Since expected inflation in future periods is not zero, the rates we observe on T-bills, for example, are risk-free rates but not *real* rates of return. T-bill rates are *nominal risk-free rates* because they contain an *inflation premium*. The approximate relation here is:

nominal risk-free rate = real risk-free rate + expected inflation rate

Securities may have one or more of several risks, and each added risk increases the required rate of return on the security. These **types of risk** are:

- *Default risk.* The risk that a borrower will not make the promised payments in a timely manner.
- *Liquidity risk.* The risk of receiving less than fair value for an investment if it must be sold for cash quickly.
- *Maturity risk.* As we will cover in detail in the section on debt securities, the prices of longer-term bonds are more volatile than those of shorter-term bonds. Longer maturity bonds have more maturity risk than shorter-term bonds and require a maturity risk premium.

Each of these risk factors is associated with a risk premium that we add to the nominal risk-free rate to adjust for greater default risk, less liquidity, and longer maturity relative to a very liquid, short-term, default risk-free rate such as that on U.S. Treasury bills. We can write:

required interest rate on a security = nominal risk-free rate
+ default risk premium
+ liquidity premium
+ maturity risk premium

LOS 6.b: Calculate and interpret the effective annual rate, given the stated annual interest rate and the frequency of compounding.

Financial institutions usually quote rates as **stated annual interest rates**, or **nominal rates**, along with a compounding frequency, as opposed to quoting rates as *periodic rates*—the rate of interest earned over a single compounding period. For example, a bank will quote a savings rate as 8 percent, compounded quarterly, rather than 2 percent per quarter. The rate of interest that investors actually realize as a result of compounding is known as the **effective annual rate** (EAR). EAR represents the annual rate of return actually being earned *after adjustments have been made for different compounding periods*.

EAR may be determined as follows:

$$\text{EAR} = (1 + \text{periodic rate})^m - 1$$

where:
periodic rate = nominal rate/m
m = the number of compounding periods per year

Obviously, the EAR for a stated rate of 8 percent *compounded annually* is not the same as the EAR for 8 percent *compounded semiannually*, or *quarterly*. Indeed, whenever compound interest is being used, the stated (nominal) rate and the actual (effective) rate of interest are equal only when interest is compounded annually. Otherwise, the greater the compounding frequency, the greater the EAR will be in comparison to the stated rate.

The computation of EAR is necessary when comparing investments that have different compounding periods. It allows for an apples-to-apples rate comparison.

Example: Computing EAR

Compute EAR if the nominal (stated) rate is 12 percent, compounded quarterly.

Answer:

Here m = 4, so the periodic rate is $\dfrac{12}{4} = 3\%$.

Thus, EAR = $(1 + 0.03)^4 - 1 = 1.1255 - 1 = 0.1255 = 12.55\%$.

This solution uses the [y^x] key on your financial calculator. The exact keystrokes on the TI for the above computation are 1.03 [y^x] 4 [=]. On the HP, the strokes are 1.03 [ENTER] 4 [y^x].

Example: Computing EARs for a range of compounding frequencies

Using a stated rate of 6 percent, **compute** EARs for semiannual, quarterly, monthly, and daily compounding.

Answer:

EAR with:				
semiannual compounding	=	$(1 + 0.03)^2 - 1$	= 1.06090 – 1 = 0.06090	= 6.090%
quarterly compounding	=	$(1 + 0.015)^4 - 1$	= 1.06136 – 1 = 0.06136	= 6.136%
monthly compounding	=	$(1 + 0.005)^{12} - 1$	= 1.06168 – 1 = 0.06168	= 6.168%
daily compounding	=	$(1 + 0.00016438)^{365} - 1$	= 1.06183 – 1 = 0.06183	= 6.183%

Notice here that the EAR increases as the compounding frequency increases.

The limit of shorter and shorter compounding periods is called continuous compounding. To convert an annual stated rate to the effective annual rate with continuous compounding, we use the formula $e^r - 1 = EAR$.

For 6 percent, we have $e^{0.06} - 1 = 6.1837\%$. The keystrokes are 0.06 [2nd] [e^x] [–] 1 [=] 0.061837.

LOS 6.c: Solve time value of money problems when compounding periods are other than annual.

While the conceptual foundations of TVM calculations are not affected by the compounding period, more frequent compounding does have an impact on FV and PV computations. Specifically, since an increase in the frequency of compounding increases the effective rate of interest, it also *increases* the FV of a given cash flow and *decreases* the PV of a given cash flow.

Example: The effect of compounding frequency on FV and PV

Compute the FV and PV of a $1,000 single sum for an investment horizon of one year using a stated annual interest rate of 6.0 percent with a range of compounding periods.

Answer:

Figure 2: Compounding Frequency Effect

Compounding Frequency	Interest Rate per Period	Effective Rate of Interest	Future Value	Present Value
Annual (m = 1)	6.000%	6.00%	$1,060.00	$943.396
Semiannual (m = 2)	3.000	6.090	1,060.90	942.596
Quarterly (m = 4)	1.500	6.136	1,061.36	942.184
Monthly (m = 12)	0.500	6.168	1,061.68	941.905
Daily (m = 365)	0.016438	6.183	1,061.83	941.769

There are two ways to use your financial calculator to compute PVs and FVs under different compounding frequencies:

1. Adjust the number of periods per year (P/Y) mode on your calculator to correspond to the compounding frequency (e.g., for quarterly, P/Y = 4). **WE DO NOT RECOMMEND THIS APPROACH!**

2. Keep the calculator in the annual compounding mode (P/Y = 1) and enter I/Y as the interest rate per compounding period, and N as the number of compounding periods in the investment horizon. Letting *m* equal the number of compounding periods per year, the basic formulas for the calculator input data are determined as follows:
    I/Y = the annual interest rate / m
    N = the number of years × m

The computations for the FV and PV amounts in Figure 2 are:

PV_A:	FV = –1,000;	I/Y = 6 / 1 = 6;	N = 1 × 1 = 1:	CPT → PV = PV_A = 943.396
PV_S:	FV = –1,000;	I/Y = 6 / 2 = 3;	N = 1 × 2 = 2:	CPT → PV = PV_S = 942.596
PV_Q:	FV = –1,000;	I/Y = 6/4 = 1.5;	N = 1 × 4 = 4:	CPT → PV = PV_Q = 942.184
PV_M:	FV = –1,000;	I/Y = 6/12 = 0.5;	N = 1 × 12 = 12:	CPT → PV = PV_M = 941.905
PV_D:	FV = –1,000;	I/Y = 6/365 = 0.016438;	N = 1 × 365 = 365:	CPT → PV = PV_D = 941.769

FV_A:	PV = –1,000;	I/Y = 6/1 = 6;	N = 1 × 1 = 1:	CPT → FV = FV_A = 1,060.00
FV_S:	PV = –1,000;	I/Y = 6/2 = 3;	N = 1 × 2 = 2:	CPT → FV = FV_S = 1,060.90
FV_Q:	PV = –1,000;	I/Y = 6/4 = 1.5;	N = 1 × 4 = 4:	CPT → FV = FV_Q = 1,061.36
FV_M:	PV = –1,000;	I/Y = 6/12 = 0.5;	N = 1 × 12 = 12:	CPT → FV = FV_M = 1,061.68
FV_D:	PV = –1,000;	I/Y = 6/365 = 0.016438;	N = 1 × 365 = 365:	CPT → FV = FV_D = 1,061.83

Example: FV of a single sum using quarterly compounding

Compute the FV of a single $2,000 cash flow at the end of five years using an interest rate of 12 percent, compounded quarterly.

Answer:

To solve this problem, enter the relevant data and compute FV:

N = 5 × 4 = 20; I/Y = 12 / 4 = 3; PV = –$2,000; CPT → FV = $3,612.22

(See Exam Flashback #1.)

Example: FV using quarterly compounding and semiannual payments

What is the FV of four semiannual $100 payments, given a nominal interest rate of 10 percent, compounded quarterly?

Answer:

The time line for this cash flow stream is shown in Figure 3.

Figure 3: FV Using Quarterly Compounding and Semiannual Payments
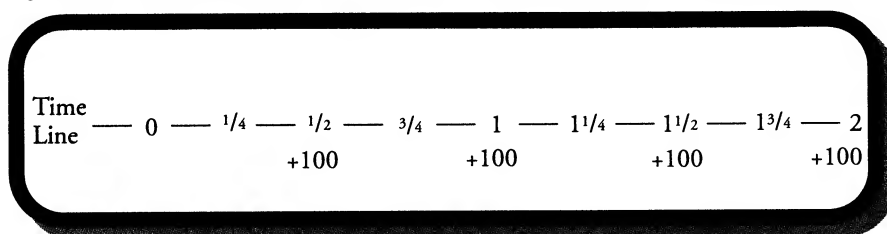
In order to solve a TVM problem using annuity techniques, the timing of the cash flows must correspond to the number of compounding periods in the year [i.e., a fixed annuity payment (receipt) will occur at the end of each compounding period]. Since m = 4 for the interest rate and m = 2 for the payments, this problem cannot be solved as an annuity. It may, however, be treated as an uneven cash flow stream.

The FV of the uneven cash flow stream described in this problem can be determined as follows:

PV = 100;	N = 6 = 3 × 2;	I/Y = 10 / 4 = 2.5%;	CPT → FV = FV_1 = 115.97
PV = 100;	N = 4 = 2 × 2;	I/Y = 10 / 4 = 2.5%;	CPT → FV = FV_2 = 110.38
PV = 100;	N = 2 = 1 × 2;	I/Y = 10 / 4 = 2.5%;	CPT → FV = FV_3 = 105.06
PV = 100;	N = 0;	I/Y = 10 / 4 = 2.5%;	CPT → FV = FV_4 = 100.00

The sum of these amounts (ΣFV) = $431.41

Note that in this example, the interest rate compounding period (quarterly) did not match the payment periods (semiannual) so we could not use our regular calculation functions for the future value of an annuity. We could convert our quarterly rate to an effective semiannual rate and then calculate the future value of the annuity. A

quarterly rate of 2.5 percent is equivalent to a semiannual rate of $1.025^2 - 1 = 5.0625$ percent. Using this as the semiannual compounding rate for the annuity yields PMT = 100, N = 4, I/Y = 5.0625, CPT → FV = −431.41. In general, when there is a mismatch between the payment frequency and the compounding frequency, we must adjust the interest rate as above. If we just double the quarterly rate to 5 percent, we would get a future value (431.01) that is too low.

LOS 6.d: Calculate the PV of a perpetuity.

A **perpetuity** is a financial instrument that pays a fixed amount of money at set intervals over an *infinite* period of time. In essence, a perpetuity is a perpetual annuity. British console bonds and most preferred stocks are examples of perpetuities since they make fixed interest or dividend payments ad infinitum. Without going into all the excruciating mathematical details, the discount factor for a perpetuity is just one divided by the appropriate rate of return (i.e., $1/r$). Given this, we can compute the PV of a perpetuity.

$$PV_{perpetuity} = \frac{PMT}{I/Y}$$

The PV of a perpetuity is the fixed periodic cash flow divided by the appropriate periodic rate of return.

As with other TVM applications, it is possible to solve for unknown variables in the $PV_{perpetuity}$ equation. In fact, you can solve for any one of the three relevant variables, given the values for the other two.

Example: PV of a perpetuity

Assume the preferred stock of Kodon Corporation pays $4.50 per year in annual dividends and plans to follow this dividend policy forever. Given an 8 percent rate of return, what is the value of Kodon's preferred stock?

Answer:

Given that the value of the stock is the PV of all future dividends, we have:

$$PV_{perpetuity} = \frac{4.50}{0.08} = \$56.25$$

Thus, if an investor requires an 8 percent rate of return, the investor should be willing to pay $56.25 for each share of Kodon's preferred stock.

Example: Rate of return for a perpetuity

Using the Kodon preferred stock described in the preceding example, **determine** the rate of return that an investor would realize if she paid $75.00 per share for the stock.

Answer:

Rearranging the equation for $PV_{perpetuity}$, we get:

$$I/Y = \frac{PMT}{PV_{perpetuity}} = \frac{4.50}{75.00} = 0.06 = 6.0\%$$

This implies that the return (yield) on a $75 preferred stock that pays a $4.50 annual dividend is 6.0 percent.

LOS 6.e: Calculate and interpret the FV and PV of a single sum of money, ordinary annuity, annuity due, or a series of uneven cash flows.

Future Value of a Single Sum

Future value is the amount to which a current deposit will grow over time when it is placed in an account paying compound interest. The FV, also called the compound value, is simply an example of compound interest at work.

The formula for the FV of a *single* cash flow is:

$$FV = PV(1 + I/Y)^N$$

where:
PV = amount of money invested today (the present value)
I/Y = rate of return per compounding period
N = total number of compounding periods

In this expression, the investment involves a single cash outflow, PV, which occurs today, at t = 0 on the time line. The single sum FV formula will determine the value of an investment at the end of N compounding periods, given that it can earn a fully compounded rate of return, I/Y, over all of the periods.

The factor $(1 + I/Y)^N$ represents the compounding rate on an investment and is frequently referred to as the *future value factor*, or the *future value interest factor*, for a single cash flow at I/Y over N compounding periods. These are the values that appear in interest factor tables, which we will not be using.

Example: FV of a single sum

Calculate the FV of a $300 investment at the end of 10 years if it earns an annually compounded rate of return of 8 percent.

Answer:

To solve this problem with your calculator, input the relevant data and compute FV.

N = 10; I/Y = 8; PV = –300; CPT → FV = $647.68

Professor's Note: Note the negative sign on PV. This is not necessary, but it makes the FV come out as a positive number. If you enter PV as a positive number, ignore the negative sign that appears on the FV.

This relatively simple problem could also be solved using the following equation.

$$FV = 300(1 + 0.08)^{10} = \$647.68$$

On the TI calculator, enter 1.08 [y^x] 10 [×] 300 [=].

(See Exam Flashback #2.)

Present Value of a Single Sum

The PV of a single sum is today's value of a cash flow that is to be received at some point in the future. In other words, it is the amount of money that must be invested today, at a given rate of return over a given period of time, in order to end up with a specified FV. As previously mentioned, the process for finding the PV of a cash flow is known as *discounting* (i.e., future cash flows are "discounted" back to the present). The interest rate used in the discounting process is commonly referred to as the *discount rate* but may also be referred to as the

opportunity cost, required rate of return, and the *cost of capital.* Whatever you want to call it, it represents the annual compound rate of return that can be earned on an investment.

The relationship between PV and FV can be seen by examining the FV expression stated earlier. Rewriting the FV equation in terms of PV, we get:

$$PV = FV \times \left[\frac{1}{(1+I/Y)^N} \right] = \frac{FV}{(1+I/Y)^N}$$

Note that for a single future cash flow, PV is always less than the FV whenever the discount rate is positive.

The quantity $1/(1 + I/Y)^N$ in the PV equation is frequently referred to as the *present value factor, present value interest factor,* or *discount factor* for a single cash flow at I/Y over N compounding periods.

Example: PV of a single sum

Given a discount rate of 9 percent, **calculate** the PV of a $1,000 cash flow that will be received in five years.

Answer:

To solve this problem, input the relevant data and compute PV.

N = 5; I/Y = 9; FV = 1,000; CPT → PV = –$649.93 (ignore the sign)

Professor's Note: With single sum PV problems, you can either enter FV as a positive number and ignore the negative sign on PV or enter FV as a negative number.

This relatively simple problem could also be solved using the following PV equation.

$$PV = \frac{1,000}{(1+0.09)^5} = \$649.93$$

On the TI, enter 1.09 [y^x] 5 [=] [1/x] [×] 1,000 [=].

The PV computed here implies that at a rate of 9 percent, an investor will be indifferent between $1,000 in five years and $649.93 today. Put another way, $649.93 is the amount that must be invested today at a 9 percent rate of return in order to generate a cash flow of $1,000 at the end of five years.

Annuities

An *annuity* is a stream of *equal cash flows* that occurs at *equal intervals* over a given period. Receiving $1,000 per year at the end of each of the next eight years is an example of an annuity. There are two types of annuities: ordinary annuities and annuities due. The *ordinary annuity* is the most common type of annuity. It is characterized by cash flows that occur at the *end* of each compounding period. This is a typical cash flow pattern for many investment and business finance applications. The other type of annuity is called an *annuity due,* where payments or receipts occur at the beginning of each period (i.e., the first payment is today at t = 0).

Computing the FV or PV of an annuity with your calculator is no more difficult than it is for a single cash flow. You will know four of the five relevant variables and solve for the fifth (either PV or FV). The difference between single sum and annuity TVM problems is that instead of solving for the PV or FV of a single cash flow, we solve for the PV or FV of a stream of equal periodic cash flows, where the size of the periodic cash flow is defined by the payment (PMT) variable on your calculator.

Example: FV of an ordinary annuity

What is the future value of an ordinary annuity that pays $150 per year at the end of each of the next 15 years, given the investment is expected to earn a 7 percent rate of return?

Answer:

This problem can be solved by entering the relevant data and computing FV.

$$N = 15; \ I/Y = 7; \ PMT = -150; \ CPT \rightarrow FV = \$3,769.35$$

Implicit here is that PV = 0, clearing the TVM functions sets.

The time line for the cash flows in this problem is depicted in Figure 4.

Figure 4: FV of an Ordinary Annuity
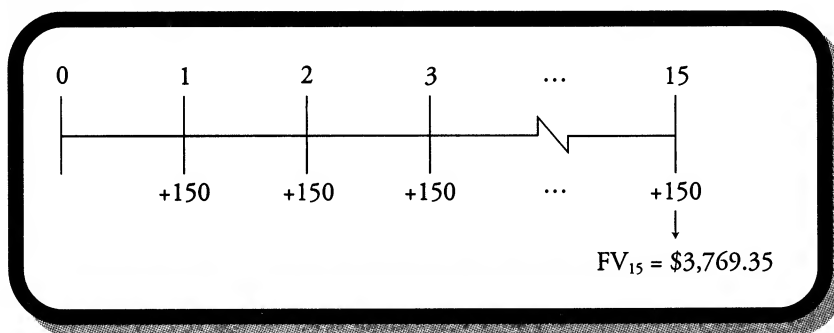
As indicated here, the sum of the compounded values of the individual cash flows in this 15-year ordinary annuity is $3,769.35. Note that the annuity payments themselves amounted to $2,250 = 15 × $150, and the balance is the interest earned at the rate of 7 percent per year.

To find the PV of an ordinary annuity, we use the future cash flow stream, PMT, that we used with FV annuity problems, but we discount the cash flows back to the present (time = 0) rather than compounding them forward to the terminal date of the annuity.

Here again, the PMT variable is a *single* periodic payment, *not* the total of all the payments (or deposits) in the annuity. The PVA_O measures the collective PV of a stream of equal cash flows received at the end of each compounding period over a stated number of periods, N, given a specified rate of return, I/Y. The following examples illustrate how to determine the PV of an ordinary annuity using a financial calculator.

Example: PV of an ordinary annuity

What is the PV of an annuity that pays $200 per year at the end of each of the next 13 years given a 6 percent discount rate?

Answer:

The payments occur at the end of the year, so this annuity is an ordinary annuity. To solve this problem, enter the relevant information and compute PV.

N = 13; I/Y = 6; PMT = –200; CPT → PV = $1,770.54

The $1,770.54 computed here represents the amount of money that an investor would need to invest *today* at a 6 percent rate of return to generate 13 end-of-year cash flows of $200 each.

Example: PV of an ordinary annuity beginning later than t = 1

What is the present value of four $100 end-of-year payments if the first payment is to be received three years from today and the appropriate rate of return is 9 percent?

Answer:

The time line for this cash flow stream is shown in Figure 5.

Figure 5: PV of an Annuity Beginning at t = 3
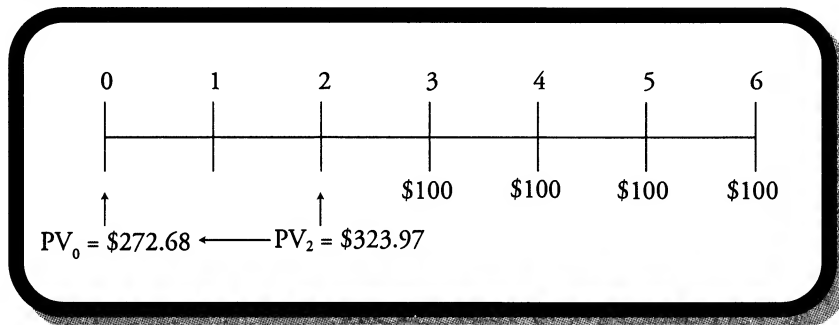
Step 1: *Find the present value of the annuity as of the end of year 2 (PV_2).*

Input the relevant data and solve for PV_2.

N = 4; I/Y = 9; PMT = –100; CPT → PV = PV_2 = $323.97

Step 2: *Find the present value of PV_2.*

Input the relevant data and solve for PV_0.

N = 2; I/Y = 9; FV = –323.97; CPT → PV = PV_0 = $272.68

In this solution, the annuity was treated as an ordinary annuity. The PV was computed one period before the first payment, and we discounted PV_2 = $323.97 over two years. We need to stress this important point. The PV annuity function on your calculator set in "END" mode gives you the value one period before the annuity begins. Although the annuity begins at t = 3, we discounted the result for only two periods to get the present (t = 0) value. *(See Exam Flashback #3.)*

Future Value of an Annuity Due

Sometimes it is necessary to find the **FV of an annuity due** (FVA_D), an annuity where the annuity payments (or deposits) occur at the beginning of each compounding period. Fortunately, our financial calculators can be used to do this, but with one slight modification—the calculator must be set to the beginning-of-period (BGN) mode.

To switch between the BGN and END modes on the TI, press [2nd] [BGN] [2nd] [SET]. When this is done, "BGN" will appear in the upper right corner of the display window. If the display indicates the desired mode, press [2nd] [QUIT]. You will normally want your calculator to be in the ordinary annuity (END) mode, so remember to switch out of BGN mode after working annuity due problems. Note that nothing appears in the upper right corner of the display window when the TI is set to the end mode. It should be mentioned that while annuity due payments are made or received at the beginning of each period, the FV of an annuity due is calculated as of the end of the last period.

Another way to compute the FV of an annuity due is to calculate the FV of an ordinary annuity, and simply multiply the resulting FV by (1 + periodic compounding rate (I/Y)). Symbolically, this can be expressed as:

$$FVA_D = FVA_O \times (1 + I/Y)$$

The following examples illustrate how to compute the FV of an annuity due.

Example: FV of an annuity due

What is the future value of an annuity that pays $100 per year at the beginning of each of the next three years, commencing today, if the cash flows can be invested at an annual rate of 10 percent? Note in the time line in Figure 6 that the FV is computed as of the end of the last year in the life of the annuity, year 3, even though the final payment occurs at the beginning of year 3 (end of year 2).

Answer:

To solve this problem, put your calculator in the BGN mode ([2nd] [BGN] [2nd] [SET] [2nd] [QUIT] on the TI or [g] [BEG] on the HP), then input the relevant data and compute FV.

N = 3; I/Y = 10; PMT = –100; CPT → FV = $364.10

Figure 6: FV of an Annuity Due
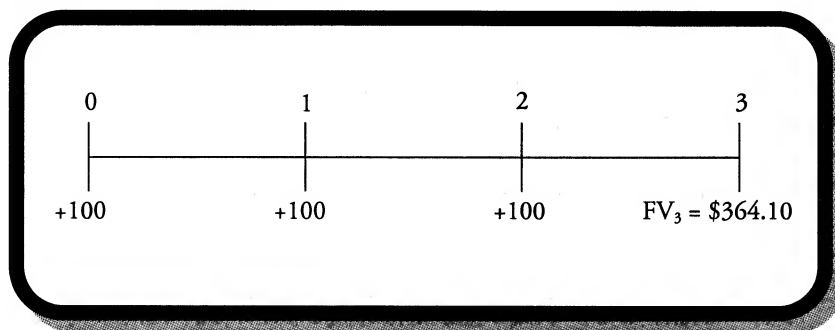
Alternatively, we could calculate the FV for an ordinary annuity and multiply it by (1 + I/Y). Leaving your calculator in the END mode, enter the following inputs:

N = 3; I/Y = 10; PMT = –100; CPT → FVA_O = $331.00

$FVA_D = FVA_O \times (1 + I/Y) = 331.00 \times 1.10 = \$364.10$

Example: FV of an annuity due

If you deposit $1,000 in the bank today and at the beginning of each of the next *three* years, how much will you have six years from today at 6 percent interest? The time line for this problem is shown in Figure 7.

Figure 7: FV for an Annuity Due
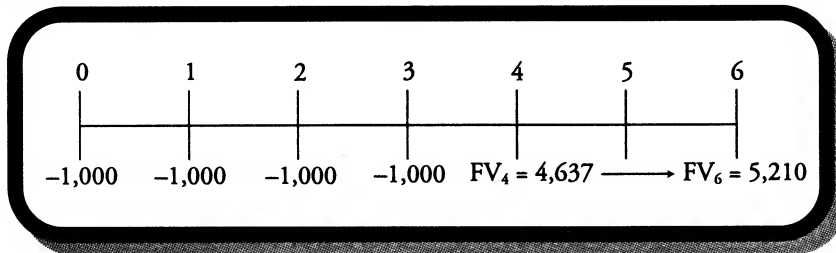
Step 1: *Compute the FV of the annuity due at the end of year 4 (FV₄).*

> Set your calculator to the annuity due (BGN) mode, enter the relevant data, and compute FV_4.

> N = 4; I/Y = 6; PMT = –1,000; CPT → FV = $4,637.09

Step 2: *Find the future value of FV₄ two years from year 4.*

> Enter the relevant data and compute FV_6.

> N = 2; I/Y = 6; PV = –4,637.09; CPT → FV =$5,210.23

(See Exam Flashbacks #4 and #5.)

Present Value of an Annuity Due

While less common than those for ordinary annuities, there may be problems (on the exam) where you have to find the **PV of an annuity due** (PVA_D). Using a financial calculator, this really shouldn't be much of a problem. With an annuity due, *there is one less discounting period* since the first cash flow occurs at t = 0 and thus is already its PV. This implies that, all else equal, the PV of an annuity due will be greater than the PV of an ordinary annuity.

As you will see in the next example, there are two ways to compute the PV of an annuity due. The first is to put the calculator in the BGN mode and then input all the relevant variables (PMT, I/Y, and N) as you normally would. The second, and far easier way, is to treat the cash flow stream as an ordinary annuity over N compounding periods, and simply multiply the resulting PV by (1 + periodic compounding rate (I/Y)). Symbolically, this can be stated as:

$$PVA_D = PVA_O \times (1 + I/Y)$$

The advantage of this second method is that you leave your calculator in the END mode and won't run the risk of forgetting to reset it. Regardless of the procedure used, the computed PV is given as of the beginning of the first period, t = 0.

Example: PV of an annuity due

Given a discount rate of 10 percent, what is the present value of a 3-year annuity that makes a series of $100 payments at the beginning of each of the next three years, starting today?

Answer:

First, let's solve this problem using the calculator's BGN mode. Set your calculator to the BGN mode ([2nd] [BGN] [2nd] [SET] [2nd] [QUIT] on the TI or [g] [BEG] on the HP), enter the relevant data, and compute PV.

$N = 3$; $I/Y = 10$; $PMT = -100$; $CPT \rightarrow PVA_D = \$273.55$

Figure 8: PV for an Annuity Due
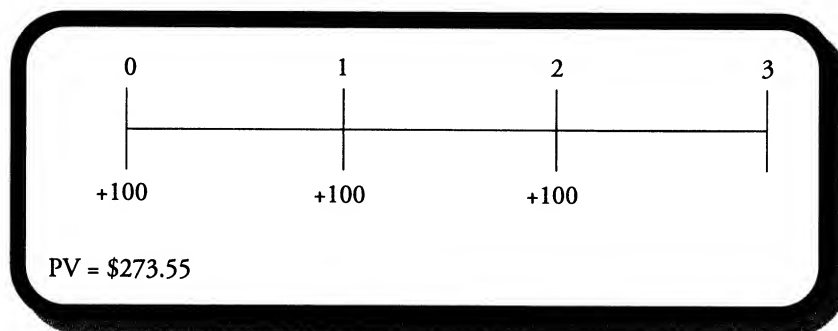
Alternatively, this problem can be solved by leaving your calculator in the END mode. First compute the PV of an ordinary 3-year annuity. Then multiply this PV by $(1 + I/Y)$. To use this approach, enter the relevant inputs and compute PV.

$N = 3$; $I/Y = 10$; $PMT = -100$; $CPT \rightarrow PVA_O = \$248.69$

$PVA_D = PVA_O \times (1 + I/Y) = \$248.69 \times 1.10 = \$273.55$

(See Exam Flashback #6.)

PV and FV of Uneven Cash Flow Series

It is not uncommon to have applications in investments and corporate finance where it is necessary to evaluate a cash flow stream that is not equal from period to period. The time line in Figure 9 depicts such a cash flow stream.

Figure 9: Time Line for Uneven Cash Flows
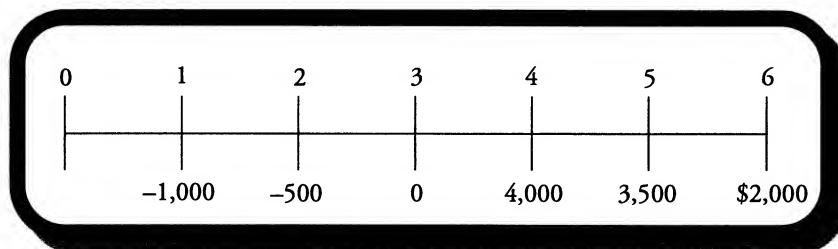
This 6-year cash flow series is not an annuity since the cash flows are different every year. In fact, there is one year with zero cash flow and two others with negative cash flows. In essence, this series of uneven cash flows is nothing more than a stream of annual single sum cash flows. Thus, to find the PV or FV of this cash flow stream, all we need to do is sum the PVs or FVs of the individual cash flows.

Example: Computing the FV of an uneven cash flow series

Using a rate of return of 10 percent, **compute** the future value of the 6-year uneven cash flow stream described above at the end of the sixth year.

Answer:

The FV for the cash flow stream is determined by first computing the FV of each individual cash flow, then summing the FVs of the individual cash flows. Note that we need to preserve the signs of the cash flows.

FV_1:   PV = –1,000;   I/Y = 10;   N = 5;   CPT → FV = FV_1 = –1,610.51

FV_2:   PV = –500;   I/Y = 10;   N = 4;   CPT → FV = FV_2 = –732.05

FV_3:   PV = 0;   I/Y = 10;   N = 3;   CPT → FV = FV_3 = 0.00

FV_4:   PV = 4,000;   I/Y = 10;   N = 2;   CPT → FV = FV_4 = 4,840.00

FV_5:   PV = 3,500;   I/Y = 10;   N = 1;   CPT → FV = FV_5 = 3,850.00

FV_6:   PV = 2,000;   I/Y = 10;   N = 0;   CPT → FV = FV_6 = 2,000.00

FV of cash flow stream = $\Sigma FV_{individual}$ = 8,347.44

Example: Computing PV of an uneven cash flow series

Compute the present value of this 6-year uneven cash flow stream described above using a 10 percent rate of return.

Answer:

This problem is solved by first computing the PV of each individual cash flow, then summing the PVs of the individual cash flows, which yields the PV of the cash flow stream. Again the signs of the cash flows are preserved.

PV_1:   FV = –1,000;   I/Y = 10;   N = 1;   CPT → PV = PV_1 = –909.09

PV_2:   FV = –500;   I/Y = 10;   N = 2;   CPT → PV = PV_2 = –413.22

PV_3:   FV = 0;   I/Y = 10;   N = 3;   CPT → PV = PV_3 = 0

PV_4:   FV = 4,000;   I/Y = 10;   N = 4;   CPT → PV = PV_4 = 2,732.05

PV_5:   FV = 3,500;   I/Y = 10;   N = 5;   CPT → PV = PV_5 = 2,173.22

PV_6:   FV = 2,000;   I/Y = 10;   N = 6;   CPT → PV = PV_6 = 1,128.95

PV of cash flow stream = $\Sigma PV_{individual}$ = $4,711.91

It is also possible to compute PV of an uneven cash flow stream by using the cash flow (CF) keys and the *net present value* (NPV) function on your calculator. This procedure is illustrated in the tables in Figures 10 and 11. In Figure 10, we have omitted the F01, F02, etc. values because they are all equal to 1. The Fn variable indicates how many times a particular cash flow amount is repeated.

Figure 10: NPV Calculator Keystrokes—TI BAII Plus®

Key Strokes	Explanation	Display
[CF] [2nd] [CLR WORK]	Clear CF Memory Registers	CF0 = 0.00000
0 [ENTER]	Initial Cash Outlay	CF0 = 0.00000
[↓] 1,000 [+/−] [ENTER]	Period 1 Cash Flow	C01 = −1,000.00000
[↓] [↓] 500 [+/−] [ENTER]	Period 2 Cash Flow	C02 = −500.00000
[↓] [↓] 0 [ENTER]	Period 3 Cash Flow	C03 =0.00000
[↓] [↓] 4,000 [ENTER]	Period 4 Cash Flow	C04 =4,000.00000
[↓] [↓] 3,500 [ENTER]	Period 5 Cash Flow	C05 =3,500.00000
[↓] [↓] 2,000 [ENTER]	Period 6 Cash Flow	C06 =2,000.00000
[NPV] 10 [ENTER]	10% Discount Rate	I = 10.00000
[↓] [CPT]	Calculate NPV	NPV = 4,711.91226

Note that the BAII Plus Professional will give the NFV of 8,347.44 also if you press the ↓ key.

Figure 11: NPV Calculator Keystrokes—HP12C®

Key Strokes	Explanation	Display
[f] [FIN] [f] [REG]	Clear Memory Registers	0.00000
0 [g] [CF$_0$]	Initial Cash Outlay	0.00000
1,000 [CHS] [g] [CF$_j$]	Period 1 Cash Flow	−1,000.00000
500 [CHS] [g] [CF$_j$]	Period 2 Cash Flow	−500.00000
0 [g] [CF$_j$]	Period 3 Cash Flow	0.00000
4,000 [g] [CF$_j$]	Period 4 Cash Flow	4,000.00000
3,500 [g] [CF$_j$]	Period 5 Cash Flow	3,500.00000
2,000 [g] [CF$_j$]	Period 6 Cash Flow	2,000.00000
10 [i]	10% Discount Rate	10.00000
[f] [NPV]	Calculate NPV	4,711.91226

LOS 6.f: Draw a time line, specify a time index, and solve problems involving the time value of money as applied, for example, to mortgages and saving for college tuition or retirement.

The Time Index

In most of the PV problems we have discussed, cash flows were discounted back to the current period. In this case, the PV is said to be indexed to t = 0, or the time index is t = 0. For example, the PV of a 3-year ordinary annuity that is indexed to t = 0 is computed at the beginning of year 1 (t = 0). Contrast this situation with another 3-year ordinary annuity that doesn't start until year 4 and extends to year 6. It would not be uncommon to want to know the PV of this annuity at the beginning of year 4, in which case the time index is t = 3. The time line for this annuity is presented in Figure 12.

Figure 12: Indexing Time Line to Other Than t = 0
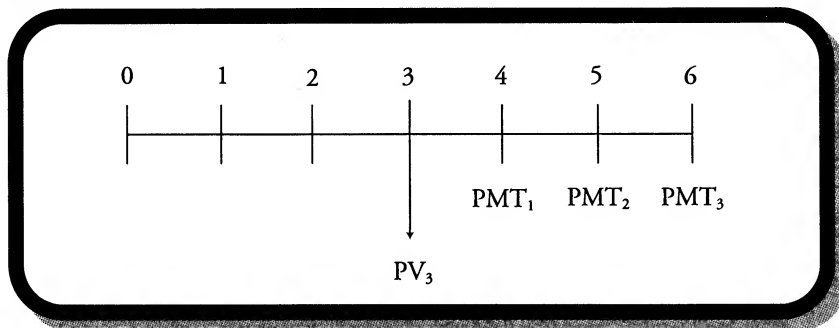
Loan Payments and Amortization

Loan amortization is the process of paying off a loan with a series of periodic loan payments, whereby a portion of the outstanding loan amount is paid off, or amortized, with each payment. When a company or individual enters into a long-term loan, the debt is usually paid off over time with a series of equal, periodic loan payments, and each payment includes the repayment of principal and an interest charge. The payments may be made monthly, quarterly, or even annually. Regardless of the payment frequency, the size of the payment remains fixed over the life of the loan. The amount of the principal and interest component of the loan payment, however, does not remain fixed over the term of the loan. Let's look at some examples to more fully develop the concept of amortization.

Example: Loan payment calculation: annual payments

A company plans to borrow $50,000 for five years. The company's bank will lend the money at a rate of 9 percent and requires that the loan be paid off in five equal end-of-year payments. **Calculate** the amount of the payment that the company must make in order to fully amortize this loan in five years.

Answer:

To determine the annual loan payment, input the relevant data and compute PMT.

$\quad$ N = 5; I/Y = 9; PV = –50,000; CPT $\rightarrow$ PMT = $12,854.62

Thus, the loan can be paid off in five equal annual payments of $12,854.62. Please note that FV = 0 in this computation; the loan will be fully paid off (amortized) after the five payments have been made.

Example: Loan payment calculation: quarterly payments

Using the loan described in the preceding example, **determine** the payment amount if the bank requires the company to make quarterly payments.

Answer:

The quarterly loan payment can be determined by inputting the relevant data and computing the payment (PMT):

N = 5 × 4 = 20; I/Y = 9 / 4 = 2.25; PV = –50,000; CPT → PMT = $3,132.10

Example: Constructing an amortization schedule

Construct an amortization schedule to show the interest and principal components of the end-of-year payments for a 10 percent, 5-year, $10,000 loan.

Answer:

The first step in solving this problem is to compute the amount of the loan payments. This is done by entering the relevant data and computing PMT:

N = 5; I/Y = 10%; PV = –$10,000; CPT → PMT = $2,637.97

Thus, the loan will be repaid via five equal $2,637.97 end-of-year payments. Each payment is made up of an interest component (profit to the lender) plus the partial recovery of loan principal, with principal recovery being scheduled so that the full amount of the loan is paid off by the end of year 5. The exact amount of the principal and interest components of each loan payment are presented and described in the amortization table shown in Figure 13.

Figure 13: Amortization Table

Period	Beginning Balance	Payment	Interest Component (1)	Principal Component (2)	Ending Balance (3)
1	$10,000.00	$2,637.97	$1,000.00	$1,637.97	$8,362.03
2	8,362.03	2,637.97	836.20	1,801.77	6,560.26
3	6,560.26	2,637.97	656.03	1,981.94	4,578.32
4	4,578.32	2,637.97	457.83	2,180.14	2,398.18
5	2,398.18	2,638.00*	239.82	2,398.18	0.00

*There is usually a slight amount of rounding error that must be recognized in the final period. The extra $0.03 associated with payment five reflects an adjustment for the rounding error and forces the ending balance to zero.

1. Interest component = beginning balance × periodic interest rate. In period 3, the interest component of the payment is $6,560.26 × 0.10 = $656.03.

2. Principal component = payment – interest. For example, the period 4 principal component is $2,637.97 – $457.83 = 2,180.14.

3. The ending balance in a given period, *t*, is the period's beginning balance minus the principal component of the payment, where the beginning balance for period *t* is the ending balance from period t – 1. For example, the period 2 ending balance equals $8,362.03 – $1,801.77 = $6,560.26, which becomes the period 3 beginning balance.

Professor's Note: Once you have solved for the payment, $2,637.97, the remaining principal on any payment date can be calculated by entering N = # of remaining payments and solving for the PV.

Example: Principal and interest component of a specific loan payment

Suppose you borrowed $10,000 at 10 percent interest to be paid semiannually over 10 years. **Calculate** the amount of the outstanding balance for the loan after the second payment is made.

Answer:

First the amount of the payment must be determined by entering the relevant information and computing the payment.

PV = –$10,000; I/Y = 10 / 2 = 5; N = 10 × 2 = 20; CPT → PMT = $802.43

The principal and interest component of the second payment can be determined using the following process:

Payment 1: Interest = ($10,000)(0.05) = $500

Principal = $802.43 – $500 = $302.43

Payment 2: Interest = ($10,000 – $302.43)(0.05) = $484.88

Principal = $802.43 – $484.88 = $317.55

Remaining balance = $10,000 – $302.43 – $317.55 = $9,380.02

The following examples will illustrate how to compute I/Y, N, or PMT in annuity problems.

Example: Computing an annuity payment needed to achieve a given FV

At an expected rate of return of 7 percent, how much must be deposited at the end of each year for the next 15 years to accumulate $3,000?

Answer:

To solve this problem, enter the three relevant known values and compute PMT.

N = 15; I/Y = 7; FV = +$3,000; CPT → PMT = –$119.38 (ignore sign)

Example: Computing a loan payment

Suppose you are considering applying for a $2,000 loan that will be repaid with equal end-of-year payments over the next 13 years. If the annual interest rate for the loan is 6 percent, how much will your payments be?

Answer:

The size of the end-of-year loan payment can be determined by inputting values for the three known variables and computing PMT.

N = 13; I/Y = 6; PV = –2,000; CPT → PMT = $225.92

Example: Computing the number of periods in an annuity

How many $100 end-of-year payments are required to accumulate $920 if the discount rate is 9 percent?

Answer:

The number of payments necessary can be determined by inputting the relevant data and computing N.

$\quad$ I/Y = 9%; FV = $920; PMT = –$100; CPT → N = 7 years

It will take seven annual $100 payments, compounded at 9 percent annually, to accrue an investment value of $920.

Professor's Note: Remember the sign convention. PMT and FV must have opposite signs or your calculator will issue an error message.

Example: Computing the number of years in an ordinary annuity

Suppose you have a $1,000 ordinary annuity earning an 8 percent return. How many annual end-of-year $150 withdrawals can be made?

Answer:

The number of years in the annuity can be determined by entering the three relevant variables and computing N.

$\quad$ I/Y = 8; PMT = 150; PV = –1,000; CPT → N = 9.9 years

Professor's Note: The HP calculator will round this to 10. This should not be a problem on the exam.

Example: Computing the rate of return for an annuity

Suppose you have the opportunity to invest $100 at the end of each of the next five years in exchange for $600 at the end of the fifth year. What is the annual rate of return on this investment?

Answer:

The rate of return on this investment can be determined by entering the relevant data and solving for I/Y.

$\quad$ N = 5; FV = $600; PMT = –100; CPT → I/Y = 9.13%

Example: Computing the discount rate for an annuity

What rate of return will you earn on an ordinary annuity that requires a $700 deposit today and promises to pay $100 per year at the end of each of the next 10 years?

Answer:

The discount rate on this annuity is determined by entering the three known values and computing I/Y.

$\quad$ N = 10; PV = –700; PMT = 100; CPT → I/Y = 7.07%

Funding a Future Obligation

There are many TVM applications where it is necessary to determine the size of the deposit(s) that must be made over a specified period in order to meet a future liability. Two common examples of this type of application are (1) setting up a funding program for future college tuition, and (2) the funding of a retirement program. In most of these applications, the objective is to determine the size of the payment(s) or deposit(s) necessary to meet a particular monetary goal.

Example: Computing the required payment to fund an annuity due

Suppose you must make five annual $1,000 payments, the first one starting at the beginning of year 4 (end of year 3). To accumulate the money to make these payments you want to make three equal payments into an investment account, the first to be made one year from today. Assuming a 10 percent rate of return, what is the amount of these three payments?

The time line for this annuity problem is shown in Figure 14.

Figure 14: Funding an Annuity Due
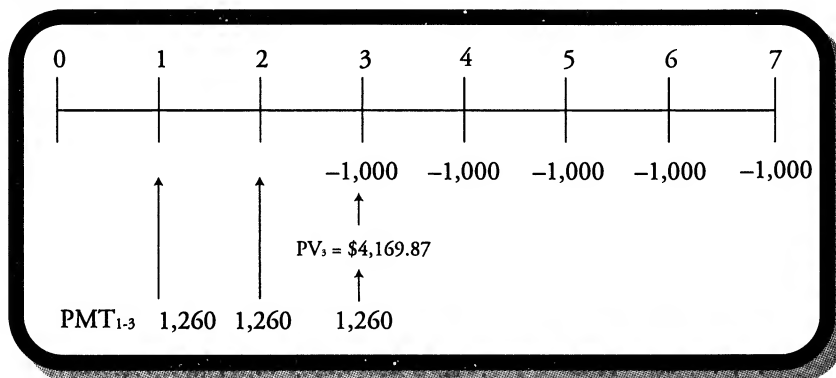
Answer:

The first step in this type of problem is to determine the amount of money that must be available at the beginning of year 4 in order to satisfy the payment requirements. This amount is the PV of a 5-year annuity due at the beginning of year 4 (end of year 3). To determine this amount, set your calculator to the BGN mode, enter the relevant data, and compute PV.

N = 5; I/Y = 10; PMT = –1,000; CPT → PV = PV_3 = $4,169.87

Alternatively, you can leave your calculator in the END mode, compute the PV of a 5-year ordinary annuity, and multiply by 1.10.

N = 5; I/Y = 10; PMT = –1,000; CPT → PV = 3,790.79 × 1.1 = PV_3 = $4,169.87

A third alternative, with the calculator in END mode, is to calculate the t = 3 value of the last four annuity payments and then add $1,000.

N = 4; I/Y = 10; PMT = –1,000; CPT → PV = 3,169.87 + 1,000 = $4,169.87 = PV_3

PV_3 becomes the FV that you need three years from today from your three equal end-of-year deposits. To determine the amount of the three payments necessary to meet this funding requirement, be sure that your calculator is in the END mode, input the relevant data, and compute PMT.

N = 3; I/Y = 10; FV = –4,169.87; CPT → PMT = $1,259.78

The second part of this problem is an ordinary annuity. If you changed your calculator to BGN mode and failed to put it back in the END mode, you will get a PMT of $1,145, which is incorrect.

Example: Funding a retirement plan

Assume a 35-year-old investor wants to retire in 25 years at the age of 60. She expects to earn 12.5 percent on her investments prior to her retirement and 10 percent thereafter. How much must she deposit at the end of each year for the next 25 years in order to be able to withdraw $25,000 per year at the beginning of each year for the 30 years from age 60 to 90?

Answer:

This is a two-step problem. First determine the amount that must be on deposit in the retirement account at the end of year 25 in order to fund the 30-year, $25,000 annuity due. Second, compute the annuity payments that must be made to achieve the required amount.

Step 1:   *Compute the amount required to meet the desired withdrawals.*

The required amount is the present value of the $25,000, 30-year annuity due at the beginning of year 26 (end of year 25). This can be determined by entering the relevant data, with the calculator in the END mode, and computing PV.

N = 29; I/Y = 10; PMT = –$25,000: CPT → PV = $234,240 (for 29 years)

Now add the first annuity payment to get $234,240 + $25,000 = $259,240. The investor will need $259,240 at the end of year 25.

Please note that we could have also performed this computation with our calculator in BGN mode as an annuity due. To do this, put your calculator in BGN mode [2nd] [BGN] [2nd] [SET] [2nd] [QUIT] on the TI or [g] [BEG] on the HP. Then enter:

N = 30; PMT = –25,000; I/Y = 10; CPT → PV = 259,240.14

If you do it this way, make certain you reset your calculator to the END mode.

Step 2:   *The annuity payment that must be made to accumulate the required amount over 25 years can be determined by entering the relevant data and computing PMT.*

N = 25; I/Y = 12.5; FV = –259,240; CPT → PMT = $1,800.02

Thus, the investor must deposit $1,800 per year at the end of each of the next 25 years in order to accumulate $259,240. With this amount she will be able to withdraw $25,000 per year for the following 30 years.

Note that all these calculations assume that the investor will earn 12.5 percent on the payments prior to retirement and 10 percent on the funds held in the retirement account thereafter.

LOS 6.g: Show and explain the connection between present values, future values, and series of cash flows.

As we have explained in the discussion of annuities and series of uneven cash flows, the sum of the present values of the cash flows is the present value of the series. The sum of the future values (at some future time = n) of a series of cash flows is the future value of that series of cash flows.

One interpretation of the present value of a series of cash flows is how much would have to be put in the bank today in order to make these future withdrawals and exhaust the account with the final withdrawal? Let's illustrate this with cash flows of $100 in year 1, $200 in year 2, $300 in year 3, and an assumed interest rate of 10 percent.

Calculate the present value of these three cash flows as:

$$\frac{100}{1.1} + \frac{200}{1.1^2} + \frac{300}{1.1^3} = \$481.59$$

If we put $481.59 in an account yielding 10 percent, at the end of the year we would have 481.59 × 1.1 = $529.75. Withdrawing $100 would leave $429.75.

Over the second year, the $429.75 would grow to 429.75 × 1.1 = $472.73. Withdrawing $200 would leave $272.73.

Over the third year, $272.73 would grow to 272.73 × 1.1 = $300, so that the last withdrawal of $300 would empty the account.

The interpretation of the future value of a series of cash flows is straightforward as well. The FV answers the question, how much would be in an account when the last of a series of deposits is made? Using the same three cash flows—$100, $200, and $300—and the same interest rate of 10 percent, we can calculate the future value of the series as:

$$100 (1.1)^2 + 200 (1.1) + 300 = \$641$$

This is simply the sum of the t = 3 value of each of the cash flows. Note that the t = 3 value and the t = 0 (present) value of the series are related by the interest rate, $481.59 (1.1)^3 = 641$.

The $100 cash flow (deposit) comes at t = 1, so it will earn interest of 10 percent compounded for two periods (until t = 3). The $200 cash flow (deposit) will earn 10 percent between t = 2 and t = 3, and the final cash flow (deposit) of $300 is made at t = 3, so $300 is the future (t = 3) value of that cash flow.

We can also look at the future value in terms of how the account grows over time. At t = 1 we deposit $100, so at time = 2 it has grown to $110 and the $200 deposit at t = 2 makes the account balance $310. Over the next period the $310 grows to 310 × 1.1 = $341 at t = 3, and the addition of the final $300 deposit puts the account balance at $641. This is, of course, the future value we calculated initially.

Professor's Note: This last view of the future value of a series of cash flows suggests a quick way to calculate the future value of an uneven cash flow series. The process described previously for the future value of a series of end-of-period payments can be written mathematically as [(100 × 1.1) + 200] × 1.1 + 300 = 641, *and this might be a quick way to do some future value problems.*

Note that questions on the future value of an *annuity due* refer to the amount in the account one period after the last deposit is made. If the three deposits considered here were made at the beginning of each period (at t = 0, 1, 2) the amount in the account at the end of three years (t = 3) would be 10 percent higher (i.e., 641 × 1.1 = $705.10).

The cash flow additivity principle refers to the fact that present value of any stream of cash flows equals the sum of the present values of the cash flows. There are different applications of this principle in time value of money problems. If we have two series of cash flows, the sum of the present values of the two series is the same as the present values of the two series taken together, adding cash flows that will be paid at the same point in time. We can also divide up a series of cash flows any way we like, and the present value of the "pieces" will equal the present value of the original series.

Example: Additivity principle

A security will make the following payments at the end of the next four years: $100, $100, $400, and $100. Calculate the present value of these cash flows using the concept of the present value of an annuity when the appropriate discount rate is 10 percent.

Answer:

We can divide the cash flows so that we have:

```
t = 1   t = 2   t = 3   t = 4

 100    100    100    100      cash flow series #1
   0      0    300      0      cash flow series #2
$100   $100   $400   $100
```

The additivity principle tells us that to get the present value of the original series, we can just add the present values of series #1 (a 4-period annuity) and series #2 (a single payment 3 periods from now).

For the annuity, N = 4, PMT = 100, FV = 0, I/Y = 10, CPT → PV = –$316.99

For the single payment: N = 3, PMT = 0, FV = 300, I/Y = 10, CPT → PV = –$225.39

The sum of these two values is 316.99 + 225.39 = $542.38.

The sum of these two (present) values is identical (except for rounding) to the sum of the present values of the payments of the original series:

$$\frac{100}{1.1} + \frac{100}{1.1^2} + \frac{400}{1.1^3} + \frac{100}{1.1^4} = \$542.38$$

KEY CONCEPTS

1. The required rate of return on a security = real risk-free rate + expected inflation + default risk premium + liquidity premium + maturity risk premium.
2. Future value: $FV = PV(1 + I/Y)^N$; present value: $PV = FV / (1 + I/Y)^N$.
3. The effective annual rate when there are m compounding periods $= \left(1 + \dfrac{\text{nominal rate}}{m}\right)^m - 1$.
4. For non-annual time value of money problems, divide the stated annual interest rate by the number of compounding periods per year, m, and multiply the number of years by the number of compounding periods per year.
5. An annuity is a series of equal cash flows that occurs at evenly spaced intervals over time.
   - Ordinary annuity cash flows occur at the end of each time period.
   - Annuity due cash flows occur at the beginning of each time period.
6. Perpetuities are annuities with infinite lives (perpetual annuities):
   $$PV_{\text{perpetuity}} = \frac{PMT}{I/Y}$$
7. A mortgage is an amortizing loan, repaid in a series of equal payments (an annuity), where each payment consists of the periodic interest and a repayment of principal.
8. The present (future) value of any series of cash flows is equal to the sum of the present (future) values of the individual cash flows.

EXAM FLASHBACKS

Required CFA Institute disclaimer: Due to CFA curriculum changes from year to year, published sample exam questions and guideline answers prior to the current year may not reflect the current curriculum.

Exam Flashback # 1
Source: Question #45 from '90 actual exam.

What is the value in five years of $100 invested today at an interest rate of 8 percent per year, compounded quarterly?
A. $144.50.
B. $146.02.
C. $148.02.
D. $148.59.

Exam Flashback # 2
Source: Question #47 from '98 sample exam.

An analyst expects a firm's earnings per share to grow at 8 percent a year. If the firm now earns $3.50 a share, its earnings per share five years from now are expected to be:
A. $4.11.
B. $4.90.
C. $5.14.
D. $5.17.

Exam Flashback # 3
Source: Question #50 from the '98 sample exam.

Doris Breen has $800,000 in her pension plan and wants to receive equal year-end payments for the next 15 years before exhausting her pension. If Doris continues to earn 9 percent on her pension, the yearly amount the pension plan will pay her will be *closest* to:
A. $53,333.
B. $58,133.
C. $91,052.
D. $99,247.

Exam Flashback # 4
Source: Question #27 from the '99–'04 sample exams.

An individual deposits $10,000 at the beginning of each of the next 10 years, starting today, into an account paying 9 percent interest compounded annually. The amount of money in the account at the end of 10 years will be *closest* to:
A. $109,000.
B. $143,200.
C. $151,900.
D. $165,600.

Exam Flashback # 5
Source: Question #20 from '92, '96 actual exams, and '97 sample exam.

An individual deposits $1,500 today and $1,500 one year from today into an interest earning account. The deposits earn 12 percent compounded annually. The total amount in the account two years from today is *closest* to:
A. $3,180.
B. $3,360.
C. $3,382.
D. $3,562.

Exam Flashback # 6
Source: Question #24 from '93, '96 actual exams, and '97 sample exams.

At an 8 percent rate of return, approximately how much must an investor have in her investment account on her 65th birthday in order that she can withdraw $30,000 on that birthday and on each of the next 19 birthdays?
A. $264,540.
B. $288,120.
C. $294,540.
D. $318,120.

CONCEPT CHECKERS: THE TIME VALUE OF MONEY

1. The amount an investor will have in 15 years if $1,000 is invested today at an annual interest rate of 9 percent will be *closest* to:
    A. $1,350.
    B. $3,518.
    C. $3,642.
    D. $9,000.

2. Fifty years ago, an investor bought a share of stock for $10. The stock has paid no dividends during this period, yet it has returned 20 percent, compounded annually, over the past 50 years. If this is true, the investment's worth is now *closest* to:
    A. $1,000.
    B. $4,550.
    C. $45,502.
    D. $91,004.

3. How much must be invested today at 0 percent to have $100 in three years?
    A. $77.75.
    B. $100.00.
    C. $126.30.
    D. $87.50.

4. How much must be invested today, at 8 percent interest, to accumulate enough to retire a $10,000 debt due seven years from today? The amount that must be invested today is *closest* to:
    A. $3,265.
    B. $5,835.
    C. $6,123.
    D. $8,794.

5. An analyst estimates that XYZ's earnings will grow from $3.00 a share to $4.50 per share over the next eight years. The rate of growth in XYZ's earnings is *closest* to:
    A. 4.9%.
    B. 5.2%.
    C. 6.7%.
    D. 7.0%.

6. If $5,000 is invested in a fund offering a rate of return of 12 percent per year, approximately how many years will it take for the investment to reach $10,000?
    A. 4 years.
    B. 5 years.
    C. 6 years.
    D. 7 years.

7. An investment is expected to produce the cash flows of $500, $200, and $800 at the end of the next three years. If the required rate of return is 12 percent, the present value of this investment is *closest* to:
   A. $835.
   B. $1,175.
   C. $1,235.
   D. $1,500.

8. Given an 8.5 percent discount rate, an asset that generates cash flows of $10 in year 1, –$20 in year 2, $10 in year 3, and is then sold for $150 at the end of year 4 has a present value of:
   A. $163.42.
   B. $150.00.
   C. $135.58.
   D. $108.29.

9. An investor has just won the lottery and will receive $50,000 per year at the end of each of the next 20 years. At a 10 percent interest rate, the present value of the winnings is *closest* to:
   A. $418,246.
   B. $425,678.
   C. $637,241.
   D. $2,863,750.

10. If $10,000 is invested today in an account that earns interest at a rate of 9.5 percent, what is the value of the equal withdrawals that can be taken out of the account at the end of each of the next five years if the investor plans to deplete the account at the end of the time period?
    A. $2,000.
    B. $2,453.
    C. $2,604.
    D. $2,750.

11. An investor is to receive a 15-year $8,000 annuity, the first payment to be received today. At an 11 percent discount rate, this annuity's worth today is *closest* to:
    A. $55,855.
    B. $57,527.
    C. $63,855.
    D. $120,000.

12. Given an 11 percent rate of return, the amount that must be put into an investment account at the end of each of the next ten years in order to accumulate $60,000 to pay for a child's education is *closest* to:
    A. $2,500.
    B. $4,432.
    C. $3,588.
    D. $6,000.

13. An investor will receive an annuity of $4,000 a year for ten years. The first payment is to be received five years from today. At a 9 percent discount rate, this annuity's worth today is *closest* to:
    A. $16,684.
    B. $18,186.
    C. $25,671.
    D. $40,000.

14. If $1,000 is invested today and $1,000 is invested at the beginning of each of the next three years at 12 percent interest (compounded annually), the amount an investor will have at the end of the fourth year will be *closest* to:
    A. $4,272.
    B. $4,779.
    C. $5,353.
    D. $6,792.

15. An investor is looking at a $150,000 home. If 20 percent must be put down and the balance is financed at 9 percent over the next 30 years, what is the monthly mortgage payment?
    A. $652.25.
    B. $799.33.
    C. $895.21.
    D. $965.55.

16. Given daily compounding, the growth of $5,000 invested for one year at 12 percent interest will be *closest* to:
    A. $5,600.
    B. $5,628.
    C. $5,637.
    D. $5,000.

17. Terry Corporation preferred stocks are expected to pay a $9 annual dividend forever. If the required rate of return on equivalent investments is 11 percent, a share of Terry preferred should be worth:
    A. $100.00.
    B. $81.82.
    C. $99.00.
    D. $122.22.

18. A share of George Co. preferred stock is selling for $65. It pays a dividend of $4.50 per year and has a perpetual life. The rate of return it is offering its investors is *closest* to:
    A. 4.5%.
    B. 6.5%.
    C. 6.9%.
    D. 14.4%.

19. If $10,000 is borrowed at 10 percent interest to be paid back over ten years, how much of the second year's payment is interest (assume annual loan payments)?
    A. $954.25.
    B. $937.26.
    C. $1,000.00.
    D. $1,037.26.

20. What is the effective annual rate for a credit card that charges 18 percent compounded monthly?
    A. 15.00%.
    B. 15.38%.
    C. 18.81%.
    D. 19.56%.

COMPREHENSIVE PROBLEMS: THE TIME VALUE OF MONEY

1. The Parks plan to take three cruises, one each year. They will take their first cruise 9 years from today, the second cruise one year after that, and the third cruise 11 years from today. A cruise of the type they will take currently costs $5,000, but they expect inflation will increase this cost by 3.5 percent per year on average. They will contribute to an account to save for these cruises that will earn 8 percent per year. What equal contributions must they make today and every year until their first cruise (10 payments) in order to have saved enough for all three cruises at that time?

2. A company's dividend in 1995 was $0.88. Over the next eight years, the dividends were $0.91, $0.99, $1.12, $0.95, $1.09, $1.25, $1.42, $1.26. Calculate the annually compounded growth rate of the dividend over the whole period.

3. An investment (a bond) will pay $1,500 at the end of each year for 25 years and on the date of the last payment will also make a separate payment of $40,000. If your required rate of return on this investment is 4 percent, how much would you be willing to pay for the bond today?

4. Your credit union offers a 2-year certificate of deposit with monthly compounding that will pay $11,180 at maturity for a $10,000 investment today. What is the stated annual rate on this CD?

5. A client has $202,971.39 in an account that earns 8 percent per year, compounded monthly. The client's 35th birthday was yesterday and she will retire when the account value is $1 million.
   A. How old will she be when she can retire if she puts no more money in the account?
   B. How old will she be when she can retire if she puts $250/month into the account every month beginning one month from today?

Answers – Exam Flashbacks: The Time Value of Money

1. **C** Given PV, FV is determined as follows: PV = –3.50; N = 5; I/Y = 8; CPT → FV = 5.14. Note in this problem that the rate of compounding is a growth rate instead of the usual interest rate.

2. **D** A stated rate of 8% compounded quarterly is equivalent to 2% per quarter. The FV after 20 quarters (five years) is determined as follows:

    N = (5)(4) = 20; I/Y = 8 / 4 = 2; PV = –100; CPT → FV = $148.59

3. **D** Solve for PMT as follows: PV = –800,000; N = 15; I/Y = 9; CPT → PMT = $99,247.

    Here we have solved for the annuity that is equivalent to a specified present value.

4. **D** Put calculator in BGN mode to solve.

    Annuity due PMT = –10,000, N = 10, I/Y = 9, CPT → FV = $165,603

    Be sure to reset your calculator to the end-of-period mode when you are done with annuity due problems.

    Alternatively, leave your calculator in END mode and perform the following steps:

    N = 10, PMT = –10,000, I/Y = 9, CPT → FV = FVA_O = $151,929

    FVA_D = FVA_O × (1 + I/Y) = $151,929 × 1.09 = $165,603

5. **D** FV on an annuity due problem. Remember that your answer is one year further out in the future than the last PMT. So PMT_1 is now, PMT_2 is one year from now, and FV is two years from now.

    Method 1: Set your calculator on the BGN mode.

    N = 2; I/Y = 12; PMT = –1,500; compute FVA_D = $3,561.60

    Remember to take your calculator out of the BGN mode.

    Alternatively, leave your calculator in END mode and perform the following steps:

    N = 2; I/Y = 12; PMT = –1,500; CPT → FVA_O = $3,180.00

    FVA_D = FVA_O × (1 + I/Y) = $3,180.00 × 1.12 = $3,561.60

    Professors Note: Read carefully for beginning and end payments. Draw a time line.

6. **D** This is an annuity due since the first payment (withdrawal) occurs on the date on which the PV is to be determined. Set your calculator on the BGN mode and proceed as follows:

    N = 20; I/Y = 8; PMT = –30,000; CPT → PV = $318,107.98 ≈ $318,120

    Alternatively, leave your calculator in the END mode and perform the following steps:

    N = 20; I/Y = 8; PMT = –30,000, CPT → PV = $294,544

    PVA_D = PVA_O × (1 + I/Y) = $294,544 × 1.08 = $318,107.98 ≈ $318,120

ANSWERS – CONCEPT CHECKERS: THE TIME VALUE OF MONEY

1. **C** N = 15; I/Y = 9; PV = –1,000; PMT = 0; CPT → FV = $3,642.48

2. **D** N = 50; I/Y = 20; PV = –10; PMT = 0; CPT → FV = $91,004.38

3. **B** Since no interest is earned, $100 is needed today to have $100 in three years.

4. **B** N = 7; I/Y = 8; FV = –10,000; PMT = 0; CPT → PV = $5,834.90

5. **B** N = 8; PV = –3; FV = 4.50; PMT = 0; CPT → I/Y = 5.1989

6. **C** PV = –5,000; I/Y = 12; FV = 10,000; PMT = 0; CPT → N = 6.12. Rule of 72 → 72/12 = six years.

    Note to HP12C users: One known problem with the HP12C is that it does not have the capability to round. In this particular question, you will come up with 7, although the correct answer is 6.1163. CFA Institute is aware of this problem, and hopefully you will not be faced with a situation like this on exam day (e.g., having to choose between two choices being so close together, like 6 and 7).

7. **B** Using your cash flow keys, CF_0 = 0; CF_1 = 500; CF_2 = 200; CF_3 = 800; I/Y = 12; NPV = $1,175.29.

    Or you can add up the present values of each single cash flow.

    PV_1 = N = 1; FV = –500; I/Y = 12; CPT → PV = 446.43
    PV_2 = N = 2; FV = –200; I/Y = 12; CPT → PV = 159.44
    PV_3 = N = 3; FV = –800; I/Y = 12; CPT → PV = 569.42

    Hence, 446.43 + 159.44 + 569.42 = $1,175.29.

8. **D** Using your cash flow keys, CF_0 = 0; CF_1 = 10; CF_2 = –20; CF_3 = 10; CF_4 = 150; I/Y = 8.5; NPV = $108.29.

9. **B** N = 20; I/Y = 10; PMT = –50,000; FV = 0; CPT → PV = $425,678.19

10. **C** PV = –10,000; I/Y = 9.5; N = 5; FV = 0; CPT → PMT = $2,604.36

11. **C** This is an annuity due. Switch to BGN mode.

    N = 15; PMT = –8,000; I/Y = 11; FV = 0; CPT → PV = 63,854.92. Switch back to END mode.

12. **C** N = 10; I/Y = 11; FV = –60,000; PV = 0; CPT → PMT = $3,588.08

13. **B** Two steps: (1) Find the PV of the 10-year annuity: N = 10; I/Y = 9; PMT = –4,000; FV = 0; CPT → PV = 25,670.63. This is the present value as of the end of year 4; (2) Discount PV of the annuity back four years: N = 4; PMT = 0; FV = –25,670.63; I/Y = 9; CPT → PV = 18,185.72.

14. **C** The key to this problem is to recognize that it is a 4-year annuity due, so switch to BGN mode: N = 4; PMT = –1,000; PV = 0; I/Y = 12; CPT → FV = 5,352.84. Switch back to END mode.

15. **D** N = 30 × 12 = 360; I/Y = 9 / 12 = 0.75; PV = –150,000(1 – 0.2) = –120,000; FV = 0; CPT → PMT = $965.55

16. **C** N = 1 × 365 = 365; I/Y = 12 / 365 = 0.0328767; PMT = 0; PV = –5,000; CPT → FV = $5,637.37

17. **B** 9 / 0.11 = $81.82

18. **C** 4.5 / 65 = 0.0692 or 6.92%

19. **B** To get the annual payment, enter PV = –10,000; FV = 0; I/Y = 10; N = 10; CPT → PMT = 1,627.45. The first year's interest is $1,000 = 10,000 × 0.10, so the principal balance going into year 2 is 10,000 – 627.45 = $9,372.55. Year 2 interest = $937.26 = $9,372.55 × 0.10.

20. **D** $\quad$ EAR $= (1 + 0.18/12)^{12} - 1 = 19.56\%$

ANSWERS – COMPREHENSIVE PROBLEMS: THE TIME VALUE OF MONEY

1.   One way to solve this is:

$$\text{cost of first cruise} = 5,000 \times 1.035^9$$
$$= 6,814.49$$
$$\text{PV of first cruise cost} = \frac{6,814.49}{(1.08)^9} = \$3,408.94$$

$$\text{PV of second cruise cost} = \frac{(1.035)^{10}}{(1.08)^{10}} \times 5,000 = \$3,266.90$$

$$\text{PV of third cruise cost} = \left(\frac{1.035}{1.08}\right)^{11} \times 5,000 = \$3,130.78$$

PV of all three = 3,408.94 + 3,266.90 + 3,130.78 = \$9,806.62. This is the amount needed in the account today so it's the PV of a 10-payment annuity due, solve for payment at 8% = \$1,353.22.

2.   Don't make easy questions hard. The dividend grew from \$0.88 to \$1.26 in eight years. $\left(\frac{1.26}{0.88}\right)^{\frac{1}{8}} - 1 = 4.589\%$.

You could also just enter $\frac{1.26}{0.88}$, press $\left[\sqrt{}\right]$ three times, get 1.045890 and see that the answer is 4.589%.

3.   By the additivity principle, we can take the present value of the payments (a regular annuity) and the present value of the \$40,000 (lump sum) and add them together. N = 25, PMT = –1,500, i = 4, CPT $\rightarrow$ PV = 23,433.12 and

$40,000 \times \left(\frac{1}{1.04}\right)^{25} = 15,004.67$, for a total value of \$38,437.79.

Alternatively, N = 25, PMT = –1,500, i = 4, FV = –40,000, CPT $\rightarrow$ PV = 38,437.79.

4.   Stated annual rate will be 12 times the monthly rate of compounding. We can solve for this rate with PV = –10,000, FV = 11,180, N = 24, PMT = 0, CPT $\rightarrow$ I/Y = 0.465837% × 12 = 5.59% APR. You could also get the

monthly compounded rate from $\left(\frac{11,180}{10,000}\right)^{\frac{1}{24}} - 1$ and multiply by 12.

5.   A. PV = –202,971.39, I/Y = 8/12 = 0.6667, PMT = 0, FV = 1,000,000, CPT $\rightarrow$ N = 240. 240 months is 20 years; she will be 55 years old.
        B. Don't clear TVM functions. PMT = –250, CPT $\rightarrow$ N = 220, which is 18.335 years, so she will be 53.

DISCOUNTED CASH FLOW APPLICATIONS

EXAM FOCUS

This topic review has a mix of topics but all are important because of their usefulness and the certainty that some if not all of these topics will be on the exam. You must be able to use the cash flow functions on your calculator to calculate NPV and IRR. We will use both of these in the Corporate Finance section and examine their strengths and weaknesses more closely there; but you must learn how to calculate them here.

The time-weighted and money-weighted return calculations are standard tools for analysis. Calculating the various yield measures and the ability to calculate one from another are must-have skills. Don't hurry here, these concepts and techniques are foundation material and will turn up repeatedly at all three levels of the CFA® curriculum.

LOS 7.a: Calculate and interpret the net present value (NPV) and the internal rate of return (IRR) of an investment.

The **net present value** (NPV) of an investment project is the present value of expected cash inflows associated with the project less the present value of the project's expected cash outflows, discounted at the appropriate cost of capital. The following procedure may be used to compute NPV.

- Identify all costs (outflows) and benefits (inflows) associated with an investment.
- Determine the appropriate discount rate or opportunity cost for the investment.
- Using the appropriate discount rate, find the PV of each cash flow. Inflows are positive and increase NPV. Outflows are negative and decrease NPV.
- Compute the NPV, the sum of the DCFs.

Mathematically, NPV is expressed as:

$$NPV = \sum_{t=0}^{N} \frac{CF_t}{(1+r)^t}$$

where:
CF_t = the expected net cash flow at time t
N = the estimated life of the investment
r = the discount rate = opportunity cost of capital

NPV is the PV of the cash flows less the initial (time = 0) outlay.

Example: Computing NPV

Calculate the NPV of an investment project with an initial cost of $5 million and positive cash flows of $1.6 million at the end of year 1, $2.4 million at the end of year 2, and $2.8 million at the end of year 3. Use 12 percent as the discount rate.

Answer:

The NPV for this project is the sum of the PVs of the project's individual cash flows and is determined as follows:

$$NPV = -\$5.0 + \frac{\$1.6}{1.12} + \frac{\$2.4}{(1.12)^2} + \frac{\$2.8}{(1.12)^3}$$

$$= -\$5.0 + \$1.42857 + \$1.91327 + \$1.99299$$

$$= \$0.3348 \text{ million, or } \$334,800$$

The procedures for calculating NPV with a TI BAII Plus® and an HP12C® hand-held financial calculator are presented in Figures 1 and 2.

Figure 1: Calculating NPV with the TI Business Analyst II Plus®

Key Strokes	Explanation	Display
[CF] [2nd] [CLR WORK]	Clear CF Memory Registers	CF0 = 0.00000
5 [+/–] [ENTER]	Initial Cash Outlay	CF0 = –5.00000
[↓]1.6 ENTER]	Period 1 Cash Flow	C01 = 1.60000
[↓] [↓] 2.4 [ENTER]	Period 2 Cash Flow	C02 = 2.40000
[↓] [↓] 2.8 [ENTER]	Period 3 Cash Flow	C03 = 2.80000
[NPV] 12 [ENTER]	12% discount rate	I =12.00000
[↓] [CPT]	Calculate NPV	NPV = 0.33482

Figure 2: Calculating NPV with the HP12C®

Key Strokes	Explanation	Display
[f] [FIN] [f] [REG]	Clear Memory Registers	0.00000
5 [CHS] [g] [CF$_0$]	Initial Cash Outlay	–5.00000
1.6 [g] [CF$_j$]	Period 1 Cash flow	1.60000
2.4 [g] [CF$_j$]	Period 2 Cash flow	2.40000
2.8 [g] [CF$_j$]	Period 3 Cash flow	2.80000
12 [i]	12% discount rate	12.00000
[f] [NPV]	Calculate NPV	0.33482

On the TI BAII Plus calculator, the sequence of [↓][↓] scrolls past the variables F01, F02, etc. The F here stands for frequency, and this is set to 1 by default. We did not enter anything because each cash flow amount occurred only once. If the period 2 cash flow, C02, were repeated three times (at t = 2, 3, and 4) we could input F02 = 3 to account for all three of these. The next input, C03 would then refer to the cash flow at t = 5.

On the HP 12C, you can also account for a repeated cash flow amount. Referring to the above example for the TI BAII Plus, the fact that the cash flow at period 2 is repeated for three periods is indicated by the sequence 3 [g] Nj immediately after the CF$_j$ keystroke to input the amount of the period 2 cash flow.

Professor's Note: The NPV function can also be used to find the present value of any series of cash flows (positive or negative) over future periods. Just set $CF_0 = 0$ and input the cash flows CF_1 through CF_N as outlined above. The NPV is the present value of these cash flows since there is now no initial negative cash flow (initial cost).

The **internal rate of return** (IRR) is defined as the rate of return that equates the PV of an investment's expected benefits (inflows) with the PV of its costs (outflows). Equivalently, the IRR may be defined as the discount rate for which the NPV of an investment is zero.

The procedure for calculating IRR requires only the identification of the relevant cash flows for the investment opportunity being evaluated. Market-determined discount rates, or any other external (market-driven) data, are not necessary with the IRR procedure. The general formula for the IRR is:

$$0 = CF_0 + \frac{CF_1}{1+IRR} + \frac{CF_2}{(1+IRR)^2} + \cdots + \frac{CF_N}{(1+IRR)^N}$$

In the majority of IRR applications to capital budgeting, the initial cash flow, CF_0, represents the initial cost of the investment opportunity, and is therefore a negative value. As such, any discount rate less than the IRR will result in a positive NPV, and a discount rate greater than the IRR will result in a negative NPV. This implies that the NPV of an investment is zero when the discount rate used equals the IRR.

Example: Computing IRR

What is the IRR for the investment described in the preceding example?

Answer:

Substituting the investment's cash flows into the previous IRR equation results in the following equation:

$$0 = -5.0 + \frac{1.6}{1+IRR} + \frac{2.4}{(1+IRR)^2} + \frac{2.8}{(1+IRR)^3}$$

Solving this equation yields an IRR = 15.52%.

It is possible to solve IRR problems through a trial and error process. That is, keep guessing IRRs until you get the one that provides an NPV equal to zero. Practically speaking, a financial calculator or an electronic spreadsheet can and should be employed. The procedures for computing IRR with the TI BA II Plus and HP12C financial calculators are illustrated in Figures 3 and 4, respectively.

Figure 3: Calculating IRR with the TI Business Analyst II Plus®

Key Strokes	Explanation	Display
[CF] [2nd] [CLR WORK]	Clear Memory Registers	CF0 = 0.00000
5 [+/−] [ENTER]	Initial Cash Outlay	CF0 = −5.00000
[↓] 1.6 [ENTER]	Period 1 Cash Flow	C01 = 1.60000
[↓] [↓] 2.4 [ENTER]	Period 2 Cash Flow	C02 = 2.40000
[↓] [↓] 2.8 [ENTER]	Period 3 Cash Flow	C03 = 2.80000
[IRR] [CPT]	Calculate IRR	IRR = 15.51757

Figure 4: Calculating IRR with the HP12C®

Key Strokes	Explanation	Display
[f] [FIN] [f] [REG]	Clear Memory Registers	0.00000
5 [CHS] [g] [CF$_0$]	Initial Cash Outlay	−5.00000
1.6 [g] [CF$_j$]	Period 1 Cash flow	1.60000
2.4 [g] [CF$_j$]	Period 2 Cash flow	2.40000
2.8 [g] [CF$_j$]	Period 3 Cash flow	2.80000
[f] [IRR]	Calculate IRR	15.51757

LOS 7.b: Contrast the NPV rule to the IRR rule.

NPV decision rule. The basic idea behind NPV analysis is that if a project has a positive NPV, this amount goes to the firm's shareholders. As such, if a firm undertakes a project with a positive NPV, shareholder wealth is increased.

The NPV decision rules are summarized:

- Accept projects with a positive NPV. Positive NPV projects will increase shareholder wealth.
- Reject projects with a negative NPV. Negative NPV projects decrease shareholder wealth.
- When two projects are mutually exclusive (only one can be accepted), the project with the higher positive NPV should be accepted.

IRR decision rule. Analyzing an investment (project) using the IRR method provides the analyst with a result in terms of a rate of return.

The following are decision rules of IRR analysis:

- Accept projects with an IRR that is greater than the firm's (investor's) required rate of return.
- Reject projects with an IRR that is less than the firm's (investor's) required rate of return.

Note that for a single project, the IRR and NPV rules lead to exactly the same accept/reject decision. If the IRR is greater than the required rate of return, the NPV is positive, and if the IRR is less than the required rate of return, the NPV is negative.

LOS 7.c: Discuss problems associated with the IRR method.

When the acceptance or rejection of one project has no effect on the acceptance or rejection of another, the two projects are considered to be independent projects. When only one of two projects may be accepted, the projects are considered to be mutually exclusive. For mutually exclusive projects, the NPV and IRR methods can give conflicting project rankings. This can happen when the projects' initial costs are of different sizes or when the timing of the cash flows is different. Let's look at an example that illustrates how NPV and IRR can yield conflicting results.

Example: Conflicting decisions between NPV and IRR

Assume NPV and IRR analysis of two mutually exclusive projects produced the results shown in Figure 5. As indicated, the IRR criteria recommends that Project A should be accepted. On the other hand, the NPV criteria indicates acceptance of Project B. Which project should be selected?

Figure 5: Ranking Reversals with NPV and IRR

Project	Investment at t = 0	Cash Flow at t = 1	IRR	NPV at 10%
A	–$5,000	$8,000	60%	$2,272.72
B	–$30,000	$40,000	33%	$6,363.64

Answer:

Investing in project A increases shareholder wealth by $2,272.72, while investing in project B increases shareholder wealth by $6,363.64. Since the overall goal of the firm is to maximize shareholder wealth, project B should be selected because it adds the most value to the firm.

Mathematically speaking, the NPV method assumes the reinvestment of a project's cash flows at the opportunity cost of capital, while the IRR method assumes that the reinvestment rate is the IRR. The discount rate used with the NPV approach represents the market-based opportunity cost of capital and is the required rate of return for the shareholders of the firm.

Given that shareholder wealth maximization is the ultimate goal of the firm, always *select the project with the greatest NPV when the IRR and NPV rules provide conflicting decisions.*

LOS 7.d: Calculate, interpret, and distinguish between the money-weighted and time-weighted rates of return of a portfolio and appraise the performance of portfolios based on these measures.

The **money-weighted return** applies the concept of internal rate of return (IRR) to investment portfolios. The money-weighted rate of return is defined as the internal rate of return on a portfolio, taking into account all cash inflows and outflows. The beginning value of the account is an inflow as are all deposits into the account. All withdrawals from the account are outflows, as is the ending value.

Example: Money-weighted rate of return

Assume an investor buys a share of stock for $100 at t = 0 and at the end of the next year (t = 1), she buys an additional share for $120. At the end of year 2, the investor sells both shares for $130 each. At the end of each year in the holding period, the stock paid a $2.00 per share dividend. What is the money-weighted rate of return?

Step 1:   Determine the timing of each cash flow and whether the cash flow is an inflow (+) or an outflow (–).

        t = 0: purchase of first share   = +$100.00

        t = 1: dividend from first share   =   –$2.00
                purchase of second share = +$120.00
                Subtotal, t = 1   +$118.00

        t = 2: dividend from two shares   =   –$4.00
                proceeds from selling shares = –$260.00
                Subtotal, t = 2   –$264.00

Step 2: Net the cash flows for each time period and set the PV of cash inflows equal to the present value of cash outflows.

$$PV_{inflows} = PV_{outflows}$$

$$\$100 + \frac{\$118}{(1+r)} = \frac{\$264}{(1+r)^2}$$

Step 3: Solve for *r* to find the money-weighted rate of return. This can be done using trial and error or by using the IRR function on a financial calculator or spreadsheet.

The intuition here is that we deposited $100 into the account at t = 0, then added $118 to the account at t = 1 (which, with the $2 dividend, funded the purchase of one more share at $120), and ended with a total value of $264.

To compute this value with a financial calculator, use these net cash flows and follow the procedure(s) described in Figures 6 or 7 to calculate the IRR.

Net cash flows: $CF_0 = -100$; $CF_1 = -120 + 2 = -118$; $CF_2 = 260 + 4 = 264$

Figure 6: Calculating Money-Weighted Return With the TI Business Analyst II Plus®

Key Strokes	Explanation	Display
[CF] [2nd][CLR WORK]	Clear Cash Flow Registers	CF0 = 0.00000
100 [+/–] [ENTER]	Initial Cash Outlay	CF0 =–100.00000
[↓] 118 [+/–] [ENTER]	Period 1 Cash Flow	C01 =–118.00000
[↓] [↓] 264 [ENTER]	Period 2 Cash Flow	C02 =264.00000
[IRR] [CPT]	Calculate IRR	IRR = 13.86122

Figure 7: Calculating Money-Weighted Return With the HP12C®

Key Strokes	Explanation	Display
[f] [FIN] [f] [REG]	Clear Memory Registers	0.00000
100 [CHS] [g] [CF$_0$]	Initial Cash Outlay	–100.00000
118 [CHS] [g] [CF$_j$]	Period 1 Cash Flow	–118.00000
264 [g] [CF$_j$]	Period 2 Cash Flow	264.00000
[f] [IRR]	Calculate IRR	13.86122

The money-weighted rate of return for this problem is 13.86 percent.

Time-weighted rate of return measures compound growth. It is the rate at which $1.00 compounds over a specified performance horizon. Time-weighting is the process of averaging a set of values over time. The *annual* time-weighted return for an investment may be computed by performing the following steps:

Step 1:   Value the portfolio immediately preceding significant addition or withdrawals. Form subperiods over the evaluation period that correspond to the dates of deposits and withdrawals.

Step 2:   Compute the holding period return (HPR) of the portfolio for each subperiod.

Step 3:   Compute the product of (1 + HPR) for each subperiod to obtain a total return for the entire measurement period [i.e., $(1 + HPR_1) \times (1 + HPR_2) \ldots (1 + HPR_n)$]. If the total investment period is greater than one year, you must take the geometric mean of the measurement period return to find the annual time-weighted rate of return.

Example: Time-weighted rate of return

A share of stock is purchased at t = 0 for $100, and at the end of the next year, t = 1, another share is purchased for $120. At the end of year 2, both shares are sold for $130 each. At the end of both years 1 and 2, the stock paid a $2.00 per share dividend. What is the time-weighted rate of return for this investment? (This is the same investment as the preceding example.)

Answer:

Step 1:   Break the evaluation period into two subperiods based on timing of cash flows.

         Holding period 1:   Beginning price = $100.00
                                    Dividends paid   = $2.00
                                      Ending price   = $120.00

         Holding period 2:   Beginning price = $240.00 (2 shares)
                                      Dividends paid   = $4.00 ($2 per share)
                                      Ending price   = $260.00 (2 shares)

Step 2:   Calculate the HPR for each holding period.

         $HPR_1 = [(\$120 + 2) / \$100] - 1 = 22\%$
         $HPR_2 = [(\$260 + 4) / \$240] - 1 = 10\%$

Step 3:   Take the geometric mean of the annual returns to find the annualized time-weighted rate of return over the measurement period.

         $(1 + \text{time-weighted rate of return})^2 = (1.22)(1.10)$
         time-weighted rate of return $= [(1.22)(1.10)]^{0.5} - 1 = 15.84\%$

In the investment management industry, *the time-weighted rate of return is the preferred method of performance measurement, because it is not affected by the timing of cash inflows and outflows.*

In the preceding examples, the time-weighted rate of return for the portfolio was 15.84 percent, while the money-weighted rate of return for the same portfolio was 13.86 percent. The difference in the results is attributable to the fact that the procedure for determining the money-weighted rate of return gave a larger weight to the year 2 HPR, which was 10 percent versus the 22 percent HPR for year 1.

If funds are contributed to an investment portfolio just before a period of relatively poor portfolio performance, the money-weighted rate of return will tend to be depressed. On the other hand, if funds are contributed to a portfolio at a favorable time, the money-weighted rate of return will increase. The use of the time-weighted return removes these distortions and thus provides a better measure of a manager's ability to select investments over the period. If a private investor has complete control over money flows into and out of an account, the money-weighted rate of return may be the more appropriate performance measure.

LOS 7.e: Calculate and interpret the bank discount yield, holding period yield, effective annual yield, and money market yield for a U.S. Treasury bill.

Pure discount instruments such as U.S. T-bills are quoted differently from U.S. government bonds. T-bills are quoted on a *bank discount basis*, which is *based on the face value* of the instrument instead of the purchase price. The **bank discount yield** is computed using the following formula:

$$r_{BD} = \frac{D}{F} \times \frac{360}{t}$$

where:
r_{BD} = the annualized yield on a bank discount basis
D = the dollar discount, which is equal to the difference between the face value of the bill and the purchase price
F = the face value (par value) of the bill
t = number of days remaining until maturity
360 = bank convention of number of days in a year

The *key distinction* of the bank discount yield is that it expresses the dollar discount from the face (par) value as a fraction of the face value, not the market price of the instrument. Another notable feature of the bank discount yield is that it is annualized by multiplying the discount-to-par by 360/t, where the market convention is to use a 360-day year versus a 365-day year. This type of annualizing method assumes no compounding (i.e., simple interest).

Example: Bank discount yield

Calculate the bank discount yield for a T-bill priced at $98,500 with a face value of $100,000 and 120 days until maturity.

Answer:

Substituting the relevant values into the bank discount yield equation in our example, we get:

$$r_{BD} = \frac{1,500}{100,000} \times \frac{360}{120} = 4.50\%$$

It is important for candidates to realize that a yield quoted on a bank discount basis is not representative of the return earned by an investor for the following reasons:

- Bank discount yield annualizes using simple interest and ignores the effects of compound interest.
- Bank discount yield is based on the face value of the bond, not its purchase price—investment returns should be evaluated relative to the amount invested.
- Bank discount yield is annualized based on a 360-day year rather than a 365-day year.

Holding period yield (HPY) or holding period return, is the total return an investor earns between the purchase date and the sale or maturity date. HPY is calculated using the following formula:

$$HPY = \frac{P_1 - P_0 + D_1}{P_0} = \frac{P_1 + D_1}{P_0} - 1$$

where:

P_0 = initial price of the instrument

P_1 = price received for instrument at maturity

D_1 = interest payment (distribution)

Example: HPY

What is the holding period yield for a T-bill priced at $98,500 with a face value of $100,000 and 120 days remaining until maturity?

Answer:

Using the HPY equation stated above, we have:

HPY = ($100,000 – $98,500) / $98,500

= $1,500 / $98,500

= 1.5228%

D_1 = 0 here because T-bills are a pure discount instrument (i.e., they make no interest payments).

The **effective annual yield** (EAY) is an annualized value, based on a 365-day year that accounts for compound interest. It is calculated using the following equation:

$$EAY = (1 + HPY)^{365/t} - 1$$

Example: EAY

Compute the EAY using the HPY of 1.5228 percent from the previous example.

Answer:

The HPY is converted to an EAY as follows:

$$EAY = (1.015228)^{365/120} - 1 = 1.047042 - 1 = 4.7042\%$$

Note that we can convert from an EAY to HPY by using the reciprocal of the exponent and

$$(1.047042)^{\frac{120}{365}} - 1 = 1.5228\%.$$

The **money market yield** is equal to the annualized holding period yield, *assuming a 360-day year.* Using the money market yield makes the quoted yield on a T-bill comparable to yield quotes for interest-bearing money market instruments that pay interest on a 360-day basis. The money market yield is $\frac{360}{t} \times HPY$.

Given the bank discount yield, r_{BD}, the money market yield, r_{MM}, may be calculated using the equation:

$$r_{MM} = \frac{360 \times r_{BD}}{360 - \left(t \times r_{BD}\right)}$$

This expression is the result of converting r_{BD} to its price-based HPY and reannualizing the HPY.

Given *HPY,* r_{MM} may be calculated directly as follows:

$$r_{MM} = HPY \times (360/t)$$

Example: Money market yield, r_{MM}

What is the money market yield for a 120-day T-bill that has a bank discount yield equal to 4.50 percent?

Answer:

Given the r_{BD} for the T-bill, the first equation for r_{MM} is applied as follows:

$$r_{MM} = \frac{360 \times 0.045}{360 - (120 \times 0.045)}$$
$$= 16.2 \ / \ 354.6$$
$$= 4.569\%$$

Alternatively, we could first calculate the HPY for the T-bill and then annualize that. Actual discount is $0.045 \times \dfrac{120}{360} = 0.015$. Based on a $1,000 face value, price = 1,000 (1 − 0.015) = 985 so that $HPY = \dfrac{1,000}{985} - 1$

and $r_{MM} = \left(\dfrac{1,000}{985} - 1\right) \times \dfrac{360}{120} = 0.04569 = 4.569\%$.

LOS 7.f: **Convert and interpret among holding period yields, money market yields, and effective annual yields.**

Once we have established HPY, EAY, or r_{MM}, we can use one as a basis for calculating the other two. Remember:

- The HPY is the actual return an investor will receive if the money market instrument is held until maturity.
- The EAY is the annualized HPY on the basis of a 365-day year and incorporates the effects of compounding.
- The r_{MM} is the annualized yield that is based on price and a *360-day year* and does not account for the effects of compounding—it assumes simple interest.

Example: Converting among EAY, HPY and r_{MM}

Assume you purchased a T-bill that matures in 150 days for a price of $98,000. The broker who sold you the T-bill quoted the money market yield at 4.898 percent. **Compute** the HPY and the EAY.

Answer:

Money market to holding period yield—r_{MM} is an annualized yield based on a 360-day year. To change the r_{MM} in this example into its HPY, we need to convert it to a 150-day holding period by multiplying it by (150 / 360). Thus:

$$HPY = r_{MM} \times (150 / 360)$$
$$= 0.04898 \times (150 / 360)$$
$$= 0.02041 = 2.041\%$$

Money market yield to effective annual yield—the EAY is equal to the annualized HPY based on a 365-day year. Now that we have computed the HPY, simply annualize using a 365-day year to calculate the EAY as follows:

$$EAY = (1 + 0.02041)^{365/150} - 1$$
$$= 1.05039 - 1 = 5.039\%$$

Note that to convert the EAY back into the HPY, apply the reciprocal of the exponent to the EAY. This is the same as taking one plus the EAY to the power (t/365). For example, we can convert the EAY we just calculated back to the HPY as follows:

$$HPY = (1.05039)^{150/365} - 1 = 2.041\%$$

Professor's Note: On the Level 1 CFA® exam, you may be asked to convert from any one of these three yields into one of the others. You should note that the EAY and r_{MM} are merely annualized versions of the HPY. If you concentrate on converting back and forth between the HPY and the other yield figures, you will be well-prepared to answer these types of questions on the Level 1 exam.

LOS 7.g: Calculate and interpret the bond equivalent yield.

The **bond-equivalent yield** refers to 2 × the semiannual discount rate. This convention stems from the fact that yields on U.S. bonds are quoted as twice the semiannual rate because the annual coupon interest is paid in two semiannual payments.

Example: Bond-equivalent yield calculation (1)

A 3-month loan has a holding period yield of 2 percent. What is the yield on a bond-equivalent basis?

Answer:

The first step is to convert the 3-month yield to a semiannual (6-month) yield:

$$1.02^2 - 1 = 4.04\%$$

The second step it to double it (2 × 4.04 = 8.08%) to get the bond-equivalent yield.

Example: Bond-equivalent yield calculation (2)

The effective annual yield on an investment is 8 percent. What is the yield on a bond-equivalent basis?

Answer:

The first step is to convert the effective annual yield to a semiannual yield.

$$1.08^{0.5} - 1 = 3.923\%$$

The second step is to double it: $2 \times 3.923 = 7.846\%$.

KEY CONCEPTS

1. The NPV is the present value of future cash flows, discounted at the firm's cost of capital, less the project's cost. IRR is the discount rate that makes the NPV = 0 (equates the PV of the expected future cash flows to the project's initial cost).

2. The NPV rule is to accept a project if NPV > 0; the IRR rule is to accept a project if IRR > required rate of return. For an independent (single) project, these rules produce the exact same decision.

3. For mutually exclusive projects, IRR rankings and NPV rankings may differ due to differences in project size or in the timing of the cash flows. Choose the project with the higher NPV.

4. The money-weighted rate of return is the IRR calculated with end-of-period account values and is also the discount rate that makes the PV of cash inflows equal to the PV of cash outflows.

5. The time-weighted rate of return is calculated as the geometric mean of the compound holding period returns.

6. The bank discount yield is the percentage discount from face value, annualized by multiplying by

$$\frac{360}{\text{days to maturity}}, \quad r_{BD} = \frac{D}{F} \times \frac{360}{\text{days}}.$$

7. The holding period yield is calculated as:

$$HPY = \frac{P_1 - P_0 + D_1}{P_0} = \frac{P_1 + D_1}{P_0} - 1$$

8. The effective annual yield converts a t-day holding period yield to a compound annual yield based on a 365-day year:

$$EAY = (1 + HPY)^{365/t} - 1$$

9. A money market yield is annualized (without compounding) based on a 360-day year:

$$r_{MM} = HPY \times \frac{360}{t}$$

10. The bond equivalent yield is two times the effective semiannual rate of return.

11. To convert a bank discount yield to a money market yield, the calculation is:

$$r_{MM} = \frac{360 \times r_{BD}}{360 - (t \times r_{BD})}$$

CONCEPT CHECKERS: DISCOUNTED CASH FLOW APPLICATIONS

1. Which of the following statements does **NOT** accurately describe the internal rate of return (IRR) and net present value (NPV) methods?
   A. The discount rate that gives an investment an NPV of zero is the investment's IRR.
   B. The IRR is the discount rate that equates the present value of cash inflows with the present value of cash outflows.
   C. If the NPV and IRR methods give conflicting decisions for mutually exclusive projects, the IRR decision should be used to select the project.
   D. The NPV method assumes that a project's cash flows will be reinvested at the cost of capital, while the IRR method assumes they will be reinvested at the IRR.

2. Which of the following statements does **NOT** accurately describe the IRR and NPV methods?
   A. A project's IRR can be positive even if the NPV is negative.
   B. A project with an IRR equal to the cost of capital will have an NPV of zero.
   C. A project's NPV may be positive even if the IRR is less than the cost of capital.
   D. Shareholder wealth will decrease if a company takes on a project with a negative NPV.

3. Which of the following statements does **NOT** accurately describe the IRR and/or NPV methods?
   A. The NPV tells how much the value of the firm has increased if you accept the project.
   B. When evaluating independent projects, the IRR and NPV methods always yield the same accept/reject decisions.
   C. When selecting between mutually exclusive projects, the NPV and IRR methods may give conflicting accept/reject decisions.
   D. When selecting between mutually exclusive projects, the project with the highest NPV should be accepted regardless of the sign of the NPV calculation.

4. A company is considering entering into a joint venture that will require an investment of $10 million. The project is expected to generate cash flows of $4 million, $3 million, and $4 million in each of the next three years, respectively. Assuming a discount rate of 10 percent, what is the project's NPV?
   A. −$879,000.
   B. −$309,000.
   C. +$243,000.
   D. −$1,523,000.

5. A company is considering entering into a joint venture that will require an investment of $10 million. The project is expected to generate cash flows of $4 million, $3 million, and $4 million in each of the next three years, respectively. Assuming a discount rate of 10 percent, what is the project's *approximate* IRR?
   A. 5%.
   B. 10%.
   C. 15%.
   D. 20%.

6. What should an analyst recommend based on the following information for two independent projects?

Project	Investment at t = 0	Cash Flow at t = 1	IRR	NPV at 12%
A	−$3,000	$5,000	66.67%	$1,464.29
B	−$10,000	$15,000	50.00%	$3,392.86

    A. Reject A and reject B.
    B. Accept A and reject B.
    C. Reject A and accept B.
    D. Accept A and accept B.

7. What should an analyst recommend based on the following information for two mutually exclusive projects?

Project	Investment at t = 0	Cash Flow at t = 1	IRR	NPV at 12%
A	−$3,000	$5,000	66.67%	$1,464.29
B	−$10,000	$15,000	50.00%	$3,392.86

    A. Reject A and reject B.
    B. Accept A and reject B.
    C. Reject A and accept B.
    D. Accept A and accept B.

8. Goodeal Inc. is considering the purchase of a new material handling system for a cost of $15 million. This system is expected to generate a positive cash flow of $1.8 million per year in perpetuity. What is the NPV of the proposed investment if the appropriate discount rate is 10.5 percent?
    A. $2,142,857.
    B. $13,200,000.
    C. $16,575,000.
    D. $17,142,857.

9. Goodeal Inc. is considering the purchase of a new material handling system for a cost of $15 million. This system is expected to generate a positive cash flow of $1.8 million per year in perpetuity. What is the IRR of the proposed investment if the appropriate hurdle rate is 10.5 percent?
    A. 8.33%.
    B. 10.5%.
    C. 12.0%.
    D. 17.1%.

10. Should a company accept a project that has an IRR of 14 percent and an NPV of $2.8 million if the cost of capital is 12 percent?
    A. Yes, based only on the NPV.
    B. Yes, based only on the IRR.
    C. Yes, based on the NPV and the IRR.
    D. No, based on both the NPV and IRR.

11. Which of the following statements does **NOT** represent a characteristic of the time-weighted rate of return? It is:
    A. not affected by the timing of cash flows.
    B. used to measure the compound rate of growth of $1 over a stated measurement period.
    C. the preferred method of performance measurement in the investment management industry.
    D. defined as the internal rate of return on an investment portfolio, taking into account all inflows and outflows.

Use the following data to answer Questions 12 and 13.

Assume you purchase a share of stock for $50 at time t = 0, and another share at $65 at time t = 1, and at the end of year 1 and year 2, the stock paid a $2.00 dividend. Also, at the end of year 2 you sold both shares for $70 each.

12. The money-weighted rate of return on the investment is:
    A. 15.45%.
    B. 18.02%.
    C. 16.73%.
    D. 15.79%.

13. The time-weighted rate of return on the investment is:
    A. 21.83%.
    B. 18.04%.
    C. 20.13%.
    D. 18.27%.

14. What is the bank discount yield for a U.S. Treasury bill (T-bill) that is selling for $99,000, with a face value of $100,000, and 95 days remaining until maturity?
    A. 1.51%.
    B. 4.50%.
    C. 3.79%.
    D. 6.00%.

15. What is the holding period yield for a T-bill that is selling for $99,000 if it has a face value of $100,000 and 95 days remaining until maturity?
    A. 2.03%.
    B. 3.48%.
    C. 3.79%.
    D. 1.01%.

16. What is the effective annual yield for a T-bill that is selling for $99,000 if it has a face value of $100,000 and 95 days remaining until maturity?
    A. 3.94%.
    B. 1.01%.
    C. 4.50%.
    D. 3.79%.

17. What is the money market yield for a T-bill that is selling for $99,000 if it has a face value of $100,000 and 95 days remaining until maturity?
    A. 3.90%.
    B. 3.83%.
    C. 3.79%.
    D. 7.52%.

18. Which of the following is **FALSE** regarding a bank discount yield?
    A. It is based on a 360 rather than a 365-day year.
    B. It ignores the opportunity to earn compound interest.
    C. It is based on the face value of the bond, not its purchase price.
    D. It reflects the nonannualized return an investor will earn over a security's life.

19. A 175-day Treasury bill has an effective annual yield of 3.80%. Its money-market yield is *closest* to:
    A. 3.80%.
    B. 3.71%.
    C. 3.65%.
    D. 1.80%.

20. Consider the following two statements:
    1) The holding period yield is greater than the bank discount yield.
    2) The money market yield is less than the bank discount yield.
    A. Statement 1) must be true and statement 2) must be false.
    B. Statement 1) may be true and statement 2) must be false.
    C. Statement 1) must be false and statement 2) may be true.
    D. Statement 1) must be false and statement 2) must be false.

ANSWERS – CONCEPT CHECKERS: DISCOUNTED CASH FLOW APPLICATIONS

1. **C** If the NPV and IRR methods give conflicting decisions when selecting among mutually exclusive projects, always select the project with the greatest positive NPV.

2. **C** A project will have a negative NPV if its IRR is less than the firm's cost of capital.

3. **D** When selecting between two mutually exclusive projects, neither project should be accepted if they both have a negative NPV.

4. **A** NPV = $4 / 1.10 + 3 / (1.10)^2 + 4 / (1.10)^3 - \$10 = -\$0.879038$ million, or –$879,038.

    Calculator approach: CF0 = –10; CF1 = 4; CF2 = 3; CF3 = 4; I = 10 → NPV = –$0.879038 (million).

5. **A** Use your test-taking skills here. You know from the previous question that the NPV is negative at 10%. Thus, the IRR must be less than 10% in order to increase the PV of the cash inflows. This leaves only choice A to be the answer. Calculator solution: IRR = 4.9%.

6. **D** Both projects should be accepted since both projects have a positive NPV and will thus increase shareholder wealth.

7. **C** When the NPV and IRR rules conflict, always select the project with the highest positive NPV in order to maximize shareholder wealth.

8. **A** NPV = PV(cash inflows) – CF_0 = (1.8 million / 0.105) – $15 million = $2,142,857. Accept the project.

9. **C** As a perpetuity, the following relationship applies: $1.8 million / IRR = $15 million. Thus, IRR = 1.8 / 15 = 12%. Since IRR > cost of capital (hurdle rate), accept the project.

10. **C** The project should be accepted on the basis of its positive NPV and its IRR, which exceeds the cost of capital.

11. **D** The money-weighted rate of return is the IRR of an investment's net cash flows.

12. **B** One way to do this problem is to set up the cash flows so that the PV of inflows = PV of outflows and plug in each of the multiple choices. $50 + 65 / (1 + r) = 2 / (1 + r) + 144 / (1 + r)^2$ → r = 18.02%. Or on your financial

    calculator, solve for IRR: $-50 - \dfrac{65-2}{1+\text{IRR}} + \dfrac{2(70+2)}{(1+\text{IRR})^2} = 0$.

Calculating Money-Weighted Return With the TI Business Analyst II Plus®		
Key Strokes	Explanation	Display
[CF] [2nd] [CLR WORK]	Clear CF Memory Registers	CF0 = 0.00000
50 [+/–] [ENTER]	Initial cash inflow	CF0 =–50.00000
[↓] 63 [+/–][ENTER]	Period 1 cash inflow	C01 =–63.00000
[↓] [↓] 144 [ENTER]	Period 2 cash outflow	C02 =144.00000
[IRR] [CPT]	Calculate IRR	IRR = 18.02210

13. **A** HPR_1 = (65 + 2) / 50 – 1= 34%, HPR_2 = (140 + 4) / 130 – 1 = 10.77%

    Time-weighted return = $[(1.34)(1.1077)]^{0.5} - 1 = 21.83\%$

14. **C** $(1{,}000 / 100{,}000) \times (360 / 95) = 3.79\%$

15. **D** $(100{,}000 - 99{,}000) / 99{,}000 = 1.01\%$

16. **A** $(1 + 0.0101)^{365/95} - 1 = 3.94\%$

17. **B** $(360 \times 0.0379) / (360 - (95 \times 0.0379)) = 3.83\%$, or $(1{,}000 / 99{,}000)(360 / 95) = 3.83\%$

18. **D**   This is actually the definition of the holding period yield. All of the other answers are true statements regarding the bank discount yield.

19. **B**   Since the effective yield is 3.8%, we know $\left[\dfrac{1{,}000}{\text{price}}\right]^{\frac{365}{175}} = 1.038$ and $\text{price} = \left[\dfrac{1{,}000}{1.038^{\frac{175}{365}}}\right] = \$982.28$ per $1{,}000 face.

The money market yield is $\left(\dfrac{360}{175}\right) \times \text{HPY} = \left(\dfrac{360}{175}\right)\left(\dfrac{1{,}000}{982.28} - 1\right) = \dfrac{360}{175}(0.01804) = 3.711\%$.

Alternatively, we can get the HPY from the EAY of 3.8% as $(1.038)^{\frac{175}{365}} - 1 = 1.804\%$.

20. **B**   Statement 1 can be true or false. For a 360-day T-bill it is certainly true because $\dfrac{\text{discount}}{\text{face} - \text{discount}} > \dfrac{\text{discount}}{\text{face}}$. For a 30-day bill, the (unannualized) holding period yield will be less than the (annualized) bank discount yield.

Statement 2 must be false since $\left(\dfrac{\text{discount}}{\text{face} - \text{discount}}\right)\left(\dfrac{360}{\text{days}}\right) > \left(\dfrac{\text{discount}}{\text{face}}\right)\left(\dfrac{360}{\text{days}}\right)$.

STATISTICAL CONCEPTS AND MARKET RETURNS

EXAM FOCUS

This quantitative review is about the uses of descriptive statistics to summarize and portray important characteristics of large sets of data. The two key areas that you should concentrate on are (1) measures of central tendency and (2) measures of dispersion. Measures of central tendency include the arithmetic mean, geometric mean, weighted mean, median, and mode. Measures of dispersion include the range, mean absolute deviation, variance, and standard deviation. These measures quantify the variability of data around its "center." When describing investments, measures of central tendency provide an indication of an investment's expected reward. Measures of dispersion indicate the riskiness of an investment. For the Level 1 exam, you should know the properties of a normal distribution and be able to assess the effects of departures from normality, such as lack of symmetry (skewness) or the extent to which a distribution is peaked (kurtosis).

LOS 8.a: Describe the nature of statistics and differentiate between descriptive statistics and inferential statistics and between a population and a sample.

LOS 8.b: Explain the concepts of a parameter and a sample statistic.

The word **statistics** is used to refer to data (e.g., the average return on XYZ stock was 8 percent over the last 10 years) and to the methods we use to analyze data. Statistical methods fall into one of two categories, *descriptive statistics* or *inferential statistics*.

Descriptive statistics are used to summarize the important characteristics of large data sets. The focus of this topic review is on the use of descriptive statistics to consolidate a mass of numerical data into useful information.

Inferential statistics, which will be discussed in subsequent topic reviews, pertain to the procedures used to make forecasts, estimates, or judgments about a large set of data on the basis of the statistical characteristics of a smaller set (a sample).

A **population** is defined as the set of all possible members of a stated group. A cross-section of the returns of all of the stocks traded on the New York Stock Exchange (NYSE) is an example of a population. A measure used to describe a characteristic of a population is referred to as a **parameter**. While many population parameters exist, investment analysis usually utilizes just a few, particularly the mean return and the standard deviation of returns.

It is frequently too costly or time consuming to obtain measurements for every member of a population, if it is even possible. In this case, a sample may be used. A **sample** is defined as a subset of the population of interest. Once a population has been defined, a sample can be drawn from the population, and the sample's characteristics can be used to describe the population as a whole. For example, a sample of 30 stocks may be selected from among all of the stocks listed on the NYSE to represent the population of all NYSE-traded stocks. In the same manner that a parameter may be used to describe a characteristic of a population, a **sample statistic** is used to measure a characteristic of a sample.

LOS 8.c: Explain the differences among the types of measurement scales.

Different statistical methods use different levels of measurement, or measurement scales. **Measurement scales** may be classified into one of four major categories:

- *Nominal scales.* Nominal scales are the least accurate level of measurement. Observations are classified or counted with no particular order. An example would be assigning the number 1 to a municipal bond fund, the number 2 to a corporate bond fund, and so on for each fund style.
- *Ordinal scales.* Ordinal scales represent a higher level of measurement than nominal scales. When working with an ordinal scale, every observation is assigned to one of several categories. Then these categories are ordered with respect to a specified characteristic. For example, the ranking of 1,000 small cap growth stocks by performance may be done by assigning the number 1 to the 100 best performing stocks, the number 2 to the next 100 best performing stocks, and so on to the assignment of the number 10 to the 100 worst performing stocks. Based on this type of measurement, it can be concluded that a stock ranked 3 is better than a stock ranked 4, but the scale reveals nothing about performance differences or whether the difference between a 3 and a 4 is the same as the difference between a 4 and a 5.
- *Interval scale.* Interval scale measurements provide relative ranking, like ordinal scales, plus the assurance that differences between scale values are equal. Temperature measurement in degrees is a prime example. Certainly, 49°C is hotter than 32°C, and the temperature difference between 49°C and 32°C is the same as the difference between 67°C and 50°C. The weakness of the interval scale is that a measurement of zero does not necessarily indicate the total absence of what we are measuring. This means that interval-scale-based ratios are meaningless. For example, 30°F is not three times as hot as 10°F.
- *Ratio scales.* Ratio scales represent the most refined level of measurement. Ratio scales provide ranking and equal differences between scale values, and they also have a true zero point as the origin. The measurement of money is a good example. If you have zero dollars, you have no purchasing power, but if you have $4.00, you have twice as much purchasing power as a person with $2.00.

LOS 8.e: Define, calculate, and interpret a holding period return (total return).

Throughout the rest of this topic review, we will apply descriptive statistics to investment returns. The analysis of investment returns usually starts with the measurement of total return over a given holding period, or simply, the **holding period return** (HPR). HPR measures the total return from holding an investment over a period of time and can be calculated using the following formula:

$$HPR_t = \frac{P_t - P_{t-1} + D_t}{P_{t-1}} = \frac{P_t + D_t}{P_{t-1}} - 1$$

where:
HPR_t = HPR over the period t = 0 to t = t
P_t = price per share (or other investment unit) at the end of time period t
P_{t-1} = price per share at the end of time period t − 1, the time period immediately preceding time period t
D_t = cash distributions received during time period t

Note that for common stocks, the cash distribution is the dividend. For bonds, the cash distribution is the coupon payment. The HPR for a given investment can be calculated for any time period (days, weeks, months, or years) simply by changing the end points of the time interval over which values and cash flows are measured.

Example: HPR

What is the HPR for a stock that is currently selling for $60 per share if it was purchased exactly one year ago for $50 and paid a $2.00 dividend during the year?

Answer:

Substituting the appropriate values into the HPR equation, we get:

$$R_t = \frac{\$60 - \$50 + \$2}{\$50} = 0.24, \text{ or } \frac{60 + 2}{50} - 1 = 0.24, \text{ which is } 24\%$$

LOS 8.d: Define and interpret a frequency distribution.

A **frequency distribution** is a tabular presentation of statistical data that aids the analysis of large data sets. Frequency distributions summarize statistical data by assigning it to specified groups, or intervals. Also, the data employed with a frequency distribution may be measured using any type of measurement scale.

Professor's Note: Intervals are also known as classes.

The following procedure describes how to **construct a frequency distribution**.

Step 1: *Define the intervals.* The first step in building a frequency distribution is to define the intervals to which data measurements (observations) will be assigned. An interval, also referred to as a class, is the set of values that an observation may take on. The range of values for each interval must have a lower and upper limit and be all-inclusive and nonoverlapping. Intervals must be *mutually exclusive* in a way that each observation can be placed in only one interval, and the total set of intervals should cover the total range of values for the entire population.

The number of intervals used is an important consideration. If too few intervals are used, the data may be too broadly summarized, and important characteristics may be lost. On the other hand, if too many intervals are used, the data may not be summarized enough.

Step 2: *Tally the observations.* After the intervals have been defined, the observations must be tallied, or assigned to their appropriate interval.

Step 3: *Count the observations.* Having tallied the data set, the number of observations that are assigned to each interval must be counted. The *absolute frequency*, or simply the frequency, is the actual number of observations that fall within a given interval.

Example: Constructing a frequency distribution

Use the data in Figure 1 to **construct** a frequency distribution for the returns on Intelco's common stock.

Figure 1: Annual Returns for Intelco, Inc. Common Stock

10.4%	22.5%	11.1%	−12.4%
9.8%	17.0%	2.8%	8.4%
34.6%	−28.6%	0.6%	5.0%
−17.6%	5.6%	8.9%	40.4%
−1.0%	−4.2%	−5.2%	21.0%

Answer:

Step 1: *Defining the interval.* For Intelco's stock, the range of returns is 69.0 percent (−28.6 percent → 40.4 percent). Using a return interval of 1 percent would result in 69 separate intervals, which in this case is too many. So let's use eight nonoverlapping intervals with a width of 10 percent. The

lowest return intervals will be $-30\% \le R_t < -20\%$, and the intervals will increase to $40\% \le R_t \le 50\%$.

Step 2: *Tally the observations and count the observations within each interval.* The tallying and counting of the observations is presented in Figure 2.

Figure 2: Tally and Interval Count for Returns Data

Interval	Tallies	Absolute Frequency
$-30\% \le R_t < -20\%$	/	1
$-20\% \le R_t < -10\%$	//	2
$-10\% \le R_t < 0\%$	///	3
$0\% \le R_t < 10\%$	///// //	7
$10\% \le R_t < 20\%$	///	3
$20\% \le R_t < 30\%$	//	2
$30\% \le R_t < 40\%$	/	1
$40\% \le R_t < 50\%$	/	1
Total		20

Tallying and counting the observations generates a frequency distribution that summarizes the pattern of annual returns on Intelco common stock. Notice that the interval with the greatest (absolute) frequency is the ($0\% \le R_t < 10\%$) interval, which includes seven return observations. For any frequency distribution, the interval with the greatest frequency is referred to as the *modal interval*.

LOS 8.f: Calculate and interpret relative frequencies and cumulative relative frequencies, given a frequency distribution.

The **relative frequency** is another useful way to present data. The relative frequency is calculated by dividing the absolute frequency of each return interval by the total number of observations. Simply stated, relative frequency is the percentage of total observations falling within each interval. Continuing with our example, the relative frequencies are presented in Figure 3.

Figure 3: Relative Frequencies

Interval	Frequency	Relative Frequency
$-30\% \le R_t < -20\%$	1	1/20 = 0.05, or 5%
$-20\% \le R_t < -10\%$	2	2/20 = 0.10, or 10%
$-10\% \le R_t < 0\%$	3	3/20 = 0.15, or 15%
$0\% \le R_t < 10\%$	7	7/20 = 0.35, or 35%
$10\% \le R_t < 20\%$	3	3/20 = 0.15, or 15%
$20\% \le R_t < 30\%$	2	2/20 = 0.10, or 10%
$30\% \le R_t < 40\%$	1	1/20 = 0.05, or 5%
$40\% \le R_t < 50\%$	1	1/20 = 0.05, or 5%
Total	20	100%

It is also possible to compute the **cumulative absolute frequency** and **cumulative relative frequency** by summing the absolute or relative frequencies starting at the lowest interval and progressing through the highest. The cumulative absolute frequencies and cumulative relative frequencies for the Intelco stock returns example are presented in Figure 4.

Figure 4: Cumulative Frequencies

Interval	Absolute Frequency	Relative Frequency	Cumulative Absolute Frequency	Cumulative Relative Frequency
$-30\% \leq R_t < -20\%$	1	5%	1	5%
$-20\% \leq R_t < -10\%$	2	10	3	15
$-10\% \leq R_t < 0\%$	3	15	6	30
$0\% \leq R_t < 10\%$	7	35	13	65
$10\% \leq R_t < 20\%$	3	15	16	80
$20\% \leq R_t < 30\%$	2	10	18	90
$30\% \leq R_t < 40\%$	1	5	19	95
$40\% \leq R_t < 50\%$	1	5	20	100
Total	20	100%		

Notice that the cumulative absolute frequency or cumulative relative frequency for any given interval is the sum of the absolute or relative frequencies up to and including the given interval. For example, the cumulative absolute frequency for the $(0\% \leq R_t < 10\%)$ interval is $13 = 1 + 2 + 3 + 7$ and the cumulative relative frequency for this interval is $5\% + 10\% + 15\% + 35\% = 65\%$.

LOS 8.g: Describe the properties of data presented as a histogram or a frequency polygon.

A **histogram** is the graphical presentation of the absolute frequency distribution. A histogram is simply a bar chart of continuous data that has been classified into a frequency distribution. The attractive feature of a histogram is that it allows us to quickly see where most of the observations are concentrated.

To construct a histogram, the intervals are scaled on the horizontal axis and the absolute frequencies are scaled on the vertical axis. The histogram for the relative frequency data in Figure 3 is provided in Figure 5.

Figure 5: Histogram of Stock Return Data
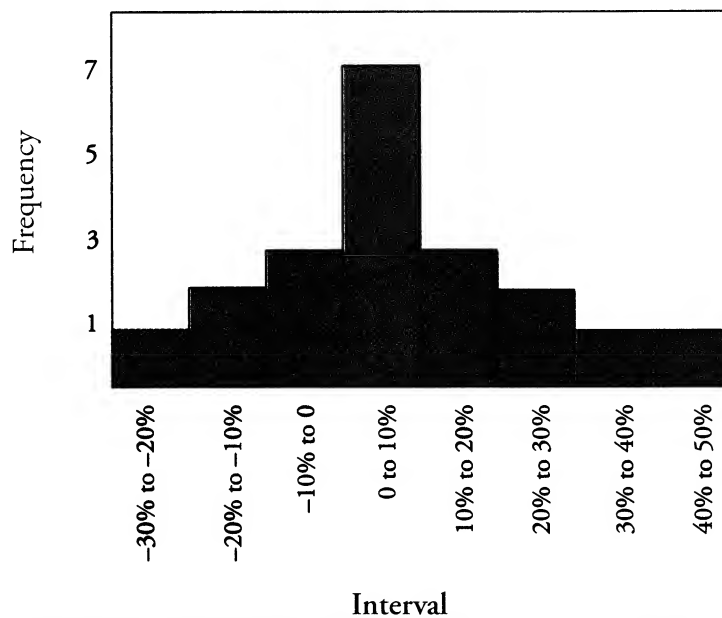
To construct a **frequency polygon**, the midpoint of each interval is plotted on the horizontal axis, and the absolute frequency for that interval is plotted on the vertical axis. Each point is then connected with a straight line. The frequency polygon for the returns data used in our example is in Figure 6.

Figure 6: Frequency Polygon of Stock Return Data
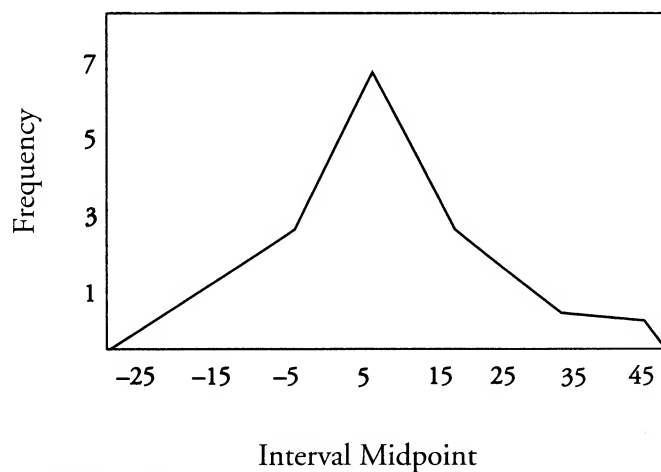
LOS 8.i: Describe and interpret quartiles, quintiles, deciles, and percentiles.

Quantile is the general term for a value at or below which a stated proportion of the data in a distribution lies. Examples of quantiles include:

- *Quartiles*—the distribution is divided into quarters.
- *Quintile*—the distribution is divided into fifths.
- *Decile*—the distribution is divided into tenths.
- *Percentile*—the distribution is divided into hundredths (percents).

Note that any quantile may be expressed as a percentile. For example, the third quartile partitions the distribution at a value such that three-fourths, or 75 percent, of the observations fall below that value. Thus, the third quartile is the 75th percentile.

The formula for the position of the observation at a given percentile, y, with n data points sorted in ascending order is:

$$L_y = (n+1)\frac{y}{100}$$

Example: Quartiles

What is the third quartile for the following distribution of returns?

8%, 10%, 12%, 13%, 15%, 17%, 17%, 18%, 19%, 23%, 24%

Answer:

The third quartile is the point below which 75 percent of the observations lie. Recognizing that there are 11 observations in the data set, the third quartile can be identified as:

$$L_y = (11+1) \times \frac{75}{100} = 9$$

When the data is arranged in ascending order, the third quartile is the ninth data point from the left, or 19 percent. This means that 75 percent of all observations lie below 19 percent.

As you will see in the next example, if L is not a whole number, linear interpolation must be used to find the quantile.

Example: Quartiles

What is the third quartile for the following distribution of returns?

8%, 10%, 12%, 13%, 15%, 17%, 17%, 18%, 19%, 23%, 24%, 26%

Answer:

With 12 observations in this data set, the third quartile can be identified as:

$$L_y = (12+1) \times \frac{75}{100} = 9.75$$

This means that when the data is arranged in ascending order, the third quartile (75th percentile) is the ninth data point from the left, plus 0.75 × (distance between the 9th and 10th data values). Specifically, the third quartile is 22 percent = [19 + 0.75 × (23 − 19)] = 22 percent, indicating that 75 percent of all observations lie below 22 percent.

LOS 8.h: Define, calculate, and interpret measures of central tendency, including the population mean, sample mean, arithmetic mean, weighted average or mean (including a portfolio return viewed as a weighted mean), geometric mean, harmonic mean, median, and mode.

Measures of central tendency identify the center, or average, of a data set. This central point can then be used to represent the typical, or expected, value in the data set.

To compute the **population mean**, all the observed values in the population are summed (ΣX) and divided by the number of observations in the population, N. Note that the population mean is unique in that a given population only has one mean. The population mean is expressed as:

$$\mu = \frac{\sum_{i=1}^{N} X_i}{N}$$

The **sample mean** is the sum of all the values in a sample of a population, ΣX, divided by the number of observations in the sample, n. It is used to make *inferences* about the population mean. The sample mean is expressed as:

$$\overline{X} = \frac{\sum_{i=1}^{n} X_i}{n}$$

Note the use of n, the sample size, versus N, the population size.

Example: Population mean and sample mean

Assume you and your research assistant are evaluating the stock of AXZ Corporation. You have calculated the stock returns for AXZ over the last 12 years to develop the data set shown below. Your research assistant has decided to conduct his analysis using only the returns from five of the years, which are displayed as the bold numbers in the data set. Given this information, **calculate** the population mean and the sample mean.

Data set: 12%, **25%**, **34%**, 15%, **19%**, 44%, **54%**, 33%, 22%, 28%, **17%**, 24%

Answer:

$$\mu = \text{population mean} = \frac{12+25+34+15+19+44+54+33+22+28+17+24}{12} = 27.25\%$$

$$\overline{X} = \text{sample mean} = \frac{25+34+19+54+17}{5} = 29.8\%$$

The population mean and sample mean are both examples of **arithmetic means**. The arithmetic mean is the sum of the observation values divided by the number of observations. It is the most widely used measure of central tendency and has the following properties:

• All interval and ratio data sets have an arithmetic mean.
• All data values are considered and included in the arithmetic mean computation.

- A data set has only one arithmetic mean (i.e., the arithmetic mean is unique).
- The sum of the deviations of each observation in the data set from the mean is always zero.
- The arithmetic mean is the only measure of central tendency for which the sum of the deviations from the mean is zero. Mathematically, this property can be expressed as follows:

$$\text{sum of mean deviations} = \sum_{i=1}^{n}(X_i - \overline{X}) = 0$$

Example: Arithmetic mean and deviations from the mean

Compute the arithmetic mean for a data set described as:

Data set: [5, 9, 4, 10]

Answer:

The arithmetic mean of these numbers is:

$$\overline{X} = \frac{5+9+4+10}{4} = 7$$

The sum of the deviations from the mean is:

$$\sum_{i=1}^{n}(X_i - \overline{X}) = (5-7)+(9-7)+(4-7)+(10-7) = -2+2-3+3 = 0$$

Two weaknesses of the arithmetic mean are:

- *Extreme values.* Unusually large or small values can have a disproportionate effect on the computed value for the arithmetic mean.
- *Unknown number of observations.* The arithmetic mean cannot be determined for an open-ended data set (i.e., *n* is unknown).

The **median** is the midpoint of a data set when the data is arranged in ascending or descending order. Half the observations lie above the median and half are below. To determine the median, arrange the data from the highest to the lowest value, or lowest to highest value, and find the middle observation.

The median is important because the arithmetic mean can be affected by extremely large or small values (outliers). When this occurs, the median is a better measure of central tendency than the mean because it is not affected by extreme values.

Example: The median using an odd number of observations

What is the median return for five portfolio managers with 10-year annualized total returns record of: 30%, 15%, 25%, 21%, and 23%?

Answer:

First, arrange the returns in descending order.

30%, 25%, **23%**, 21%, 15%

Then, select the observation that has an equal number of observations above and below it—the one in the middle. For the given data set, the third observation, 23 percent, is the median value.

Example: The median using an even number of observations

Suppose we add a sixth manager to the previous example with a return of 28 percent. What is the median return?

Answer:

Arranging the returns in descending order gives us:

> 30%, 28%, 25%, 23%, 21%, 15%

With an even number of observations, there is no single middle value. The median value in this case is the arithmetic mean of the two middle observations, 25 percent and 23 percent. Thus, the median return for the six managers is 24.0% = 0.5(25 + 23).

The **mode** is the value that occurs most frequently in a data set. A data set may have more than one mode or even no mode. When a distribution has one value that appears most frequently, it is said to be *unimodal*. When a set of data has two or three values that occur most frequently, it is said to be *bimodal* or *trimodal*, respectively.

Example: The mode

What is the mode of the following data set?

> Data set: [30%, 28%, 25%, 23%, 28%, 15%, 5%]

Answer:

The mode is 28 percent because it is the value appearing most frequently.

(See Exam Flashbacks #1 and #2.)

The **geometric mean** is often used when calculating investment returns over multiple periods or when measuring compound growth rates. The general formula for the geometric mean, *G*, is as follows:

$$G = \sqrt[n]{X_1 \times X_2 \times ... \times X_n} = \left(X_1 \times X_2 \times ... \times X_n\right)^{1/n}$$

Note that this equation has a solution only if the product under the radical sign is non-negative.

When calculating the geometric mean for a returns data set, it is necessary to add 1.0 to each value under the radical and then subtract 1.0 from the result.

The geometric mean return (R_G) can be computed using the following equation:

$$1 + R_G = \sqrt[n]{(1 + R_1) \times (1 + R_2) \times ... \times (1 + R_n)}$$

where:
R_t = the HPR for period *t*

Example: Geometric mean return

For the last three years, the returns for Acme Corporation common stock have been –9.34%, 23.45%, and 8.92%. Compute the geometric mean return.

Answer:

$$1 + R_G = \sqrt[3]{(-0.0934 + 1) \times (0.2345 + 1) \times (0.0892 + 1)}$$

$$1 + R_G = \sqrt[3]{0.9066 \times 1.2345 \times 1.0892} = \sqrt[3]{1.21903} = (1.21903)^{1/3} = 1.06825$$

$$R_G = 1.06825 - 1 = 6.825\%$$

Solving this type of problem with your calculator is done as follows:

- On the TI, enter 1.21903; [y^x] 0.33333; [=]
- On the HP, enter 1.21903; [ENTER]; 0.33333; [y^x]

Note: The 0.33333 represents the one-third power.

(See Exam Flashback #3.)

Professor's Note: The geometric mean is always less than or equal to the arithmetic mean, and the difference increases as the dispersion of the observations increases from period to period. The only time the arithmetic and geometric means are equal is when there is no variability in the observations (i.e., all observations are equal).

The computation of a **weighted mean** recognizes that different observations may have a disproportionate influence on the mean. The weighted mean of a set of numbers is computed with the following equation:

$$\overline{X}w = \sum_{i=1}^{n} w_i X_i = (w_1 X_1 + w_2 X_2 + \ldots + w_n X_n)$$

where:
$X_1, X_2, \ldots X_n$ = observed values
$w_1, w_2, \ldots w_n$ = corresponding weights associated with each of the observations such that $\Sigma w_i = 1$

A **harmonic mean** is used for certain computations, such as the average cost of shares purchased over time. The harmonic mean is calculated as $\dfrac{N}{\sum\limits_{i=1}^{N} \dfrac{1}{X_i}}$, where there are N values of X_i.

Example: Calculating average cost with the harmonic mean

An investor purchases $1,000 of stock each month, and over the last three months the prices paid per share were $8, $9, and $10. What is the average cost per share for the shares acquired?

Answer:

$$\overline{X}_H = \frac{3}{\frac{1}{8} + \frac{1}{9} + \frac{1}{10}} = \$8.926 \text{ per share}$$

To check this result, calculate the total shares purchased as $\dfrac{1,000}{8}+\dfrac{1,000}{9}+\dfrac{1,000}{10}=336.11$ shares. The average price is $\dfrac{\$3,000}{336.11}=\$8.926$ per share.

The previous example illustrates the interpretation of the harmonic mean in its most common application. Note that the average price paid per share ($8.93) is less than the arithmetic average of the share prices, $\dfrac{8+9+10}{3}=9$.

Example: Weighted mean as a portfolio return

A portfolio consists of 50 percent common stocks, 40 percent bonds, and 10 percent cash. If the return on common stocks is 12 percent, the return on bonds is 7 percent, and the return on cash is 3 percent, what is the return to the portfolio?

Answer:

$$\overline{X}_W = w_{stock}R_{stock} + w_{bonds}R_{bonds} + w_{cash}R_{cash}$$

$$\overline{X}_W = (0.50 \times 0.12) + (0.40 \times 0.07) + (0.10 \times 0.03) = 0.091, \text{ or } 9.1\%$$

The example illustrates an extremely important investments concept: *the return for a portfolio is the weighted average of the returns of the individual assets in the portfolio.* Asset weights are market weights, the market value of the asset relative to the market value of the entire portfolio.

(See Exam Flashback #4.)

LOS 8.j: Define, calculate, and interpret 1) a range and mean absolute deviation, and 2) a sample and a population variance and standard deviation.

Dispersion is defined as the *variability around the central tendency.* The common theme in finance and investments is the tradeoff between reward and variability, where the central tendency is the measure of the reward and dispersion is a measure of risk.

The **range** is a relatively simple measure of variability, but when used with other measures it provides extremely useful information. The range is the distance between the largest and the smallest value in the data set, or:

range = maximum value – minimum value

Example: The range

What is the range for the 5-year annualized total returns for five investment managers if the managers' individual returns were 30 percent, 12 percent, 25 percent, 20 percent, and 23 percent?

Answer:

range = 30 – 12 = 18%

The **mean absolute deviation** (MAD) is the average of the absolute values of the deviations of individual observations from the arithmetic mean.

$$MAD = \frac{\sum_{i=1}^{n} |X_i - \overline{X}|}{n}$$

The computation of the MAD uses the absolute values of each deviation from the mean because the sum of the actual deviations from the arithmetic mean is zero.

Example: MAD

What is the MAD of the investment returns for the five managers discussed in the preceding example? How is it interpreted?

Answer:

annualized returns: [30%, 12%, 25%, 20%, 23%]

$$\overline{X} = \frac{[30 + 12 + 25 + 20 + 23]}{5} = 22\%$$

$$MAD = \frac{\left[|30 - 22| + |12 - 22| + |25 - 22| + |20 - 22| + |23 - 22| \right]}{5}$$

$$MAD = \frac{[8 + 10 + 3 + 2 + 1]}{5} = 4.8\%$$

This result can be interpreted to mean that, on average, an individual return will deviate ±4.8 percent from the mean return of 22 percent.

The **population variance** is defined as the average of the squared deviations from the mean. The population variance (σ^2) uses the values for all members of a population and is calculated using the following formula:

$$\sigma^2 = \frac{\sum_{i=1}^{N} (X_i - \mu)^2}{N}$$

Example: Population variance, σ^2

Assume the 5-year annualized total returns for the five investment managers used in the earlier example represent *all* of the managers at a small investment firm. What is the population variance of returns?

Answer:

$$\mu = \frac{[30 + 12 + 25 + 20 + 23]}{5} = 22\%$$

$$\sigma^2 = \frac{\left[(30-22)^2 + (12-22)^2 + (25-22)^2 + (20-22)^2 + (23-22)^2\right]}{5} = 35.60\left(\%^2\right)$$

Interpreting this result, we can say that the average variation from the mean return is 35.60 percent squared. Had we done the calculation using decimals instead of whole percents, the variance would be 0.00356. What is a percent squared? Yes, this is nonsense, but let's see what we can do so that it makes more sense.

As you have just seen, a major problem with using the variance is the difficulty of interpreting it. The computed variance, unlike the mean, is in terms of squared units of measurement. How does one interpret squared percents, squared dollars, or squared yen? This problem is mitigated through the use of the standard deviation. The **population standard deviation**, σ, is the square root of the population variance and is calculated as follows:

$$\sigma = \sqrt{\frac{\sum_{i=1}^{N}(X-\mu)^2}{N}}$$

Example: Population standard deviation, σ

Using the data from the preceding example, **compute** the population standard deviation.

Answer:

$$\sigma = \sqrt{\frac{[(30-22)^2 + (12-22)^2 + (25-22)^2 + (20-22)^2 + (23-22)^2\,]}{5}}$$

$$= \sqrt{35.60} = 5.97\%$$

Calculated with decimals instead of whole percents, we would get:

$$\sigma^2 = 0.00356 \text{ and } \sigma = \sqrt{0.00356} = 0.05966 = 5.97\%$$

Since the population standard deviation and population mean are both expressed in the same units (percent), these values are easy to relate. The outcome of this example indicates that the mean return is 22 percent and the standard deviation about the mean is 5.97 percent.

The **sample variance**, s^2, is the measure of dispersion that applies when we are evaluating a sample of n observations from a population. The sample variance is calculated using the following formula:

$$s^2 = \frac{\sum_{i=1}^{n}(X_i - \overline{X})^2}{n-1}$$

Notice that the formula for sample variance, s^2, is nearly the same as that for the population variance, σ^2. One difference is the use of the sample mean, $\overline{X}$, instead of the population mean, μ. The most noteworthy difference is that the denominator for s^2 is n – 1, one less than the sample size, n, versus the entire population size, N. Based on the mathematical theory behind statistical procedures, the use of the entire number of sample observations, n, instead of n – 1 as the divisor in the computation of s^2, will systematically *underestimate* the population parameter, σ^2, particularly for small sample sizes. This systematic underestimation causes the sample variance to be what is referred to as a *biased estimator* of the population variance. Using n – 1 instead of n in the

denominator, however, improves the statistical properties of s^2 as an estimator of σ^2. Thus, s^2, as expressed in the equation above, is considered to be an unbiased estimator of σ^2.

Example: Sample variance

Assume that the 5-year annualized total returns for the five investment managers used in the preceding examples represent only a sample of the managers at a large investment firm. What is the sample variance of these returns?

Answer:

$$\overline{X} = \frac{[30 + 12 + 25 + 20 + 23]}{5} = 22\%$$

$$s^2 = \frac{\left[(30-22)^2 + (12-22)^2 + (25-22)^2 + (20-22)^2 + (23-22)^2\right]}{5-1} = 44.5\left(\%^2\right)$$

Thus, the sample variance of $44.5(\%^2)$ can be interpreted to be an unbiased estimator of the population variance.

As with the population standard deviation, the **sample standard deviation** can be calculated by taking the square root of the sample variance. The sample standard deviation, s, is defined as:

$$s = \sqrt{\frac{\sum_{i=1}^{n}(X_i - \overline{X})^2}{n-1}}$$

Example: Sample standard deviation

Compute the sample standard deviation based on the result of the preceding example.

Answer:

Since the sample variance for the preceding example was computed to be $44.5(\%^2)$, the sample standard deviation is:

$$s = [44.5(\%^2)]^{1/2} = 6.67\%$$

The results shown here mean that the sample standard deviation, $s = 6.67$ percent, can be interpreted as an unbiased estimator of the population standard deviation, σ.

LOS 8.k: Contrast variance to semivariance and target semivariance.

Recall that variance measures the average squared deviation from the mean and that the average is based on $n - 1$ when calculating the variance of a sample.

Semivariance is calculated in the same manner, but only those observations that fall below the mean are included in the calculation. The formula is then:

$$\frac{\sum_{\text{All } X_i < \overline{X}}\left(X_i - \overline{X}\right)^2}{\left(\# \text{ of Xs less than } \overline{X}\right) - 1}$$

Semivariance is sometimes described as a measure of "downside risk" in an investments context. Note that for a symmetric distribution, semivariance will be equivalent to variance and therefore contains no additional information. For skewed distributions, the semivariance can provide additional information that variance does not. Even though semivariance has the appealing intuition of measuring downside risk, mathematically it does not have the attractive properties that variance does (e.g., we cannot sum semivariance for a portfolio of assets).

A related measure, **target semivariance**, is based on observations below a specific value. For example, we may be concerned with only negative returns values or values below an arbitrary returns "target," such as 4 percent. The calculation is the same as that of semivariance, with only observations below the target rate included.

LOS 8.l: Calculate and interpret the proportion of observations falling within a specified number of standard deviations of the mean, using Chebyshev's inequality.

Chebyshev's inequality states that for any set of observations, whether sample or population data and regardless of the shape of the distribution, the percentage of the observations that lie within k standard deviations of the mean is *at least* $1 - 1/k^2$ for all k > 1. Given the standard deviation, Chebyshev's inequality can be used to measure the *maximum* amount of dispersion, regardless of the shape of the distribution.

> **Example: Chebyshev's inequality**
>
> What is the minimum percentage of any distribution that will lie within ±2 standard deviations of the mean?
>
> **Answer:**
>
> Applying Chebyshev's inequality, we have:
>
> $$1 - 1/k^2 = 1 - 1/2^2 = 1 - 1/4 = 0.75 \text{ or } 75\%$$

According to Chebyshev's inequality, the following relationships hold for any distribution. At least:

- 36% of observations lie within ±1.25 standard deviations of the mean.
- 56% of observations lie within ±1.50 standard deviations of the mean.
- 75% of observations lie within ±2 standard deviations of the mean.
- 89% of observations lie within ±3 standard deviations of the mean.
- 94% of observations lie within ±4 standard deviations of the mean.

LOS 8.m: Define, calculate, and interpret the coefficient of variation and the Sharpe ratio.

A direct comparison between two or more measures of dispersion may be difficult. For instance, suppose you are comparing the annual returns distribution for retail stocks with a mean of 8 percent and an annual returns distribution for a real estate portfolio with a mean of 16 percent. A direct comparison between the dispersion of the two distributions is not meaningful because of the relatively large difference in their means. To make a meaningful comparison, a relative measure of dispersion must be used. *Relative dispersion* is the amount of variability in a distribution relative to a reference point or benchmark. Relative dispersion is commonly measured with the **coefficient of variation** (CV), which is computed as:

$$CV = \frac{s_x}{\overline{X}} = \frac{\text{standard deviation of x}}{\text{average value of x}}$$

CV measures the amount of dispersion in a distribution relative to the distribution's mean. It is useful because it enables us to make a direct comparison of dispersion across different sets of data. In an investments setting, the CV is used to measure the risk (variability) per unit of expected return (mean).

Example: Coefficient of variation

You have just been presented with a report that indicates that the mean monthly return on T-bills is 0.25 percent with a standard deviation of 0.36 percent, and the mean monthly return for the S&P 500 is 1.09 percent with a standard deviation of 7.30 percent. Your unit manager has asked you to **compute** the CV for these two investments and to **interpret** your results.

Answer:

$$CV_{\text{T-bills}} = \frac{0.36}{0.25} = 1.44$$

$$CV_{\text{S\&P 500}} = \frac{7.30}{1.09} = 6.70$$

These results indicate that there is less dispersion (risk) per unit of monthly return for T-bills than there is for the S&P 500 (1.44 versus 6.70).

Professor's Note: In order to remember the formula for CV, remember that the coefficient of variation is a measure of variation, so standard deviation goes in the numerator. CV is variation per unit of return.

(See Exam Flashback #5.)

The Sharpe Ratio

The **Sharpe measure** (a.k.a., the *Sharpe ratio* or *reward-to-variability ratio*) is widely used for investment performance measurement to measure *excess* return per unit of risk. The Sharpe measure appears over and over throughout the CFA® curriculum. It is defined according to the following formula:

$$\text{Sharpe ratio} = \frac{\overline{r_p} - \overline{r_f}}{\sigma_p}$$

where:

$\overline{r_p}$ = portfolio return

$\overline{r_f}$ = risk-free return

σ_p = standard deviation of portfolio returns

Notice that the numerator of the Sharpe ratio uses a measure for a risk-free return. As such, the quantity $(\overline{r_p} - \overline{r_f})$, referred to as the *excess return* on Portfolio p, measures the extra reward that investors receive for exposing themselves to risk. Portfolios with large Sharpe ratios are preferred to portfolios with smaller ratios because it is assumed that rational investors prefer return and dislike risk.

Example: The Sharpe ratio

Assume that the mean monthly return on T-bills is 0.25 percent and that the mean monthly return and standard deviation for the S&P 500 are 1.30 percent and 7.30 percent, respectively. Using the T-bill return to represent the risk-free rate, as is common in practice, **compute** and **interpret** the Sharpe ratio.

Answer:

$$\text{Sharpe ratio} = \frac{1.30 - 0.25}{7.30} = 0.144$$

The Sharpe ratio of 0.144 indicates that the S&P 500 earned 0.144 percent of excess return per unit of risk, where risk is measured by standard deviation of portfolio returns.

WARM-UP: SYMMETRICAL DISTRIBUTIONS

A distribution is **symmetrical** if it is shaped identically on both sides of its mean. Distributional symmetry implies that intervals of losses and gains will exhibit the same frequency. For example, a symmetrical distribution with a mean return of zero will have losses in the –6 percent to –4 percent interval as frequently as it will have gains in the +4 percent to +6 percent interval.

The *normal distribution* is the most commonly encountered frequency distribution. It is the bell-shaped curve we so often hear about. The most obvious property of a normal distribution is that it is symmetrical about its mean. Other properties of a normal distribution include the following:

- The mean and median are equal.
- It can be completely described by its mean and variance.
- Approximately 68 percent of normally distributed observations lie within ±1 standard deviation of the mean; 95 percent of the observations lie within ±2 standard deviations of the mean; and 99 percent of the observations lie within ±3 standard deviations of the mean.

LOS 8.n: Define and interpret skew, explain the meaning of a positively or negatively skewed return distribution, and describe the relative locations of the mean, median, and mode for a nonsymmetrical distribution.

The extent to which a returns distribution is symmetrical is important because the degree of symmetry tells analysts if deviations from the mean are more likely to be positive or negative.

Skewness, or skew, refers to the extent to which a distribution is not symmetrical. Nonsymmetrical distributions may be either positively or negatively skewed and result from the occurrence of outliers in the data set. *Outliers* are observations with extraordinarily large values, either positive or negative.

- A *positively skewed* distribution is characterized by many outliers in the upper region, or right tail. A positively skewed distribution is said to be skewed right because of its relatively long upper (right) tail.
- A *negatively skewed* distribution has a disproportionately large amount of outliers that fall within its lower (left) tail. A negatively skewed distribution is said to be skewed left because of its long lower tail.

Mean, Median, and Mode for a Nonsymmetrical Distribution

Skewness affects the **location of the mean, median, and mode** of a distribution as summarized in the following bulleted list.

- For a symmetrical distribution, the mean, median, and mode are equal.
- For a positively skewed distribution, the mode is less than the median, which is less than the mean. The mean is affected by outliers; in a positively skewed distribution, there are large, positive outliers which will tend to "pull" the mean upward, or more positive. An example of a positively skewed distribution is that of housing prices. Suppose that you live in a neighborhood with 100 homes; 99 of them sell for $100,000, and one sells for $1,000,000. The median and the mode will be $100,000, but the mean will be $109,000. Hence, the mean has been "pulled" upward (to the right) by the existence of one home (outlier) in the neighborhood.
- For a negatively skewed distribution, the mean is less than the median, which is less than the mode. In this case, there are large, negative outliers which tend to "pull" the mean downward (to the left).

Professor's Note: The key to remembering how measures of central tendency are affected by skewed data is to recognize that skew affects the mean more than the median and mode, and the mean is "pulled" in the direction of the skew. The relative location of the mean, median, and mode for different distribution shapes is shown in Figure 7.

Figure 7: Effect of Skewness on Mean, Median, and Mode

Symmetrical
(Mean = Median = Mode)

Mean
Median
Mode

Positive (right) skew
(Mean > Median > Mode)

Mean
Median
Mode

Negative (left) skew
(Mean < Median < Mode)

Mean
Median
Mode
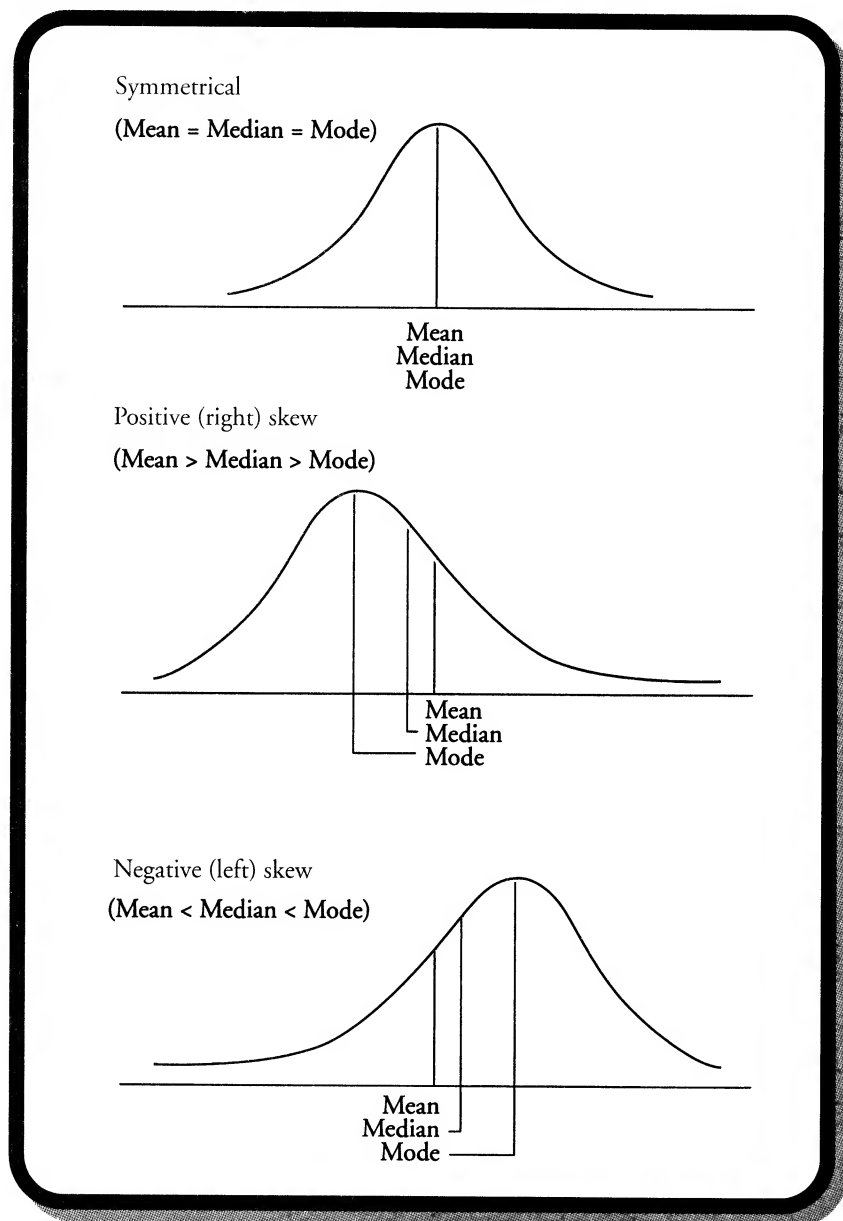
(See Exam Flashbacks #6 and #7.)

LOS 8.o: Define and interpret kurtosis, and measures of population and sample skew and kurtosis.

Kurtosis is a measure of the degree to which a distribution is more or less "peaked" than a normal distribution. *Leptokurtic* describes a distribution that is more peaked than a normal distribution, whereas *platykurtic* refers to a distribution that is less peaked, or flatter than a normal distribution.

As indicated in Figure 8, a leptokurtic return distribution will have more returns clustered around the mean and more returns with large deviations from the mean (fatter tails). Relative to a normal distribution, a leptokurtic

distribution will have a greater percentage of small deviations from the mean and a greater percentage of extremely large deviations from the mean. This means that there is a relatively greater probability of an observed value being either close to the mean or far from the mean. With regard to an investment returns distribution, a greater likelihood of a large deviation from the mean return is often perceived as an increase in risk.

Figure 8: Kurtosis
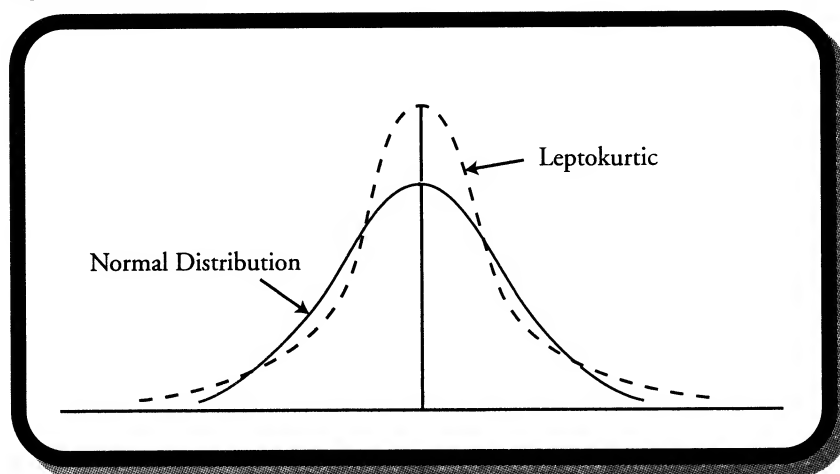
A distribution is said to exhibit **excess kurtosis** if it has either more or less kurtosis than the normal distribution. The computed kurtosis for all normal distributions is three. Statisticians, however, sometimes report excess kurtosis, which is defined as kurtosis minus three. Thus, a normal distribution has excess kurtosis equal to zero, a leptokurtic distribution has excess kurtosis greater than zero, and platykurtic distributions will have excess kurtosis less than zero.

Kurtosis is critical in a risk management setting. Most research of the distribution of securities returns has shown that returns are not normally distributed. Actual securities returns tend to exhibit both skewness and kurtosis. Skewness and kurtosis are critical concepts for risk management because when securities returns are modeled using an assumed normal distribution, the predictions from the models will not take into account the potential for extremely large, negative outcomes. In fact, most risk managers put very little emphasis on the mean and standard deviation of a distribution and focus more on the distribution of returns in the tails of the distribution—that is where the risk is.

Measures of Sample Skew and Kurtosis

Sample skewness is equal to the sum of the cubed deviations from the mean divided by the cubed standard deviation and by the number of observations. Sample skewness for large samples is computed as:

$$\text{sample skewness}(S_K) = \frac{1}{n} \frac{\sum_{i=1}^{n}\left(X_i - \overline{X}\right)^3}{s^3}$$

where:
s = sample standard deviation

Note that the denominator is always positive but that the numerator can be positive or negative, depending on whether observations above the mean or observations below the mean tend to be further from the mean on average. When a distribution is right skewed, sample skewness is positive because the deviations above the mean are larger on average. A left-skewed distribution has a negative sample skewness.

Dividing by standard deviation cubed standardizes the statistic and allows **interpretation of the skewness measure**. If relative skewness is equal to zero, the data is not skewed. Positive levels of relative skewness imply a positively skewed distribution, whereas negative values of relative skewness imply a negatively skewed distribution. Values of S_K in excess of 0.5 in absolute value indicate significant levels of skewness.

Sample kurtosis is measured using deviations raised to the *fourth power.*

$$\text{sample kurtosis} = \frac{1}{n} \frac{\sum_{i=1}^{n} (X_i - \overline{X})^4}{s^4}$$

where:
s = sample standard deviation

To **interpret kurtosis**, note that it is measured relative to the kurtosis of a normal distribution, which is 3. Positive values of excess kurtosis indicate a distribution that is leptokurtic (more peaked, fat tails), whereas negative values indicate a platykurtic distribution (less peaked, thin tails). Excess kurtosis values that exceed 1.0 in absolute value are considered large. We can calculate kurtosis relative to that of a normal distribution as:

excess kurtosis = sample kurtosis − 3

KEY CONCEPTS

1. Descriptive statistics summarize the characteristics of a data set; inferential statistics are used to make probabilistic statements about a population based on a sample.
2. A population includes all members of a specified group, while a sample is a subset of the population used to draw inferences about the population.
3. Any measurable characteristic of a population is called a parameter; a characteristic of a sample is given by a sample statistic.
4. Data may be measured using different scales.
   - Nominal scale—data is put into a category with no particular order.
   - Ordinal scale—data is categorized and ordered with respect to some characteristic.
   - Interval scale—the difference in data values is meaningful, but zero does not represent the absence of what is being measured.
   - Ratio scale—the difference between observed values is meaningful, and a true zero point is the origin.
5. An interval is the set of return values, or range, that an observation falls within. A frequency distribution is a grouping of raw data into classes, or intervals.
6. The HPR measures the total return, R_t, for holding an investment over a specified period of time.

   $$R_t = \frac{P_t - P_{t-1} + D_t}{P_{t-1}} = \frac{P_t + D_t}{P_{t-1}} - 1$$

7. Relative frequency is the percentage of total observations falling within each interval; cumulative relative frequency is the sum of the relative frequencies up to a point.
8. Histograms and frequency polygons are graphical tools used for portraying frequency distributions.
9. The arithmetic mean is $\overline{X} = \dfrac{\sum_{i=1}^{n} X_i}{n}$. The geometric mean is $G = \sqrt[n]{X_1 \times X_2 \times \ldots \times X_n}$. The weighted mean is $\overline{X}_W = \sum_{i=1}^{n} w_i X_i$, and the harmonic mean is $\overline{X}_H = \dfrac{N}{\sum_{i=1}^{N} \dfrac{1}{x_i}}$.

10. The median is the midpoint of a data set when the data is arranged from largest to smallest, and the mode of a data set is the value that appears most frequently.

11. Quantile is the general term for a value at or below which a stated proportion of the data in a distribution lies. Examples of quantiles include:
    - Quartiles—the distribution is divided into quarters.
    - Quintile—the distribution is divided into fifths.
    - Decile—the distribution is divided into tenths.
    - Percentile—the distribution is divided into hundredths (percents).

12. The range is the difference between the largest value and the smallest value in a data set.

13. Mean absolute deviation (MAD) is the average of the absolute values of the deviations from the arithmetic mean:

$$MAD = \frac{\sum\limits_{i=1}^{n} |X_i - \overline{X}|}{n}$$

14. The variance is defined as the mean of the squared deviations from the arithmetic mean.
    - Population variance $= \sigma^2 = \dfrac{\sum\limits_{i=1}^{N}(X_i - \mu)^2}{N}$, where μ = population mean and N = size
    - Sample variance $= s^2 = \dfrac{\sum\limits_{i=1}^{n}(X_i - \overline{X})^2}{n-1}$, where $\overline{X}$ = sample mean and n = sample size

15. Standard deviation is the positive square root of the variance and is frequently used as a quantitative measure of risk.

16. Target semivariance is the variance calculated only for the observations that are below the stated target.

17. Chebyshev's inequality states that the proportion of the observations within k standard deviations of the mean is at least $1 - 1/k^2$ for all k > 1.

18. The coefficient of variation, $CV = \dfrac{s}{\overline{X}}$, expresses dispersion (risk) relative to the mean of a distribution.

19. The Sharpe measure (ratio) measures excess return per unit of risk:

$$Sharpe\ ratio = \frac{(\overline{r_p} - \overline{r_f})}{\sigma_p}$$

20. Skewness describes the degree to which a distribution is nonsymmetric about its mean.
    - A right-skewed distribution has positive sample skewness and a mean that is higher than the median that is higher than the mode.
    - A left-skewed distribution has negative skewness and a mean that is lower than the median that is lower than the mode.

21. Kurtosis measures the peakedness of a distribution and the probability of extreme outcomes.
    - Excess kurtosis is measured relative to a normal distribution, which has a kurtosis of 3.
    - Positive values of excess kurtosis indicate a distribution that is leptokurtic (fat tails, more peaked).
    - Negative values of excess kurtosis indicate a platykurtic distribution (thin tails, less peaked).
    - Excess kurtosis with an absolute value greater than 1 is considered large.

EXAM FLASHBACKS

Required CFA Institute Disclaimer: Due to CFA curriculum changes from year to year, published sample exam questions and guideline answers prior to the current year may not reflect the current curriculum.

Exam Flashback # 1
Source: Question #32 from '98 sample exam.

The price of a firm's common stock at the end of each month was:

Month	Stock Price	Month	Stock Price	Month	Stock Price
January	$20	May	$20	September	$50
February	25	June	25	October	42
March	31	July	24	November	35
April	35	August	25	December	40

The arithmetic mean, median, and mode of this distribution of stock prices are:

	Mean	Median	Mode
A.	$25	$28	$31
B.	$28	$25	$31
C.	$31	$28	$25
D.	$31	$24	$28

Exam Flashback # 2
Source: Question #21 from '92, '96 actual exams.

What are the mean, median, and mode, respectively, of the following data series: 1, 2, 3, 5, 8, 8, 15?
A. 6,5,8.
B. 6,8,5.
C. 8,5,6.
D. 5,8,6.

Exam Flashback # 3
Source: Question #34 from '98 sample exam.

A portfolio had an initial market value of $100,000. At the end of the next two years, the market value was $110,000 and $88,000, respectively. The portfolio's arithmetic and geometric mean returns, respectively, excluding any dividends, were:

	Arithmetic Mean	Geometric Mean
A.	–12.00%	– 6.00%
B.	–6.00%	– 6.19%
C.	–5.00%	– 6.19%
D.	–5.00%	–12.00%

Exam Flashback # 4
Source: Question #35 from '98 sample exam.

An investor bought common stock of Blackstone Company on several occasions at the following prices.

Number of Shares	Price per Share
100	$34
200	$30
400	$28

The average price per share at which the investor bought these shares of common stock was *closest* to:
A. $28.00.
B. $29.43.
C. $30.67.
D. $31.00.

Exam Flashback # 5
Source: Question #25 from '99–'02 sample exams.

A stock with a coefficient of variation of 0.50 has a(n):
A. variance equal to half the stock's expected return.
B. expected return equal to half the stock's variance.
C. expected return equal to half the stock's standard deviation.
D. standard deviation equal to half the stock's expected return.

Exam Flashback # 6
Source: Question #24 from '99 sample exam.

An analyst develops the following graph of a set of observations:
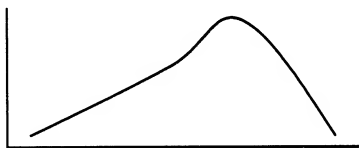
Based on the graph above, which of the following statements about the probability distribution is/are **TRUE?**
  I. The mean of the graphed data is larger than the median and mode.
 II. The probability distribution of observations is symmetrically distributed about the mean.
III. The probability distribution of observations is negatively skewed.
A. I only.
B. II only.
C. III only.
D. I and III.

Exam Flashback # 7
Source: Question #110 from '90 actual exam.

The probability distribution having the characteristics described below is:

Mean = 50
Highest possible value = 100
Lowest possible value = 10

A. normally distributed.
B. symmetric.
C. skewed.
D. continuous.

CONCEPT CHECKERS: STATISTICAL CONCEPTS AND MARKET RETURNS

1. The intervals in a frequency distribution should always have which of the following characteristics? The intervals should always:
   A. be truncated.
   B. be open ended.
   C. have a width of 10.
   D. be nonoverlapping.

Use the following frequency distribution for Questions 2 through 4.

Return, R	Frequency
−10% up to 0%	3
0% up to 10%	7
10% up to 20%	3
20% up to 30%	2
30% up to 40%	1

2. The number of intervals in this frequency table is:
   A. 1.
   B. 5.
   C. 16.
   D. 50.

3. The sample size is:
   A. 1.
   B. 5.
   C. 16.
   D. 50.

4. The relative frequency of the second class is:
   A. 10.0%.
   B. 43.8%.
   C. 0.0%.
   D. 16.0%.

Use the following data to answer Questions 5 through 13.

XYZ Corp. Annual Stock Prices

1995	1996	1997	1998	1999	2000
22%	5%	−7%	11%	2%	11%

5. What is the arithmetic mean return for XYZ stock?
   A. 7.3%.
   B. 8.0%.
   C. 11.0%.
   D. −7.0%.

6. What is the median return for XYZ stock?
   A. 7.3%.
   B. 8.0%.
   C. 11.0%.
   D. –7.0%.

7. What is the mode return for XYZ stock?
   A. 7.3%.
   B. 8.0%.
   C. 11.0%.
   D. –7.0%.

8. What is the range for XYZ stock returns?
   A. –7.0%.
   B. 22.0%.
   C. 11.0%.
   D. 29.0%.

9. What is the mean absolute deviation for XYZ stock returns?
   A. 0.00%.
   B. 5.20%.
   C. 7.33%.
   D. 29.0%.

10. Assuming that the distribution of XYZ stock returns is a population, what is the population variance?
    A. $5.0\%^2$.
    B. $6.8\%^2$.
    C. $7.7\%^2$.
    D. $80.2\%^2$.

11. Assuming that the distribution of XYZ stock returns is a population, what is the population standard deviation?
    A. 5.02%.
    B. 6.84%.
    C. 8.96%.
    D. 46.22%.

12. Assuming that the distribution of XYZ stock returns is a sample, the sample variance is *closest* to:
    A. $5.0\%^2$.
    B. $7.4\%^2$.
    C. $72.4\%^2$.
    D. $96.3\%^2$.

13. Assuming that the distribution of XYZ stock returns is a sample, what is the sample standard deviation?
    A. 7.4%.
    B. 9.8%.
    C. 72.4%.
    D. 96.3%.

14. For a skewed distribution, what is the minimum percentage of the observations that will lie between ±2.5 standard deviations of the mean based on Chebyshev's Inequality?
   A. 16%.
   B. 56%.
   C. 75%.
   D. 84%.

Use the following data to answer Questions 15 and 16.

The annual returns for FJW's common stock over the years 1998, 1999, 2000, and 2001 were 15 percent, 19 percent, –8 percent, and 14 percent, respectively.

15. What is the arithmetic mean return for FJW's common stock?
   A. 8.62%.
   B. 10.00%.
   C. 14.00%.
   D. 15.25%.

16. What is the geometric mean return for FJW's common stock?
   A. 9.45%.
   B. 10.00%.
   C. 14.21%.
   D. It cannot be determined because the 2000 return is negative.

17. A distribution of returns that has a greater percentage of small deviations from the mean and a greater percentage of extremely large deviations from the mean:
   A. is positively skewed.
   B. is a symmetric distribution.
   C. has positive excess kurtosis.
   D. has negative excess kurtosis.

18. Which of the following is *most accurate* regarding a distribution of returns that has a mean greater than its median?
   A. It is positively skewed.
   B. It is a symmetric distribution.
   C. It has positive excess kurtosis.
   D. It has negative excess skewness.

19. For a normal distribution, what *approximate* percentage of the observations fall within ±1 standard deviation of the mean?
   A. 50.
   B. 68.
   C. 95.
   D. 99.

20. For a normal distribution, what *approximate* percentage of the observations fall within ±2 standard deviations of the mean?
   A. 50%.
   B. 68%.
   C. 95%.
   D. 99%.

21. The harmonic mean of 3, 4, and 5 is:
   A. 4.
   B. 3.83.
   C. 3.74.
   D. 4.12.

COMPREHENSIVE PROBLEMS: STATISTICAL CONCEPTS AND MARKET RETURNS

1. Year-end prices and dividends for Nopat Mutual Fund for each of six years are listed below along with the actual yield (return) on a money market fund called Emfund.

Year	Nopat Fund Year-End Price	Nopat Fund Year-End Dividend	Nopat Annual Holding Period Return	Emfund Return for the Year
1999	$28.50	$0.14		3.00%
2000	$26.80	$0.15	?	4.00%
2001	$29.60	$0.17	?	4.30%
2002	$31.40	$0.17	?	5.00%
2003	$34.50	$0.19	?	4.10%
2004	$37.25	$0.22	?	6.00%

Average risk free rate over the 5 years 2000 – 2004 is 2.8%.

A. Calculate the annual holding period returns for a beginning-of-year investment in Nopat fund for each of the five years over the period 2000–2004 (% with 2 decimal places).

B. What is the arithmetic mean annual total return on an investment in Nopat fund shares (dividends reinvested) over the period 2000–2004?

C. What is the average compound annual rate of return on an investment in Nopat fund made at year end 1999 if it was held (dividends reinvested) until the end of 2004?

D. What is the median annual return on an Emfund investment over the 6-year period 1999–2004?

E. What is the sample standard deviation of the annual returns on money market funds over the 6-year period, using the Emfund returns as a sample?

F. What is the holding period return on a 6-year investment in Emfund made at the beginning of 1999?

G. If an investor bought $10,000 of Nopat Fund shares at the end of the year in each of the three years 2002–2004, what is the average price paid per share? What is the arithmetic mean of the three year-end prices?

H. What would have been the 1-year holding period return on a portfolio that had $60,000 invested in Nopat Fund and $40,000 invested in Emfund as of the beginning of 2004?

I. What is the coefficient of variation of the Nopat Fund annual total returns 2000–2004 and of the Emfund annual returns for the six years 1999–2004? Which is riskier?

J. What is the Sharpe ratio for an investment in the Nopat Fund over the five years from 2000–2004? What is the Sharpe ratio for an investment in the Emfund over the six years 1999–2004?

K. Calculate the range and mean absolute deviation of returns for an investment in the Emfund over the 6-year period 1999–2004.

L. Calculate the semivariance of returns on Emfund over the 6-year period.

M. What is the annual growth rate of dividends on Nopat Fund over the period from 1999–2004?

2. Identify the type of scale for each of the following:

A. Cars ranked as heavy, medium, or light.

B. Birds divided into categories of songbirds, birds of prey, scavengers, and game birds.

C. The height of each of the players on a baseball team.

D. The average temperature on 20 successive days in January in Chicago.

E. Interest rates on T-bills each year for 60 years.

ANSWERS – EXAM FLASHBACKS

1. **C** First, list the numbers in either ascending or descending order.

    {20, 20, 24, 25, 25, **25**, **31**, 35, 35, 40, 42, 50}

    The mode is most frequently occurring number. In this case, it is 25.

    $$\text{Mean} = \overline{X} = \frac{\sum_{i=1}^{n} X_i}{n} = \frac{372}{12} = 31$$

    Median: If there is an odd number of observations, the median is the number in the middle. If there is an even number of observations, the median is the average of the two observations in the middle, which in this problem is (25 + 31)/2 = 28.

    Exam trick: Always find the mode first, which is 25 in this question. Since C is the only choice with a mode of 25, C has to be the answer.

2. **A** First, list the numbers from smallest to largest, or vice versa:

    {1, 2, 3, 5, 8, 8, 15}

    The mode is the observation that occurs most often, 8 in this case.

    $$\text{Mean} = \overline{X} = \frac{\sum_{i=1}^{n} X_i}{n} = \frac{42}{7} = 6$$

    If there is an odd number of observations, the median is the number in the middle, 5 in this problem. If there is an even number of observations, the median is the average of the two observations in the middle.

    Exam trick: You can readily observe that the mode is 8. Since only choice A has 8 listed as the mode, the answer has to be A. There is no other possible answer.

3. **C** $\text{HPY}_1 = 10{,}000/100{,}000 = 0.10$, or 10%

    $\text{HPY}_2 = -22{,}000/110{,}000 = -0.2$, or –20%

    Arithmetic mean = (0.10 – 0.20) / 2 = –0.05, or –5%

    Geometric mean = $[(1.0 + 0.1)(1.0 - 0.2)]^{0.5} - 1 = -0.0619$, or –6.19%

    Professor's Note: The exam question will most likely require you to calculate both the arithmetic and geometric mean. Remember that the geometric mean is less than the arithmetic mean. Knowing this, you may be able to rule out some of the choices.

4. **B** Weighted average = $\overline{X}_W = \sum_{i=1}^{n} w_i X_i = (w_1 X_1 + w_2 X_2 + \ldots + w_n X_n)$

    The share weights here, w_i, are 100/700, 200/700, and 400/700. So the weighted average share price is:

    (100/700)(34) + (200/700)(30) + (400/700)(28) = $29.43

    Professor's Note: This problem may also be solved using the following procedure, but this is simply a rearrangement of the initial procedure.

Weighted average = total value of all shares / total number of shares
= [Σ (shares)(price)] / Σ (shares)
= [(100)(34) + (200)(30) + (400)(28)] / (700) = $29.43

5. **D** CV = (σ / expected return)

σ = CV(expected return)

σ = 0.5(expected return), or standard deviation is one-half the expected return.

Professor's Note: This is merely an exercise in algebra. The trick is to recognize that all of the choices have the standard deviation (or variance) in common. Thus, you need to express the CV relationship in terms of the standard deviation, then pick the choice that describes the result. Also note that in this question, the mean was given as an expected return. Don't let this throw you off.

6. **C** A left skew pulls the mean below the median and mode.

Professor's Note: The skew pulls the mean first, then the median, and lastly, the mode in the direction of the skew. Note that these are in alphabetical order (mean, median, mode).

7. **C** When a distribution is normal, the dispersion of the observations to the left of the mean is identical to the dispersion to the right of the mean. In this problem, the highest number, 100, is 50 units to the right of the mean. The lowest number, 10, is 40 units to the left of the mean. Hence, the distribution is not symmetrical—it is skewed to the right.

Professor's Note: Sketching this out will help you see what's going on.

ANSWERS – CONCEPT CHECKERS: STATISTICAL CONCEPTS AND MARKET RETURNS

1. **D** Intervals within a frequency distribution should always be nonoverlapping and closed ended so that each data value can be placed into only one interval. Intervals have no set width and should be set at a width so that data is adequately summarized without losing valuable characteristics.

2. **B** An interval is the set of return values that an observation falls within. Simply count the return intervals on the table—there are five of them.

3. **C** The sample size is the sum of all of the frequencies in the distribution, or 3 + 7 + 3 + 2 + 1 = 16.

4. **B** The relative frequency is found by dividing the frequency of the interval by the total number of frequencies.

$$\frac{7}{16} = 43.8\%$$

5. **A** [22% + 5% + −7% + 11% + 2% +11%] / 6 = 7.3%

6. **B** To find the median, rank the returns in order and take the middle value. −7%, 2%, 5%, 11%, 11%, 22%. In this case, because there is an even number of observations, the median is the average of the two middle values, or (5% + 11%) / 2 = 8.0%.

7. **C** The mode is the value that appears most often, or 11%.

8. **D** The range is calculated by taking the highest value minus the lowest value.

22% − (−7%) = 29.0%

9. **C** The mean absolute deviation is found by taking the mean of the absolute values of the deviations from the mean.

(|22 − 7.3| + |5 − 7.3| + |−7 − 7.3| + |11 − 7.3| + |2 − 7.3| + |11 − 7.3|) / 6 = 7.33%

10. **D**   The population variance, σ^2, is found by taking the mean of all squared deviations from the mean.

$\sigma^2 = [(22 - 7.3)^2 + (5 - 7.3)^2 + (-7 - 7.3)^2 + (11 - 7.3)^2 + (2 - 7.3)^2 + (11 - 7.3)^2] / 6 = 80.2\%^2$

11. **C**   The population standard deviation, σ, is found by taking the square root of the population variance.

$\sigma = \{[(22 - 7.3)^2 + (5 - 7.3)^2 + (-7 - 7.3)^2 + (11 - 7.3)^2 + (2 - 7.3)^2 + (11 - 7.3)^2] / 6\}^{\frac{1}{2}} = (80.2\%^2)^{0.5}$
$= 8.96\%$

12. **D**   The sample variance, s^2, uses n − 1 in the denominator.

$s^2 = [(22 - 7.3)^2 + (5 - 7.3)^2 + (-7 - 7.3)^2 + (11 - 7.3)^2 + (2 - 7.3)^2 + (11 - 7.3)^2] / (6 - 1) = 96.3\%^2$

13. **B**   The sample standard deviation, s, is the square root of the sample variance.

$s = \{[(22 - 7.3)^2 + (5 - 7.3)^2 + (-7 - 7.3)^2 + (11 - 7.3)^2 + (2 - 7.3)^2 + (11 - 7.3)^2] / (6 - 1)\}^{0.5}$
$= (96.3\%^2)^{0.5} = 9.8\%$

14. **D**   Applying Chebyshev's inequality, $1 - [1 / (2.5)^2] = 0.84$, or 84%.

15. **B** $(15\% + 19\% + (-8\%) + 14\%) / 4 = 10\%$

16. **A** $(1.15 \times 1.19 \times 0.92 \times 1.14)^{0.25} - 1 = 9.45\%$

Professor's Note: This question could have been answered very quickly since the geometric mean must be less than the arithmetic mean computed in the preceding problem.

17. **C**   A distribution that has a greater percentage of small deviations from the mean and a greater percentage of extremely large deviations from the mean will be leptokurtic and will exhibit excess kurtosis (positive). The distribution will be taller and have fatter tails than a normal distribution.

18. **A**   A distribution with a mean greater than its median is positively skewed, or skewed to the right. The skew "pulls" the mean. *Note:* Kurtosis deals with the height of the distribution and not the skewness.

19. **B**   For normal distributions, approximately 68% of the observations fall within ±1 standard deviation of the mean.

20. **C**   For normal distributions, approximately 95% of the observations fall within ±2 standard deviations of the mean.

21. **B** $\overline{X}_H = \dfrac{3}{\frac{1}{3} + \frac{1}{4} + \frac{1}{5}} = 3.83$

ANSWERS – COMPREHENSIVE PROBLEMS: STATISTICAL CONCEPTS AND MARKET RETURNS

1. A. The annual holding period returns (total returns) are given in the table and are each calculated as (year-end price + year-end dividend)/previous year-end price − 1.

Year	Nopat Fund Year-End Price	Nopat Fund Year-End Dividend	Nopat Annual Holding Period Return	Emfund Return for the Year
1999	$28.50	$0.14		3.00%
2000	$26.80	$0.15	−5.44%	4.00%
2001	$29.60	$0.17	11.08%	4.30%
2002	$31.40	$0.17	6.66%	5.00%
2003	$34.50	$0.19	10.48%	4.10%
2004	$37.25	$0.22	8.61%	6.00%

B. The arithmetic mean of the holding period returns is 6.28%.

C. $((1 − 0.0544)(1.1108)(1.0666)(1.1048)(1.0861))^{1/5} − 1 = 6.10\%$

D. Median = (4.3 + 4.1) / 2 = 4.2%.

E. Sample standard deviation of Emfund returns over the six years is:

$$\{[(3 − 4.4)^2 + (4 − 4.4)^2 + (4.3 − 4.4)^2 + (5 − 4.4)^2 + (4.1 − 4.4)^2 + (6 − 4.4)^2] / 5\}^{1/2} = \left(\frac{5.14}{5}\right)^{1/2} = 1.01\%$$

F. $(1.03)(1.04)(1.043)(1.05)(1.041)(1.06) − 1 = 29.45\%$

G. The harmonic mean is 3/(1/31.4 + 1/34.5 + 1/37.25) = $34.22 average purchase price per share. Arithmetic mean price = (31.4 + 34.5 + 37.25)/ 3 = $34.38.

H. The portfolio return is a weighted average, 0.6 × 8.61% + 0.4 × 6% = 7.57%.

I. CV for Nopat = 6.77/6.28 = 1.08. CV for Emfund = 1.01/4.4 = 0.23. Emfund is less risky by this measure.

J. Sharpe ratio for Nopat is (6.28 − 2.8)/ 6.77 = 0.51. Sharpe measure for Emfund is (4.4 − 2.8) / 1.01 = 1.58.

K. Range is 6% − 3% = 3%. MAD is 0.73% = [(4.4% − 3%) + (4.4% − 4%) +(4.4% − 4.3%) +(5% − 4.4%) +(4.4% − 4.1%) +(6% − 4.4%)] / 6. Remember to use absolute values; we show all differences as positive to reflect that.

L. Semivariance = $\{[(3 − 4.4)^2 +(4 − 4.4)^2 +(4.3 − 4.4)^2 +(4.1 − 4.4)^2] / 3\}^{1/2} = 0.86\%$.

M. Average annual growth rate of dividends is the geometric mean: $(0.22/0.14)^{1/5} − 1 = 9.46\%$.

2. A. An ordinal scale.

    B. A nominal scale.

    C. A ratio scale.

    D. An interval scale.

    E. A ratio scale.

PROBABILITY CONCEPTS

EXAM FOCUS

This topic review covers important terms and concepts associated with probability theory. Random variables, events, outcomes, conditional probability, and joint probability are described. Probability rules such as the addition rule and multiplication rule are introduced. These rules are frequently used by finance practitioners, so your understanding of and ability to apply probability rules is likely to be tested on the exam.

Expected value, standard deviation, covariance, and correlation for individual asset and portfolio returns are discussed. A well-prepared candidate will be able to calculate and interpret these widely used measures.

This review also discusses counting rules, which lay the foundation for the binomial probability distribution that is covered in the next topic review.

LOS 9.a: Define a random variable, an outcome, an event, mutually exclusive events, and exhaustive events.

- A **random variable** is an uncertain quantity/number.
- An **outcome** is the realization of a random variable.
- An **event** is a single outcome or a set of outcomes.
- **Mutually exclusive events** are events that cannot both happen at the same time.
- **Exhaustive events** are those that include all possible outcomes.

Consider rolling a six-sided die. The number that comes up is a *random variable*. If you roll a four, that is an *outcome*. Rolling a 4 is an *event*, and rolling an even number is an *event*. Rolling a 4 and rolling a 6 are *mutually exclusive events*. Rolling an even number and rolling an odd number is a set of mutually exclusive and *exhaustive events*.

LOS 9.b: Explain the two defining properties of probability.

There are **two defining properties of probability.**

- The probability of occurrence of any event (E_i) is between zero and 1 (i.e., $0 \leq P(E_i) \leq 1$).
- If a set of events, E_1, E_2, ... E_n, is mutually exclusive and exhaustive, the probabilities of those events sum to 1 (i.e., $\Sigma P(E_i) = 1$).

The first of the defining properties introduces the term $P(E_i)$, which is shorthand for the "probability of event i." If $P(E_i) = 0$, the event will never happen. If $P(E_i) = 1$, the event is certain to occur, and the outcome is not random.

The probability of rolling any one of the numbers 1–6 with a fair die is $1/6 = 0.1667 = 16.7\%$. The set of events—rolling a number equal to 1, 2, 3, 4, 5, or 6—is exhaustive, and the individual events are mutually exclusive, so the probability of this set of events is equal to 1. We are certain that one of the values in this set of events will occur.

LOS 9.c: Distinguish among empirical, subjective, and a priori probabilities.

An **empirical probability** is established by analyzing past data. An **a priori probability** is determined using a formal reasoning and inspection process. A **subjective probability** is the least formal method of developing probabilities and involves the use of personal judgment.

The following are examples of statements that use *empirical, a priori,* and *subjective probabilities* for developing probabilities.

- *Empirical probability.* "Historically, the Dow Jones Industrial Average (DJIA) has closed higher than the previous close two out of every three trading days. Therefore, the probability of the Dow going up tomorrow is two-thirds, or 66.7 percent."
- *A priori probability.* "Yesterday, 24 of the 30 DJIA stocks increased in value. Thus, if one of the thirty stocks is selected at random, there is an 80 percent (= 24/30) probability that its value increased yesterday."
- *Subjective probability.* "It is my personal feeling that the probability the DJIA will close higher tomorrow is 90 percent."

LOS 9.d: State the probability of an event in terms of odds for or against the event.

Stating the odds that an event will or will not occur is an alternative way of expressing probabilities. Consider an event that has a probability of occurrence of 0.125, which is one-eighth. The *odds* that the event will occur are

$$\frac{0.125}{(1-0.125)} = \frac{1/8}{7/8} = \frac{1}{7}$$, which we state as 'the odds for the event occurring are one to seven'. The *odds against* the

event occurring are the reciprocal of 1/7, which is seven to one.

We can also get the probability of an event from the odds by reversing these calculations. If we know that the

odds for an event are one to six, we can compute the probability of occurrence as $\frac{1}{1+6} = \frac{1}{7} = 0.1429 = 14.29\%$.

Alternatively, the probability that the event will not occur is $\frac{6}{1+6} = \frac{6}{7} = 0.8571 = 85.71\%$.

Professor's Note: While I am quite familiar with the use of odds rather than probabilities at the horse track, I can't remember encountering odds for a stock or bond. The use of odds at the horse track lets you know how much you will win per $1 bet on a horse (less the track's percentage). If you bet on a 15-1 long shot and the horse wins, you will receive $15 and your $1 bet will be returned, so the profit is $15. Of course, if the horse loses, you would lose the $1 you bet and the "profit" is -$1.

One last point is that the expected return on the bet is zero, based on the probability of winning expressed in the odds. The prob-

ability of the horse winning when the odds are 15 to 1 is $\frac{1}{15+1} = \frac{1}{16}$ and the probability of the horse losing is 15/16. The

expected profit is $\frac{1}{16} \times \$15 + \frac{15}{16} \times (-\$1) = 0$.

LOS 9.e: Describe the investment consequences of probabilities that are mutually inconsistent.

With respect to investment opportunities, **inconsistent probabilities** exist when two assets are priced on the basis of probabilities that are different but assigned to the same event.

Let's look at an example of *profiting from inconsistent probabilities.* Assume that the occurrence of event Q will increase the return of both Stocks A and B. Further, assume that the pricing of Stock A incorporates a higher probability of Q than the pricing of Stock B. All else equal, Stock A is overpriced when compared to Stock B. In this situation, a savvy investor will lower her holdings of Stock A and increase her holdings of Stock B. An aggressive investor might even engage in a *pairs arbitrage trade,* which involves short-selling Stock A and using the proceeds to buy Stock B.

Now let's look at an example of the effects of *inconsistent probabilities on future prices.* Assume that the monetary authority is going to meet tomorrow to announce whether it will change interest rates. If rates are lowered, the operations of firms A and B will benefit equally. Despite this, it appears that the market's pricing of these firms has incorporated a different probability of a change in interest rates. Specifically:

- Stock A seems to be priced such that P(*decrease in interest rates*) = 0.7.
- Stock B seems to be priced such that P(*decrease in interest rates*) = 0.4.

Under these conditions, if the monetary authority lowers interest rates, Stock A will not increase in value by the same amount as Stock B because more "anticipation" of a rate reduction is already built into the price of Stock A. Also, if the decrease in interest rates does not occur, Stock A will decline in value more than Stock B.

LOS 9.f: Distinguish between unconditional and conditional probabilities.

- **Unconditional probability** (a.k.a., *marginal probability*) refers to the probability of an event regardless of the past or future occurrence of other events. If we are concerned with the probability of an economic recession, regardless of the occurrence of changes in interest rates or inflation, we are concerned with the unconditional probability of a recession.
- A **conditional probability** is one where the occurrence of one event affects the probability of the occurrence of another event. For example, we might be concerned with the probability of a recession *given* that the monetary authority increases interest rates. This is a conditional probability. The key word to watch for here is "given." Using probability notation, "the probability of A *given* the occurrence of B" is expressed as P(A | B), where the vertical bar (|) indicates "given," or "conditional upon." For our interest rate example above, the probability of a recession *given* an increase in interest rates is expressed as P(*recession|increase in interest rates*).

LOS 9.g: Define a joint probability and calculate and interpret the joint probability of two events.

The **joint probability** of two events is the probability that they will both occur. We can calculate this from the conditional probability that A will occur given B occurs (a conditional probability) and the probability that B will occur (the unconditional probability of B). This calculation is sometimes referred to as the multiplication rule of probability. Using the notation for conditional and unconditional probabilities we can express this rule as:

$$P(AB) = P(A \mid B) \times P(B)$$

This expression is read as follows: "The joint probability of A and B, P(AB), is equal to the conditional probability of A *given* B, P(A | B), times the unconditional probability of B, P(B)."

This relationship can be rearranged to define the conditional probability of A given B as follows:

$$P(A \mid B) = \frac{P(AB)}{P(B)}$$

Example: Multiplication rule

Consider the following information:

- P(I) = 0.4, the probability of the monetary authority increasing interest rates (I) is 40 percent.
- P(R | I) = 0.7, the probability of a recession (R) given an increase in interest rates is 70 percent.

What is P(RI), the joint probability of a recession *and* an increase in interest rates?

Answer:

Applying the multiplication rule, we get the following result:

$$P(RI) = P(R \mid I) \times P(I)$$
$$P(RI) = 0.7 \times 0.4$$
$$P(RI) = 0.28$$

Don't let the cumbersome notation obscure the simple logic of this result. If an interest rate increase will occur 40 percent of the time and lead to a recession 70 percent of the time when it occurs, the joint probability of an interest rate increase and a resulting recession is (0.4)(0.7) = (0.28) = 28 percent.

LOS 9.h: Calculate the probability that at least one of two events will occur, given the probability of each and the joint probability of the two events.

The addition rule for probabilities is used to determine the **probability that at least one of two events will occur.** For example, given two events, A and B, the addition rule can be used to determine the probability that either A or B will occur. If the events are *not* mutually exclusive, double counting must be avoided by subtracting the joint probability that *both* A and B will occur from the sum of the unconditional probabilities. This is reflected in the following general expression for the addition rule:

$$P(A \text{ or } B) = P(A) + P(B) - P(AB)$$

For mutually exclusive events, where the joint probability, P(AB), is zero, the probability that either A or B will occur is simply the sum of the unconditional probabilities for each event, P(A or B) = P(A) + P(B).

Figure 1 illustrates the addition rule with a Venn Diagram and highlights why the joint probability must be subtracted from the sum of the unconditional probabilities.

Figure 1: Venn Diagram
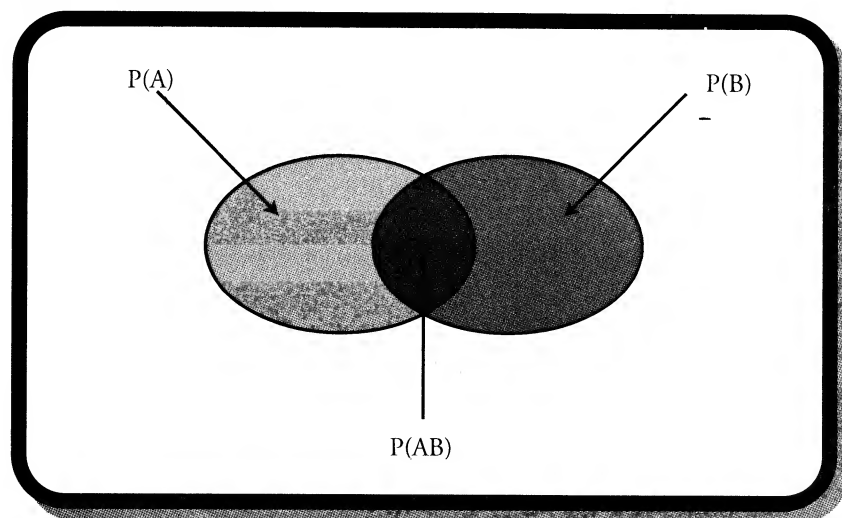
Example: Addition rule

Using the information in our previous interest rate and recession example and the fact that the unconditional probability of a recession, P(R), is 34 percent, **determine** the probability that either interest rates will increase *or* a recession will occur.

Answer:

Given that P(R) = 0.34, P(I) = 0.40, and P(RI) = 0.28, we can compute P(R or I) as follows:

P(R or I) = P(R) + P(I) − P(RI)
P(R or I) = 0.34 + 0.40 − 0.28
P(R or I) = 0.46

(See Exam Flashback #1.)

LOS 9.i: Distinguish between dependent and independent events.

Independent events refer to events for which the occurrence of one has no influence on the occurrence of the others. The definition of independent events can be expressed in terms of conditional probabilities. Events A and B are independent if and only if:

P(A | B) = P(A), or equivalently, P(B | A) = P(B)

If this condition is not satisfied, the events are **dependent events** (i.e., the occurrence of one is dependent on the occurrence of the other).

In our interest rate and recession example, recall that events I and R are not independent: the occurrence of I affects the probability of the occurrence of R. In this example, the independence conditions for I and R are violated because:

P(R) = 0.34, but P(R | I) = 0.7, the probability of a recession is greater when there is an increase in interest rates.

The best examples of independent events are found with the a priori probabilities of dice tosses or coin flips. A die has "no memory." Therefore, the event of rolling a 4 on the second toss is independent of rolling a 4 on the first toss. This idea may be expressed as:

P(4 on second toss | 4 on first toss) = P(4 on second toss) = 1/6 or 0.167

The idea of independent events also applies to flips of a coin:

P(heads on first coin | heads on second coin) = P(heads on first coin) = 1/2 or 0.50

Multiplication rule for independent events. For independent events A and B, P(A | B) = P(A), and the general form of the multiplication rule for probabilities can be simplified as:

P(A | B) × P(B) = P(A) × P(B) = P(AB)   [i.e., P(A and B) = P(A) × P(B)]
P(B | A) × P(A) = P(B) × P(A) = P(AB)   [i.e., P(B and A) = P(B) × P(A)]

LOS 9.j: Calculate a joint probability of any number of independent events.

On the roll of two dice, the **joint probability** of getting two 4s is calculated as:

P(4 on first die and 4 on second die) = P(4 on first die) × P(4 on second die) = 1/6 × 1/6 = 1/36 = 0.0278

On the flip of two coins, the probability of getting two heads is:

P(heads on first coin and heads on second coin) = 1/2 × 1/2 = 1/4 = 0.25

Hint: When dealing with *independent events*, the word *and* indicates multiplication, and the word *or* indicates addition. In probability notation:

P(A **or** B) = P(A) + P(B), and P(A **and** B) = P(A) × P(B)

The multiplication rule we used to calculate the joint probability of two independent events may be applied to any number of independent events as the following examples illustrate.

Example: Joint probability for more than two independent events

What is the probability of rolling three 4s in one simultaneous toss of three dice?

Answer:

Since the probability of rolling a 4 for each die is 1/6, the probability of rolling three 4s is:

P(three 4s on the roll of three dice) = 1/6 × 1/6 × 1/6 = 1/216 = 0.00463

Similarly:

P(four heads on the flip of four coins) = 1/2 × 1/2 × 1/2 × 1/2 = 1/16 = 0.0625

Example: Joint probability for more than two independent events

Using empirical probabilities, suppose we observe that the DJIA has closed higher on two-thirds of all days in the past few decades. Furthermore, it has been determined that up and down days are independent. Based on this information, **compute** the probability of the DJIA closing higher for five consecutive days.

Answer:

P(DJIA up five days in a row) = 2/3 × 2/3 × 2/3 × 2/3 × 2/3 = $(2/3)^5$ = 0.132

Similarly:

P(DJIA down five days in a row) = 1/3 × 1/3 × 1/3 × 1/3 × 1/3 = $(1/3)^5$ = 0.004

(See Exam Flashback #2.)

LOS 9.k: Calculate, using the total probability rule, an unconditional probability.

The **total probability rule** highlights the relationship between unconditional and conditional probabilities of mutually exclusive and exhaustive events. It is used to explain the unconditional probability of an event in terms of probabilities that are conditional upon other events.

In general, the unconditional probability of event R, $P(R) = P(R \mid S_1) \times P(S_1) + P(R \mid S_2) \times P(S_2) + \ldots + P(R \mid S_N) \times P(S_N)$, where the set of events $\{S_1, S_2, \ldots S_N\}$ is mutually exclusive and exhaustive.

Example: An investment application of unconditional probability

Building upon our ongoing example about interest rates and economic recession, we can assume that a recession can only occur with either of the two events—interest rates increase (I) or interest rates do not increase (I^C)—since these events are mutually exclusive and exhaustive. It is logical, therefore, that the sum of the two joint probabilities must be the unconditional probability of a recession. This can be expressed as follows:

$$P(R) = P(RI) + P(RI^C)$$

Applying the multiplication rule, we may restate this expression as:

$$P(R) = P(R \mid I) \times P(I) + P(R \mid I^C) \times P(I^C)$$

Assume that $P(R \mid I) = 0.70$, $P(R \mid I^C)$, the probability of recession if interest rates do not rise, is 10 percent and that $P(I) = 0.40$ so that $P(I^C) = 0.060$. The unconditional probability of a recession can be calculated as follows:

$$P(R) = P(R \mid I) \times P(I) + P(R \mid I^C) \times P(I^C)$$
$$= (0.70)(0.40) + (0.10)(0.60)$$
$$= 0.28 + 0.06 = 0.34$$
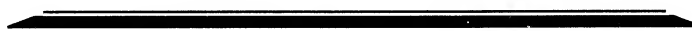
WARM-UP: EXPECTED VALUE, VARIANCE, AND COVARIANCE

Now that we have developed some probability concepts and tools for working with probabilities, we can apply this knowledge to determine the average value for a random variable that results from multiple experiments. This average is called an expected value. In any given experiment, the observed value for a random variable may not equal its expected value, and even if it does, the outcome from experiment to experiment will be different. The degree of dispersion of outcomes around the expected value of a random variable is measured using the variance and standard deviation. When pairs of random variables are being observed, the covariance and correlation are used to measure the extent of the relationship between the observed values for the two variables from one experiment to another.

LOS 9.m: Calculate an expected value using the total probability rule for expected value.

The **expected value** is the weighted average of the possible outcomes of a random variable, where the weights for each outcome is the probability that the outcome will occur. The mathematical representation for the expected value of random variable X is:

$$E(X) = \Sigma P(x_i)x_i = P(x_1)x_1 + P(x_2)x_2 + \ldots + P(x_n)x_n$$

Here, E is referred to as the expectations operator and is used to indicate the computation of a probability-weighted average. The symbol x_1 represents the first observed value (observation) for random variable X; x_2 is the second observation, and so on through the nth observation. The concept of expected value may be demonstrated using the a priori probabilities associated with a coin toss. On the flip of one coin, the occurrence of the event "heads" may be used to assign the value of one to a random variable. Alternatively, the event "tails" means the random variable equals zero. Statistically, we would formally write:

if heads, then X = 1
if tails, then X = 0

For a fair coin, P(heads) = P(X = 1) = 0.5, and P(tails) = P(X = 0) = 0.5. The expected value can be computed as follows:

$$E(X) = \Sigma P(x_i)x_i = P(X = 0)(0) + P(X = 1)(1) = (0.5)(0) + (0.5)(1) = 0.5$$

In any individual flip of a coin, *X* cannot assume a value of 0.5. Over the long term, however, the average of all the outcomes is expected to be 0.5. Similarly, the expected value of the roll of a fair die, where X = number that faces up on the die, is determined to be:

$$E(X) = \Sigma P(x_i)x_i = (1/6)(1) + (1/6)(2) + (1/6)(3) + (1/6)(4) + (1/6)(5) + (1/6)(6)$$
$$E(X) = 3.5$$

We can never roll a 3.5 on a die, but over the long term, 3.5 should be the average value of all outcomes.

The expected value is, statistically speaking, our "best guess" of the outcome of a random variable. While a 3.5 will never appear when a die is rolled, the average amount by which our guess differs from the actual outcomes is minimized when we use the expected value calculated this way.

Professor's Note: When we had historical data in an earlier topic review, we calculated the mean or simple arithmetic average and used deviations from the mean to calculate the variance and standard deviation. The calculations given here for the expected value (or weighted mean) is based on a probability model, whereas our earlier calculations were based on a sample or population of outcomes. Note that when the probabilities are equal, the simple mean is the expected value. For the roll of a die, all six outcomes are equally likely, so $\frac{1+2+3+4+5+6}{6} = 3.5$ gives us the same value as the probability model. However, with a probability model, the probabilities of the possible outcomes need not be equal and the simple mean is not necessarily the expected outcome as the following example illustrates.

Example: Expected value for stock returns

In the past, Tillard Corporation has fallen short, met, or exceeded analysts' earnings forecasts with the relative frequencies of 20 percent, 45 percent, and 35 percent, respectively. Historically, on the day Tillard announces earnings, the stock has fallen 3 percent, increased 1 percent, or increased 4 percent, depending on whether the earnings announcement fell short of, met, or exceeded the forecast earnings, respectively. What is the expected value of Tillard's stock return?

Answer:

The computation of the expected value of Tillard's stock return is demonstrated in Figure 2. As indicated in Figure 2, the expected value is the sum of the values in the column of the $P(x_i)x_i$ calculations.

Figure 2: Expected Value Computation

Event	Probability of Event $P(x_i)$	Stock Return x_i	$P(x_i)x_i$
Fall short of forecast	0.20	−0.03	−0.0060
Meet forecast	0.45	0.01	0.0045
Exceed forecast	0.35	0.04	0.0140
		Expected Value = $\Sigma P(x_i)x_i$ = 0.0125	

(See Exam Flashback #3.)

LOS 9.n: Diagram an investment problem, using a tree diagram.

You might well wonder where the returns and probabilities used in calculating expected values come from. A general framework called a tree diagram is used to show the probabilities of various outcomes. In Figure 3, we have shown estimates of EPS for four different outcomes: a good economy and relatively good results at the company, a good economy and relatively poor results at the company, a poor economy and relatively good results at the company, a poor economy and relatively poor results at the company. Using the rules of probability we can calculate the probabilities of each of the four EPS outcomes shown in the boxes on the right-hand side of the 'tree'.

Figure 3: A Tree Diagram
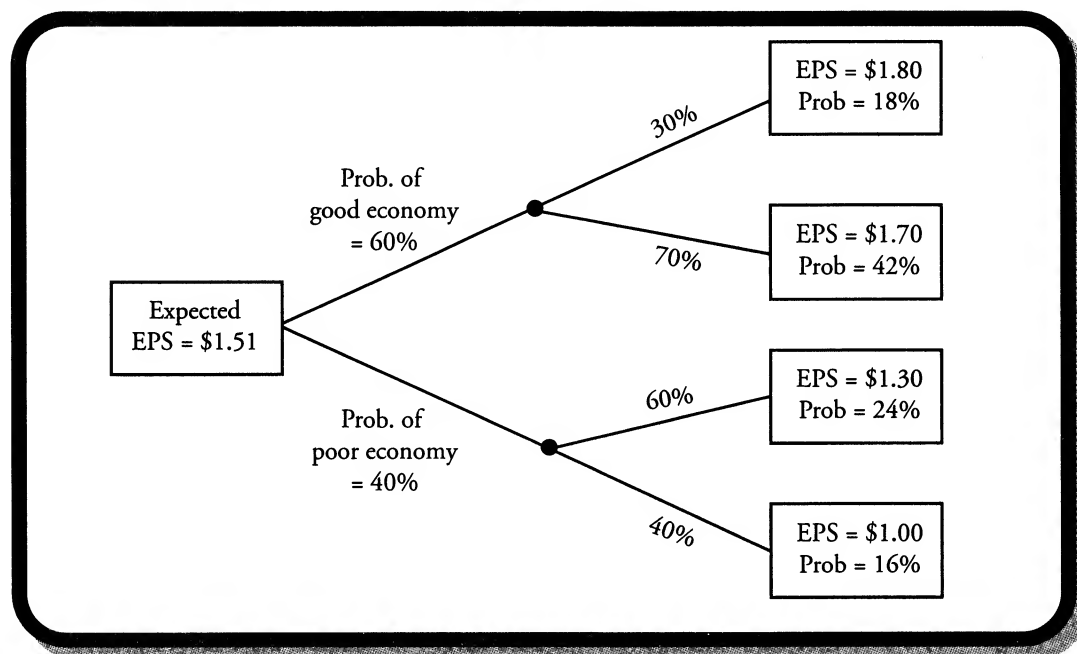
The expected EPS of $1.51 is simply calculated as:

0.18 × 1.80 + 0.42 × 1.70 + 0.24 × 1.30 + 0.16 × 1.00 = $1.51

Note that the probabilities of the four possible outcomes sum to 1.

LOS 9.l: Explain the use of conditional expectation in investment applications.

Conditional expected values are calculated using conditional probabilities. In investments, forecasts are frequently made using the expected value for a stock's return, earnings, and dividends. After the initial forecast, new and relevant information may surface that can affect the forecasted value(s). When this happens, the original forecast must be refined, and it is done using conditional expected values. As the name implies, *conditional expected values* are expected values that are contingent upon the occurrence of some other event. Let's look at an example to more fully develop this idea.

Example: Conditional expected value

Continuing with the preceding example, suppose the probability of the company's earnings announcement actually falling short of, meeting, or exceeding the earnings forecasts depends upon (is *conditional* upon) situations such as the general state of the economy when the announcement is made. Given a "good" economy at the time of the earnings announcement, the conditional probabilities for the level of announced earnings relative to forecasted earnings are as follows:

P(fall short of forecasted earnings | good economy) = 0.10
P(meet forecasted earnings | good economy) = 0.50
P(exceed forecasted earnings | good economy) = 0.40

Given a "poor" economy at the time of the announcement, the corresponding probabilities are:

P(fall short of forecasted earnings | poor) = 0.30
P(meet forecasted earnings | poor) = 0.40
P(exceed forecasted earnings | poor) = 0.30

Using these conditional probabilities, it is now possible to **compute** the conditional expected value for each possible state of the economy.

Answer:

The expected value of Tillard's stock returns given a "good" economy at the time earnings are announced is:

$$E(X \mid \text{good economy}) = (0.10)(-0.03) + (0.50)(0.01) + (0.40)(0.04) = 0.018$$

Similarly, the expected value of Tillard's stock return, given a "poor" economy at the time earnings are announced is:

$$E(X \mid \text{poor economy}) = (0.30)(-0.03) + (0.40)(0.01) + (0.30)(0.04) = 0.007$$

(See Exam Flashback #4.)

WARM-UP: COVARIANCE AND CORRELATION

The variance and standard deviation measure the dispersion, or movement, of only one variable. In many finance situations, however, we are interested in how two random variables move together. For investment applications, one of the most frequently analyzed pairs of random variables is the returns of two assets. Investors and managers frequently ask questions such as, "what is the relationship between the return for Stock A and Stock B?" or "what is the relationship between the performance of the S&P 500 and that of the automotive industry?" As you will soon see, the covariance and correlation are measures that provide useful information about how two random variables, such as asset returns, are related.

LOS 9.o: Define, calculate and interpret covariance and correlation.

Covariance is a measure of how two assets move together. It is the expected value of the product of the deviations of the two random variables from their respective expected values. A common symbol for the covariance between random variables X and Y is $\text{Cov}(X,Y)$. Since we will be mostly concerned with the covariance of asset returns, the following formula has been written in terms of the covariance of the return of asset i, R_i, and the return of asset j, R_j:

$$Cov(R_i, R_j) = E\{[R_i - E(R_i)][R_j - E(R_j)]\}$$

The following are *properties of the covariance:*

- The covariance is a general representation of the same concept as the variance. That is, the variance measures how a random variable moves with itself, and the covariance measures how one random variable moves with another random variable.
- The covariance of R_A with itself is equal to the variance of R_A; that is, $Cov(R_A, R_A) = Var(R_A)$.
- The covariance may range from negative infinity to positive infinity.

To aid in the interpretation of covariance, consider the returns of a stock and of a put option on the stock. These two returns will have a negative covariance because they move in opposite directions. The returns of two automotive stocks would likely have a positive covariance, and the returns of a stock and a riskless asset would have a zero covariance because the riskless asset's returns never move, regardless of movements in the stock's return.

Example: Covariance

Assume that the economy can be in three possible states (S) next year: boom, normal, or slow economic growth. An expert source has calculated that P(boom) = 0.30, P(normal) = 0.50, and P(slow) = 0.20. The returns for Stock A, R_A, and Stock B, R_B, under each of the economic states are provided in Figure 4. What is the covariance of the returns for Stock A and Stock B?

Answer:

First, the expected returns for each of the stocks must be determined.

$$E(R_A) = (0.3)(0.20) + (0.5)(0.12) + (0.2)(0.05) = 0.13$$
$$E(R_B) = (0.3)(0.30) + (0.5)(0.10) + (0.2)(0.00) = 0.14$$

The covariance can now be computed using the procedure described in Figure 4.

Figure 4: Covariance Computation

Event	P(S)	R_A	R_B	$P(S) \times [R_A - E(R_A)] \times [R_B - E(R_B)]$
Boom	0.3	0.20	0.30	$(0.3)(0.2 - 0.13)(0.3 - 0.14) = 0.00336$
Normal	0.5	0.12	0.10	$(0.5)(0.12 - 0.13)(0.1 - 0.14) = 0.00020$
Slow	0.2	0.05	0.00	$(0.2)(0.05 - 0.13)(0 - 0.14) = 0.00224$
$Cov(R_A, R_B) = \sum P(S) \times [R_A - E(R_A)] \times [R_B - E(R_B)] = \mathbf{0.00580}$				

The preceding example illustrates the use of a joint probability function. A *joint probability function* for two random variables gives the probability of the joint occurrence of specified outcomes. In this case we only had three joint probabilities:

$$P(R_A = 0.2 \text{ and } R_B = 0.3) = 0.30$$
$$P(R_A = 0.12 \text{ and } R_B = 0.1) = 0.50$$
$$P(R_A = 0.05 \text{ and } R_B = 0.0) = 0.20$$

Joint probabilities are often presented in a table such as the one shown in Figure 5. According to Figure 5, $P(R_A = 0.12$ and $R_B = 0.10) = 0.50$. This is the boldfaced probability represented in the cell at the intersection of the column labeled $R_B = 0.10$ and the row labeled $R_A = 0.12$. Similarly, $P(R_A = 0.20$ and $R_B = 0.10) = 0$.

Figure 5: Joint Probability Table

Joint Probabilities	$R_B = 0.30$	$R_B = 0.10$	$R_B = 0.00$
$R_A = 0.20$	0.30	0	0
$R_A = 0.12$	0	**0.50**	0
$R_A = 0.05$	0	0	0.20

In more complex applications, there would likely be positive values where the zeros appear in Figure 5. In any case, the sum of all the probabilities in the cells on the table must equal 1.

In practice, the covariance is difficult to interpret. This is mostly because it can take on extremely large values, ranging from negative to positive infinity, and, like the variance, these values are expressed in terms of square units.

To make the covariance of two random variables easier to interpret, it may be divided by the product of the random variable's standard deviations. The resulting value is called the **correlation coefficient**, or simply, correlation. The relationship between covariances, standard deviations, and correlations can be seen in the following expression for the correlation of the returns for asset i and j:

$$\text{Corr}\left(R_i, R_j\right) = \frac{\text{Cov}\left(R_i, R_j\right)}{\sigma\left(R_i\right)\sigma\left(R_j\right)}, \text{ which implies } \text{Cov}\left(R_i, R_j\right) = \text{Corr}\left(R_i, R_j\right)\sigma\left(R_i\right)\sigma\left(R_j\right)$$

The correlation between two random return variables may also be expressed as Corr R_i, R_j, $\rho(R_i, R_j)$, or $\rho_{i,j}$.

Properties of correlation of two random variables R_i and R_j are summarized here:

- Correlation measures the strength of the linear relationship between two random variables.
- Correlation has no units.
- The correlation ranges from −1 to +1.
- That is, $-1 \leq \text{Corr}(R_i, R_j) \leq +1$
- If $\text{Corr}(R_i, R_j) = 1.0$, the random variables have perfect positive correlation. This means that a movement in one random variable results in an exact measurable positive movement in the other.
- If $\text{Corr}(R_i, R_j) = -1.0$, the random variables have perfect negative correlation. This means that a movement in one random variable results in an exact measurable negative movement in the other.
- If $\text{Corr}(R_i, R_j) = 0$, there is no linear relationship between the variables, indicating that prediction of R_i cannot be made on the basis of R_j using linear methods.

Example: Correlation

Using our previous example, **compute** and **interpret** the correlation of the returns for stocks A and B given that $\sigma^2(R_A) = 0.0028$ and $\sigma^2(R_B) = 0.0124$ and recalling that $\text{Cov}(R_{A,}R_B) = 0.0058$.

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Answer:

First, it is necessary to convert the variances to standard deviations.

$$\sigma(R_A) = (0.0028)^{\frac{1}{2}} = 0.0529$$
$$\sigma(R_B) = (0.0124)^{\frac{1}{2}} = 0.1114$$

Now, the correlation between the returns of Stock A and Stock B can be computed as:

$$\text{Corr}(R_A, R_B) = \frac{0.0058}{(0.0529)(0.1114)} = 0.9842$$

The closeness of this value to +1 indicates that the strength of the linear relationship is positive and very strong.

LOS 9.p: Calculate and interpret the expected value, variance, and standard deviation particularly for return on a portfolio.

The expected value and variance for a portfolio of assets can be determined using the properties of the individual assets in the portfolio. To do this, it is necessary to establish the portfolio weight for each asset. As indicated in the formula below, the weight, w, of portfolio asset i is simply the market value currently invested in the asset divided by the current market value of the entire portfolio.

$$w_i = \frac{\text{market value of investment in asset i}}{\text{market value of the portfolio}}$$

Portfolio expected value. The expected value of a portfolio composed of n assets with weights, w_i, and expected values, R_i, can be determined using the following formula:

$$E(R_p) = \sum_{i=1}^{N} w_i E(R_i) = w_1 E(R_1) + w_2 E(R_2) + \dots + w_n E(R_n)$$

More often, we have expected returns (rather than expected prices). When the R_i are returns, the expected return for a portfolio, $E(R_p)$, is calculated using the asset weights and the same formula as above.

Portfolio variance. The variance of the portfolio return uses the portfolio weights also, but in a more complicated way:

$$\text{Var}(R_p) = \sum_{i=1}^{N} \sum_{j=1}^{N} w_i w_j \text{Cov}(R_i, R_j)$$

The way this formula works, particularly in its use of the double summation operator, $\sum\sum$, is best explained using two-asset and three-asset portfolio examples.

Example: Variance of a two-asset portfolio

Symbolically express the variance of a portfolio composed of risky asset A and risky asset B.

Answer:

Application of the variance formula provides the following:

$$Var(R_p) = w_A w_A Cov(R_A,R_A) + w_A w_B Cov(R_A,R_B) + w_B w_A Cov(R_B,R_A) + w_B w_B Cov(R_B,R_B)$$

Now, since $Cov(R_A,R_B) = Cov(R_B,R_A)$, and $Cov(R_A,R_A) = \sigma^2(R_A)$, this expression reduces to the following:

$$Var(R_p) = w_A^2 \sigma^2(R_A) + w_B^2 \sigma^2(R_B) + 2w_A w_B Cov(R_A,R_B)$$

Professor's Note: Know this formula!

Since $Cov(R_A,R_B) = \sigma(R_B)\sigma(R_A)\rho(R_A,R_B)$, another way to present this formula is:

$$Var(R_p) = w_A^2 \sigma^2(R_A) + w_B^2 \sigma^2(R_B) + 2w_A w_B \sigma(R_A)\sigma(R_B)\rho(R_A,R_B)$$

Example: Variance of a three-asset portfolio

A portfolio composed of risky assets A, B, and C will have a variance of return determined as:

$$\begin{aligned}
Var(R_p) =\ & w_A w_A Cov(R_A,R_A) + w_A w_B Cov(R_A,R_B) + w_A w_C Cov(R_A,R_C) \\
& + w_B w_A Cov(R_B,R_A) + w_B w_B Cov(R_B,R_B) + w_B w_C Cov(R_B,R_C) \\
& + w_C w_A Cov(R_C,R_A) + w_C w_B Cov(R_C,R_B) + w_C w_C Cov(R_C,R_C)
\end{aligned}$$

which can be reduced to the following expression:

$$\begin{aligned}
Var(R_p) =\ & w_A^2 \sigma^2(R_A) + w_B^2 \sigma^2(R_B) + w_C^2 \sigma^2(R_C) \\
& + 2w_A w_B Cov(R_A,R_B) + 2w_A w_C Cov(R_A,R_C) + 2w_B w_C Cov(R_B,R_C)
\end{aligned}$$

A portfolio composed of four assets will have four $w_i^2 Var(R_i)$ terms and six $2w_i w_j Cov(R_i,R_j)$ terms. A portfolio with five assets will have five $w_i^2 Var(R_i)$ terms and ten $2w_i w_j Cov(R_i,R_j)$ terms. In fact, the expression for the variance of an *n*-asset portfolio will have n $w_i^2 Var(R_i)$ terms and $n(n-1)/2$ $2w_i w_j Cov(R_i,R_j)$ terms.

The following formula is useful when we want to compute covariances, given correlations and variances.

$$Cov(R_i,R_j) = \sigma(R_i)\sigma(R_j)\rho(R_i,R_j)$$

LOS 9.q: Calculate covariance given a joint probability function.

Example: Expected value, variance, and covariance

What is the expected value, variance, and covariance(s) for a portfolio that consists of $400 in Asset A and $600 in Asset B? The joint probabilities of the returns of the two assets are in Figure 6.

Figure 6: Probability Table

Joint Probabilities	$R_B = 0.40$	$R_B = 0.20$	$R_B = 0.00$
$R_A = 0.20$	0.15	0	0
$R_A = 0.15$	0	0.60	0
$R_A = 0.04$	0	0	0.25

Answer:

The asset weights are:

w_A = $400 / ($400 + $600) = 0.40
w_B = $600 / ($400 + $600) = 0.60

The expected return of the individual assets is determined as:

$E(R_A) = P(R_{A1},R_{B1})R_{A1} + P(R_{A2},R_{B2})R_{A2} + P(R_{A3},R_{B3})R_{A3}$
$E(R_A) = (0.15)(0.20) + (0.60)(0.15) + (0.25)(0.04) = 0.13$
$E(R_B) = P(R_{B1},R_{A1})R_{B1} + P(R_{B2},R_{A2})R_{B2} + P(R_{B3},R_{A3})R_{B3}$
$E(R_B) = (0.15)(0.40) + (0.60)(0.20) + (0.25)(0.00) = 0.18$

The variance for the individual asset returns is determined as:

$$Var(R_A) = P(R_{A1},R_{B1})[(R_{A1} - E(R_A)]^2 + P(R_{A2},R_{B2})[(R_{A2} - E(R_A)]^2 + P(R_{A3},R_{B3})[(R_{A3} - E(R_A)]^2$$
$$Var(R_A) = (0.15)(0.20 - 0.13)^2 + (0.6)(0.15 - 0.13)^2 + (0.25)(0.04 - 0.13)^2 = 0.0030$$
$$Var(R_B) = P(R_{B1},R_{A1})[(R_{B1} - E(R_B)]^2 + P(R_{A2},R_{B2})[(R_{B2} - E(R_B)]^2 + P(R_{B3},R_{A3})[(R_{B3} - E(R_B)]^2$$
$$Var(R_B) = (0.15)(0.40 - 0.18)^2 + (0.6)(0.20 - 0.18)^2 + (0.25)(0.00 - 0.18)^2 = 0.0156$$

The covariance of the individual asset returns is determined as:

$$\begin{aligned} Cov(R_A, R_B) = &\ P(R_{A1},R_{B1})[R_{A1} - E(R_A)][(R_{B1} - E(R_B)] \\ &+ P(R_{A2},R_{B2})[R_{A2} - E(R_A)][(R_{B2} - E(R_B)] \\ &+ P(R_{A3},R_{B3})[R_{A3} - E(R_A)][(R_{B3} - E(R_B)] \end{aligned}$$

$$\begin{aligned} Cov(R_A, R_B) = &\ 0.15(0.20 - 0.13)(0.40 - 0.18) \\ &+ 0.60(0.15 - 0.13)(0.20 - 0.18) \\ &+ 0.25(0.04 - 0.13)(0.00 - 0.18) \\ &= 0.0066 \end{aligned}$$

Using the weights $w_A = 0.40$ and $w_B = 0.60$, the expected return and variance of the portfolio are computed as:

$$E(R_P) \quad = w_A E(R_A) + w_B E(R_B) = (0.4)(0.13) + (0.6)(0.18) = 0.16$$

$$\begin{aligned} Var(R_p) \quad &= (0.40)^2(0.003) + (0.60)^2(0.0156) + 2(0.4)(0.60)(0.0066) \\ &= 0.009264 \end{aligned}$$

Please note that as tedious as this example was, if more of the cells in the joint probability matrix were not zero, it could have been even more tedious.

Example: Correlation and covariance

Consider a portfolio of three assets, X, Y, and Z, where the individual market value of these assets is $600, $900, and $1,500, respectively. The market weight, expected return, and variance for the individual assets are presented below. The correlation matrix for the asset returns are shown in Figure 7. Using this information, **compute** the variance of the portfolio return.

$E(R_X) = 0.10$ $Var(R_X) = 0.0016$ $w_X = 0.2$
$E(R_Y) = 0.12$ $Var(R_Y) = 0.0036$ $w_Y = 0.3$
$E(R_Z) = 0.16$ $Var(R_Z) = 0.0100$ $w_Z = 0.5$

Figure 7: Stock X, Y, and Z Returns Correlation

Correlation Matrix			
Returns	R_X	R_Y	R_Z
R_X	1.00	0.46	0.22
R_Y	0.46	1.00	0.64
R_Z	0.22	0.64	1.00

Answer:

The expected return for the portfolio may be determined as:

$$E(R_p) = \sum w_i E(R_i) = w_1 E(R_1) + w_2 E(R_2) + w_3 E(R_3)$$
$$E(R_p) = (0.20)(0.10) + (0.30)(0.12) + (0.50)(0.16)$$
$$E(R_p) = 0.136$$

The variance of a three-asset portfolio return is determined using the formula:

$$\begin{aligned} Var(R_p) = \; & w_X^2 \sigma^2(R_X) + w_Y^2 \sigma^2(R_Y) + w_Z^2 \sigma^2(R_Z) + 2w_X w_Y Cov(R_X,R_Y) + 2w_X w_Z Cov(R_X,R_Z) \\ & + 2w_Y w_Z Cov(R_Y,R_Z) \end{aligned}$$

Here we must make use of the relationship $Cov(R_i,R_j) = \sigma(R_i)\sigma(R_j)\rho(R_i,R_j)$, since we are not provided with the covariances.

Let's solve for the covariances, then substitute the resulting values into the portfolio return variance equation.

$$Cov(R_X,R_Y) = (0.0016)^{\frac{1}{2}}(0.0036)^{\frac{1}{2}}(0.46) = 0.001104$$
$$Cov(R_X,R_Z) = (0.0016)^{\frac{1}{2}}(0.0100)^{\frac{1}{2}}(0.22) = 0.000880$$

$$\text{Cov}(R_Y, R_Z) = (0.0036)^{\frac{1}{2}}(0.0100)^{\frac{1}{2}}(0.64) = 0.003840$$

Professor's Note: We raised the variance terms to the ½ power to get its square root, which is the standard deviation.

Now we can solve for the variance of the portfolio returns as:

$$\text{Var}(R_P) = (0.20)^2(0.0016) + (0.30)^2(0.0036) + (0.50)^2(0.01) + (2)(0.2)(0.3)(0.001104)$$
$$+ (2)(0.2)(0.5)(0.00088) + (2)(0.3)(0.5)(0.00384)$$

$$\text{Var}(R_P) = 0.004348$$

Example: Covariance matrix

Assume you have a portfolio that consists of Stock S and a put option, O, on Stock S. The corresponding weights of these portfolio assets are $w_S = 0.90$ and $w_O = 0.10$. Using the covariance matrix provided in Figure 8, **calculate** the variance of the return for the portfolio.

Figure 8: Returns Covariance for Stock S and Put O

Covariance Matrix		
Returns	R_S	R_O
R_S	0.0011	−0.0036
R_O	−0.0036	0.016

Answer:

This is the simplest type of example because the most tedious calculations have already been performed. Simply extract the appropriate values from the covariance matrix and insert them into the variance formula.

Recall that the covariance of an asset with itself is the variance. Thus, the terms along the diagonal in the covariance matrix are return variances.

The portfolio return variance can be computed as:

$$\text{Var}(R_P) = (0.90)^2(0.0011) + (0.10)^2(0.016) + 2(0.90)(0.10)(-0.0036) = 0.000403$$

LOS 9.r: Calculate and interpret an updated probability, using Bayes' formula.

Bayes' formula is used to update a given set of prior probabilities for a given event in response to the arrival of new information. The rule for updating prior probability of an event is:

$$\text{updated probability} = \frac{\text{probability of new information for a given event}}{\text{unconditional probability of new information}} \times \text{prior probability of event}$$

Note in the following example of the application of Bayes' formula that we can essentially reverse a given set of conditional probabilities. This means that given $P(B)$, $P(A \mid B)$, and $P(A \mid B^C)$, it is possible to use Bayes' formula to compute $P(B \mid A)$.

Example: Bayes' formula

Electcomp Corporation manufactures electronic components for computers and other devices. There is speculation that Electcomp is about to announce a major expansion into overseas markets. The expansion will occur, however, only if Electcomp's managers estimate overseas demand to be sufficient to support the necessary sales. Furthermore, if demand is sufficient and overseas expansion occurs, Electcomp is likely to raise its prices.

Using O to represent the event of overseas expansion, I to represent a price increase, and I^c to represent no price increase, an industry analyst has estimated the unconditional and conditional probabilities shown as follows:

$$P(I) = 0.3$$
$$P(I^C) = 0.7$$
$$P(O \mid I) = 0.6$$
$$P(O \mid I^C) = 0.4$$

The analyst's estimates for P(I) and $P(I^C)$ are called the *priors* because they reflect what is already known. They do not reflect the current information about the possible overseas expansion.

Application of Bayes' formula allows us to compute P(I | O), the probability that prices will increase given that Electcomp announces that it will expand overseas (the new information). Using the multiplication rule, we can express the joint probability of I and O:

$$P(O \mid I) = P(IO) / P(I), \text{ and } P(IO) = P(I \mid O) \times P(O)$$

Based on these relationships, Bayes' formula can be expressed using the information from this example as indicated below [i.e., substitute P(IO) from the second equation into the first and solve for P(O | I)]:

$$P(I \mid O) = \frac{P(O \mid I)}{P(O)} \times P(I)$$

In order to solve this equation, P(O) must be determined. This can be done using the total probability rule:

$$P(O) = P(O \mid I) \times P(I) + P(O \mid I^C) \times P(I^C)$$
$$P(O) = (0.6 \times 0.3) + (0.4 \times 0.7)$$
$$P(O) = 0.46$$

Now the updated probability of the increase in prices given that Electcomp expands overseas can be computed:

$$P(I \mid O) = \frac{0.60}{0.46} \times 0.30 = 0.3913$$

This means that if the new information of "expand overseas" is announced, the prior probability estimate of P(I) = 0.30 must be increased to 0.3913.

Another illustration of the use of Bayes' formula may make it easier to remember and apply. Consider the following possibilities: There is a 60 percent probability the economy will outperform, and if it does, there is a 70 percent chance a stock will be up and a 30 percent chance the stock will go down. There is a 40 percent chance the economy will underperform, and if it does, there is a 20 percent chance the stock in question will appreciate and an 80 percent chance it will not. Let's diagram this situation.

Figure 9: A Probability Model
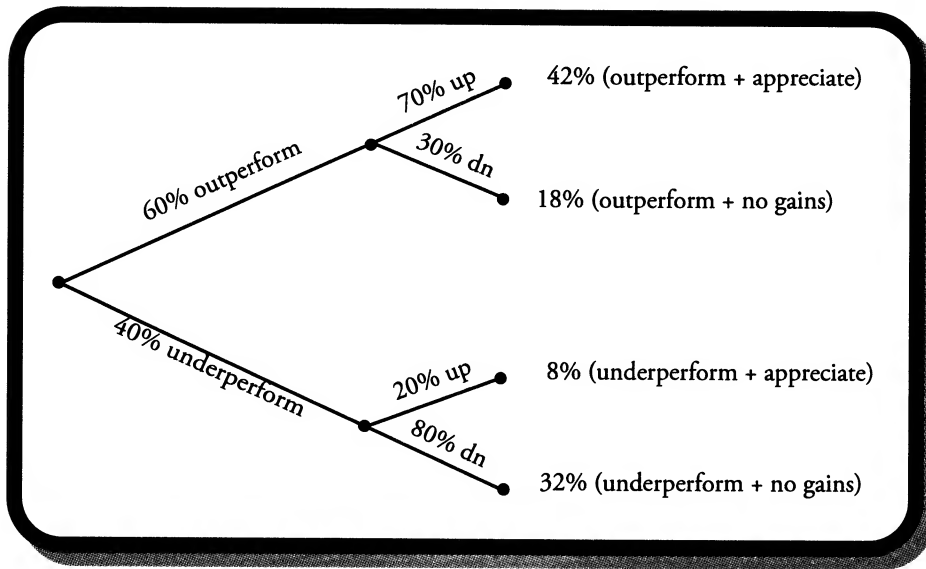
In Figure 9, we have multiplied the probabilities to calculate the probabilities of each of the four outcome pairs. Note that these sum to 1. Given that the stock has gains, what is our updated probability of an outperforming economy? We sum the probability of stock gains in both states (outperform and underperform) to get 42% + 8 percent = 50%. Given that the stock has gains, the probability that the economy has outperformed is

$\dfrac{42\%}{50\%} = 84\%$. In the previous notation the priors are as follows:

probability of economic outperformance = P(O) = 60%, the probability of stock gains given economic outperformance is P(G | O) = 70%, and the (unconditional) probability of a gain in stock price is 50%.

We are seeking P(O | G), the probability of outperformance given gains. Bayes' formula says:

$$P\big(O\big|G\big) = \frac{P\big(G\big|O\big) \times P\big(O\big)}{P\big(G\big)}, \text{ which is our } \frac{42\%}{50\%} = 84\%$$

LOS 9.s: Calculate and interpret the number of ways a specified number of tasks can be performed using the multiplication rule of counting.

The **multiplication rule of counting** applies when there is a series of k decisions to make, where each decision, i, can be made in n_i ways. That is, the first decision can be made in n_1 ways, the second decision, given the first, can be made in n_2 ways, and so on through the kth decision. The total number of ways that the k decisions can be made is $n_1 \times n_2 \times \ldots \times n_k$.

Example: Multiplication rule of counting

An analyst is interested in whether a firm decides to raise its prices, lower prices, or keep prices the same. The analyst is also interested in whether the firm decides to expand overseas. How many possible ways can these two decisions be made?

Answer:

Each of these events represents separate decisions that can occur in different ways, since there are three ways the prices can change, $n_1 = 3$. Also, the decision to expand can be made in two ways, expand or not expand, so $n_2 = 2$. This gives $n_1 \times n_2 = 3 \times 2 = 6$ possible ways the decisions can be made. To verify this result, let's list the possible pairs of decisions: (raise prices, expand), (raise prices, not expand), (lower prices, expand), (lower prices, not expand), (keep prices the same, expand), (keep prices the same, not expand).

If a third issue is up for consideration, for which there are n_3 decision choices, there will be $n_1 \times n_2 \times n_3$ decision combinations.

LOS 9.t: Solve counting problems using the factorial, combination, and permutation notations.

Labeling refers to the situation where there are n items that can each receive one of k different labels. The number of items that receives label 1 is n_1 and the number that receive label 2 is n_2, and so on such that $n_1 + n_2 + n_3 + \ldots + n_k = n$. The total number of ways that the labels can be assigned is:

$$\frac{n!}{(n_1!) \times (n_2!) \times \ldots \times (n_k!)}$$

where:
the symbol "!" stands for **factorial**. For example, $4! = 4 \times 3 \times 2 \times 1 = 24$, and $2! = 2 \times 1 = 2$.

The general expression for n factorial is:

$n! = n \times (n - 1) \times (n - 2) \times (n - 3) \times \ldots \times 1$, where by definition, $0! = 1$

Calculator help: On the TI, factorial is [2nd] [x!] (above the multiplication sign). On the HP, factorial is [g] [n!]. To compute 4! on the TI, enter [4][2nd][x!] = 24. On the HP, press [4][ENTER][g][n!].

Example: Labeling

Consider a portfolio consisting of eight stocks. Your goal is to designate four of the stocks as "long-term holds," three of the stocks as "short-term holds," and one stock as "sell." How many ways can these eight stocks be labeled?

Answer:

There are $8! = 40{,}320$ total possible sequences that can be followed to assign the three labels to the eight stocks. However, the order that each stock is assigned a label does not matter. For example, it does not matter which of the first three stocks labeled "long-term" is the first to be labeled. Thus, there are $4!$ ways to assign the long-term label. Continuing this reasoning to the other categories, there are $4! \times 3! \times 1!$ equivalent sequences for assigning the labels. To eliminate the counting of these redundant sequences, the total number of possible sequences ($8!$) must be divided by the number of redundant sequences ($4! \times 3! \times 1!$).

Thus, the number of *different* ways to label the eight stocks is:

$$\frac{8!}{4! \times 3! \times 1!} = \frac{40{,}320}{24 \times 6 \times 1} = 280$$

LOS 9.u: Calculate the number of ways to choose *r* objects from a total of *n* objects, when the order in which the *r* objects are listed matters, and calculate the number of ways to do so when the order does not matter.

A special case of labeling arises when the number of labels equals 2 (k = 2). That is, the *n* items can only be in one of two groups, and $n_1 + n_2 = n$. In this case, we can let $r = n_1$ and $n_2 = n - r$. Since there are only two categories, we usually talk about choosing *r* items. Then (n − r) are not chosen. The general formula for labeling when k = 2 is called the **combination formula** (or *binomial formula*) and is expressed as:

$$_nC_r = \frac{n!}{(n-r)!r!} \, ,$$

where $_nC_r$ is the number of possible ways (combinations) of selecting *r* items from a set of *n* items when the order of selection is not important. This is also written $\binom{n}{r}$ and read "n choose r."

Another useful formula is the **permutation formula**. A permutation is a specific ordering of a group of objects. The question of how many different groups of size *r* in specific order can be chosen from *n* objects is answered by the permutation formula. The number of permutations of *r* objects from *n* objects = $\frac{n!}{(n-r)!}$. We will give an example using this formula shortly.

Professor's Note: The combination formula $_nC_r$ and the permutation formula $_nP_r$ are both available on the TI calculator. To calculate the number of different groups of three stocks from a list of eight stocks (i.e., $_8C_3$) the sequence is 8 [2nd] [$_nC_r$] 3 [=] which yields 56. If we want to know the number of differently ordered groups of three that can be selected from a list of eight, we enter 8 [2nd] [$_nP_r$] 3 [=] to get 336 which is the number of permutations, $\frac{8!}{(8-3)!}$.

This function is not available on the HP calculator. Remember, current policy permits you to bring both calculators to the exam if you choose.

Example: Number of choices in any order

How many ways can three stocks be sold from an eight-stock portfolio?

Answer:

This is similar to the preceding labeling example. Since order does not matter, we take the total number of possible ways to select three of the eight stocks and divide by the number of possible redundant selections. Thus, the answer is:

$$\frac{8!}{5! \times 3!} = 56$$

In the preceding two examples, ordering did not matter. The order of selection could, however, be important. For example, suppose we want to liquidate only one stock position per week over the next three weeks. Once we choose three particular stocks to sell, the order in which they are sold must be determined. In this case, the concept of permutation comes into play. The *permutation formula* is:

$$_nP_r = \frac{n!}{(n-r)!} \, ,$$

where $_nP_r$ is the number of possible ways (permutations) to select r items from a set of n items when the order of selection is important. The permutation formula implies that there are r! *more* ways to choose r items if the *order* of selection *is important* than if order is not important.

Example: Permutation

How many ways are there to sell three stocks out of eight if the order of the sales is important?

Answer:

$$_nP_r = {}_8P_3 = \frac{8!}{(8-3)!} = \frac{8!}{5!} = 336$$

This is 3! times the 56 possible combinations computed in the preceding example for selecting the three stocks when the order was not important.

LOS 9.v: Identify which counting method is appropriate to solve a particular counting problem.

There are five guidelines that may be used to determine which counting method to employ when dealing with counting problems.

- The *multiplication rule* of counting is used when there are *two or more groups*. The key is that only *one* item may be selected from each group.
- *Factorial* is used by itself when there are *no groups*—we are only arranging a given set of n items. Given n items, there are n! ways of arranging them.
- The *labeling formula* applies to *three or more sub-groups* of predetermined size. Each element of the entire group must be assigned a place, or label, in one of the three or more sub-groups.
- The *combination formula* applies to *only two groups* of predetermined size. Look for the word "choose" or "combination."
- The *permutation formula* applies to *only two groups* of predetermined size. Look for a specific reference to "order" being important.

KEY CONCEPTS

1. A random variable is an uncertain outcome determined by chance.
2. The two properties of probability are:
   - The sum of the probabilities of all possible mutually exclusive events is 1.
   - The probability of any event is not larger than 1 or smaller than zero.
3. A priori probability measures probabilities based on well-defined inputs; empirical probability measures probability from observations or experiments; and subjective probability is an informed guess.
4. If the probabilities of an event's occurrence reflected in two securities' prices are inconsistent, a profitable trading strategy results.
5. Unconditional probability (marginal probability) is the probability of an event occurring; conditional probability, P(A|B), is the probability of an event (A) occurring given that another event (B) has occurred.
6. The general rule of multiplication is used to find the probability that two events will occur when one event is conditional on the other is P(A and B) = P(A|B) × P(B) which is P(A and B) = P(A) × P(B) for independent events.
7. The general rule of addition is that P(A or B) = P(A) + P(B) − P(AB).
8. The probability of an independent event is unaffected by the occurrence of other events, but the probability of a dependent event is changed by the occurrence of another event.
9. The probability that any one of a set of independent events will occur is the sum of their probabilities, and the probability that they will all occur is the product of their probabilities.
10. Using the total probability rule, the expected value is the probability-weighted average of the conditional expected values: $E(X) = \sum_{i=1}^{n}[P_i(S_i)] \times E(X_i \mid S_i)$, where S_i is a set of mutually exclusive and exhaustive events.
11. The expected value of a random variable, E(X), equals $\sum_{i=1}^{n}P(x_i)X_i$, and the variance of a random variable, Var(X), equals $\sum_{i=1}^{n}P(x_i)\,[X_i - E(X)]^2 = \sigma_X^2$.
12. Conditional expectations are used in investments to update expectations when a conditioning event has occurred.
13. Correlation is a standardized measure of association between two random variables; it ranges in value from −1 to +1 and is equal to $\dfrac{Cov(A,B)}{\sigma_A \sigma_B}$.
14. The expected returns and variance of a two-asset portfolio are given by:
$$E(R_p) = w_1 E(R_1) + w_2 E(R_2)$$
$$Var(R_p) = w_1^2 \sigma_1^2 + w_2^2 \sigma_2^2 + 2w_1 w_2 Cov_{12}$$
$$= w_1^2 \sigma_1^2 + w_2^2 \sigma_2^2 + 2w_1 w_2 \sigma_1 \sigma_2 \rho_{12}$$
15. Bayes' formula for updating probabilities based on the occurrence of an event O is:
$$P(I \mid O) = \frac{P(O \mid I)}{P(O)} \times P(I)$$
16. The multiplication rule of counting states that for a series of k tasks, each of which can be done in n_i ways, the total number of different ways the series of tasks can be performed is $(n_1)(n_2)\ldots(n_k)$.
17. The number of ways to order n objects is n factorial, n! = n × (n −1) × (n − 2) × … × 1.
18. The number of ways to choose a subset of r from a set of n when order doesn't matter is $\dfrac{n!}{(n-r)!r!}$; when order matters, there are $\dfrac{n!}{(n-r)!}$ permutations.

EXAM FLASHBACKS

Required CFA Institute disclaimer: Due to CFA curriculum changes from year to year, published sample exam questions and guideline answers prior to the current year may not reflect the current curriculum.

Exam Flashback # 1
Source: Question #42 from '98 sample exam.

The probability of Company X's stock price rising 10 percent is 5 percent, and the probability of Company Y's stock price rising 10 percent is 25 percent. The probability that the stock price of at least one of the companies will rise 10 percent is:
A. 1.25%.
B. 20.00%.
C. 28.75%.
D. 30.00%.

Exam Flashback # 2
Source: Question #41 from '98 sample exam.

An analyst estimates that the stock of Company A has a 10 percent probability of declining 20 percent in price while the stock of Company B has a 15 percent probability of declining 20 percent. The probability of both these events occurring is:
A. 1.5%.
B. 5.0%.
C. 23.4%.
D. 25.0%.

Exam Flashback # 3
Source: Question #24 from the '94 and '96 actual exams and '97 sample exam.

An analyst estimates that a stock has the following probabilities of return depending on the state of the economy.

Economy	Probability	Return
Good	0.1	15%
Normal	0.6	13%
Poor	0.3	7%

The expected return of the stock is:
A. 7.8%.
B. 11.4%.
C. 11.7%.
D. 13.0%.

Exam Flashback # 4

Source: Question #43 from '94 actual exam and '98 sample exam.

An analyst constructs the following probability table for the market and Company M's stock:

State of the Economy	Probability of the Economic State	Stock Performance	Conditional Probability of Stock Performance
Good	0.40	Good Neutral Poor	0.50 0.25 0.25
Neutral	0.30	Good Neutral Poor	0.50 0.30 0.20
Poor	0.30	Good Neutral Poor	0.40 0.40 0.20

The total probability of good performance for Company M's stock is:
A. 0.10.
B. 0.20.
C. 0.40.
D. 0.47.

CONCEPT CHECKERS: PROBABILITY CONCEPTS

1. Given the conditional probabilities in the table below and the unconditional probabilities $P(Y = 1) = 0.3$ and $P(Y = 2) = 0.7$, what is the expected value of X?

x_i	$P(x_i / Y = 1)$	$P(x_i / Y = 2)$
0	0.2	0.1
5	0.4	0.8
10	0.4	0.1

A. 4.3.
B. 5.0.
C. 5.3.
D. 5.7.

Use the following data to answer Questions 2 through 6.

Joint Probabilities			
Returns	$R_B = 0.5$	$R_B = 0.0$	$R_B = 0.0$
$R_A = -0.1$	0.4	0	0
$R_A = 0.1$	0	0.3	0
$R_A = 0.3$	0	0	0.3

2. Given the joint probability table, the expected return of Stock A is *closest* to:
    A. 0.12.
    B. 0.08.
    C. 0.20.
    D. 0.15.

3. Given the joint probability table, the standard deviation of Stock B is *closest* to:
    A. 0.060.
    B. 0.212.
    C. 0.045.
    D. 0.245.

4. Given the joint probability table, the variance of Stock A is *closest* to:
    A. 0.0276.
    B. 0.1661.
    C. 0.0450.
    D. 0.0129.

5. Given the joint probability table, the covariance between A and B is *closest* to:
    A. 0.03690.
    B. 0.00129.
    C. −0.03600.
    D. −0.00129.

6. Given the joint probability table, the correlation between R_A and R_B is *closest* to:
    A. −0.88.
    B. −0.33.
    C. +0.33.
    D. +0.50.

7. The probability that the Dow Jones Industrial Average (DJIA) will increase tomorrow is 2/3. The probability of an increase in the DJIA stated as odds is:
    A. two-to-one.
    B. one-to-three.
    C. one-to-two.
    D. two-to-three.

8. A discrete uniform distribution (each event has an equal probability of occurrence) has the following possible outcomes for X [1, 2, 3, 4]. The variance of this distribution is *closest* to:
    A. 0.00.
    B. 1.00.
    C. 1.25.
    D. 2.00.

9. If events A and B are mutually exclusive, then:
    A. $P(A \mid B) = P(A)$.
    B. $P(A \mid B) = P(B)$.
    C. $P(AB) = P(A) \times P(B)$.
    D. $P(A \text{ or } B) = P(A) + P(B)$.

10. At a charity ball, 800 names were put into a hat. Four of the names are identical. On a random draw, what is the probability that one of these four names will be drawn?
    A. 0.004.
    B. 0.005.
    C. 0.010.
    D. 0.025.

11. Among 900 taxpayers with incomes below $100,000, 35 were audited by the IRS. The probability that a randomly chosen individual with an income below $100,000 was audited is *closest* to:
    A. 0.039.
    B. 0.125.
    C. 0.350.
    D. 1.000.

12. Which of the following values *cannot* be the probability of an event?
    A. 0.00.
    B. 0.78.
    C. 1.25.
    D. 1.00.

13. Two mutually exclusive events:
    A. always occur together.
    B. cannot occur together.
    C. can sometimes occur together.
    D. occur together based only upon mutual consent.

14. Two events are said to be independent if the occurrence of one event:
    A. means that the second event cannot occur.
    B. means that the second event is certain to occur.
    C. affects the probability of the occurrence of the other event.
    D. does not affect the probability of the occurrence of the other event.

Use the following conditional probabilities to answer Questions 15 through 18.

State of the Economy	Probability of the Economic State	Stock Performance	Conditional Probability of Stock Performance
Good	0.30	Good Neutral Poor	0.60 0.30 0.10
Neutral	0.50	Good Neutral Poor	0.30 0.40 0.30
Poor	0.20	Good Neutral Poor	0.10 0.60 0.30

15. What is the conditional probability of having good stock performance in a poor economic environment?
    A. 0.02.
    B. 0.03.
    C. 0.10.
    D. 0.30.

16. What is the joint probability of having a good economy and a neutral stock performance?
    A. 0.09.
    B. 0.20.
    C. 0.30.
    D. 1.30.

17. What is the total probability of having a good performance in the stock?
    A. 0.20.
    B. 0.35.
    C. 0.65.
    D. 1.00.

18. Given that the stock had good performance, the probability the state of the economy was good is *closest* to:
    A. 0.35.
    B. 0.46.
    C. 0.51.
    D. 1.00.

19. Consider a universe of ten bonds from which an investor will ultimately purchase six bonds for his portfolio. If the order in which he buys these bonds is not important, how many potential six-bond combinations are there?
    A. 7.
    B. 210.
    C. 5,040.
    D. Cannot be determined.

20. The correlation of returns between Stocks A and B is 0.50. The covariance between these two securities is 0.0043, and the standard deviation of the return of Stock B is 26 percent. The variance of returns for Stock A is:
    A. 0.0331.
    B. 0.0011.
    C. 0.2656.
    D. 0.0112.

21. There are ten sprinters in the Olympic finals. How many different ways can the gold, silver, and bronze medals be awarded?
    A. 120.
    B. 720.
    C. 1,440.
    D. 604,800.

22. Which of the following is **NOT** a probability distribution?

    A. $X = [1,2,3,4]$; Prob $[X_i] = \dfrac{X_i}{10}$.

    B. $X = [1,2,3,4]$; Prob $[X_i] = \dfrac{X_i^2}{30}$.

    C. $X = [5,10]$; Prob $[X_i] = \dfrac{8 - X_i}{5}$.

    D. $X = [5,10]$; Prob $[X_i] = \dfrac{X_i - 3}{9}$.

COMPREHENSIVE PROBLEMS: PROBABILITY CONCEPTS

1. Given the following probability data for the return on the market and the return on Best Oil, calculate the covariance of returns between Best Oil and the market.

    Joint Probability Table

	$R_{Best} = 20\%$	$R_{Best} = 10\%$	$R_{Best} = 5\%$
$R_{Mkt} = 15\%$	0.4	0	0
$R_{Mkt} = 10\%$	0	0.2	0
$R_{Mkt} = 0\%$	0	0	0.4

2. The correlation of returns between the returns on Cape Products and Dogger Industries is 0.6. The standard deviation of returns for Cape is 15 percent and the standard deviation of returns for Dogger is 20 percent. The expected return for Dogger is 18 percent and the expected return for Cape is 12 percent. Calculate the expected returns and standard deviation of returns on a portfolio that has $300,000 invested in Dogger and $200,000 invested in Cape.

3. M. Atwood, an analyst, has developed a scoring system for bonds and found that if the score from a bond is less than 20, there is a probability of 85 percent that it will default within five years. If a bond's score is greater than or equal to 20, there is only a 40 percent chance that it will default within five years. Given that a randomly chosen bond currently has a 25 percent probability of a score less than 20, what is the probability that a bond that defaults within the next five years has a score of 20 or higher?

ANSWERS – EXAM FLASHBACKS

1. **C** When calculating total probability, you are solving for the total area enclosed in the event space.

   Total area = area of event 1 + area of event 2 – the overlap, where the overlap is the joint probability of events 1 and 2 (i.e., P(1)×P(2)).

   Total area = P(1) + P(2) – P(1)P(2) = (0.05) + (0.25) – (0.05)(0.25) = 0.2875 or 28.75%.

2. **A** With joint probabilities, you are solving for the overlap, or the probability of one event AND the other. The "AND" indicates multiplication, so P(event 1 AND event 2) = P(1) × P(2) = (0.1)(0.15) = 0.015 or 1.5%.

3. **B** Expected return = $\Sigma(P_i)(R_i)$

   E(R) = (0.1)(15%) + (0.6)(13%) + (0.3)(7%)

   E(R) = 1.5 + 7.8 + 2.1 = 11.4%

   Professor's Note: I can't stress how important this simple relationship is throughout the entire CFA curriculum.

4. **D** The total probability of one stock outcome (good) across all states (S_i) = $\Sigma P[(S_i) \times P(\text{"good" stock given state}_i)]$.

   S_i = (0.4)(0.5) + (0.3)(0.5) + (0.3)(0.4) = 0.47

ANSWERS – CONCEPT CHECKERS: PROBABILITY CONCEPTS

1. **C** E(X|Y = 1) = (0.2)(0) + (0.4)(5) + (0.4)(10) = 6 and E(X|Y = 2) = (0.1)(0) + (0.8)(5) + (0.1)(10) = 5

   E(X) = (0.3)(6) + (0.7)(5) = 5.30

2. **B** $E(R_A)$ = (0.4)(–0.1) + (0.3)(0.1) + (0.3)(0.3) = 0.08

3. **D** Expected return of Stock B = (0.4)(0.5) = 0.20

   $VAR(R_B)$ = $0.4(0.5 - 0.2)^2 + 0.3(0 - 0.2)^2 + 0.3(0 - 0.2)^2$ = 0.06

   Standard deviation = $(0.06)^{1/2}$ = 0.2449

4. **A** $E(R_A)$ = (0.4)(–0.1) + (0.3)(0.1) + (0.3)(0.3) = 0.08

   $VAR(R_A)$ = $0.4(-0.1 - 0.08)^2 + 0.3(0.1 - 0.08)^2 + 0.3(0.3 - 0.08)^2$ = 0.0276

5. **C** $COV(R_A, R_B)$ = 0.4(–0.1 – 0.08)(0.5 – 0.2) + 0.3(0.1 – 0.08)(0 – 0.2) + 0.3(0.3 – 0.08)(0 – 0.2) = –0.036

6. **A** $CORR(R_A, R_B)$ = $COV(R_A, R_B)$ / $\sigma(R_A)\sigma(R_B)$ = –0.036 / (0.1661 × 0.24494) = –0.036 / 0.0406862 = –0.8849

7. **A** Odds for E = P(E) / [1 – P(E)] = $\dfrac{2/3}{1/3}$ = 2/1 = two-to-one

8. **C** Expected value = (1/4)(1 + 2 + 3 + 4) = 2.5

   Variance = 1.25 = $(1/4)[(1 - 2.5)^2 + (2 - 2.5)^2 + (3 - 2.5)^2 + (4 - 2.5)^2]$

   Note that since each observation is equally likely, each has 25% (1/4) chance of occurrence.

9. **D** There is no intersection of events when events are mutually exclusive. $P(A|B) = P(A) \times P(B)$ is only true for independent events. Note that since A and B are mutually exclusive (cannot both happen), $P(A|B)$ and $P(AB)$ must both be equal to zero, making answers A, B, and C incorrect.

10. **B** P(name 1 or name 2 or name 3 or name 4) = 1/800 + 1/800 + 1/800 + 1/800 = 4/800 = 0.005

11. **A** 35 / 900 = 0.0389

12. **C** Probabilities may range from zero (meaning no chance of occurrence) through 1 (which means a sure thing).

13. **B** One or the other may occur, not both.

14. **D** Two events are said to be independent if the occurrence of one event does not affect the probability of the occurrence of the other event.

15. **C** Go to the poor state and read off the probability of good performance [i.e., P(poor performance | good economy) = 0.10].

16. **A** P(good economy and neutral performance) = P(good economy)P(neutral performance | good economy) = (0.3)(0.3) = 0.09

17. **B** (0.3)(0.6) + (0.5)(0.3) + (0.2)(0.1) = 0.35. This is the sum of all the joint probabilities for good performance over all states [i.e., $\sum P(\text{economic state}_i) \, P(\text{good performance} \mid \text{economic state}_i)$].

18. **C** P(good economy | good performance) = Prob(good stock performance | good economy) × Prob(good economy)/ P(good stock performance).

$$= \frac{(0.6)(0.3)}{(0.3)(0.6)+ (0.5)(0.3) + (0.2)(0.1)} = \frac{0.18}{0.35} = 0.5143$$

19. **B** $_nC_r = \dfrac{n!}{(n-r)!r!} = \;_{10}C_6 = \dfrac{10!}{(10-6)!6!} = \dfrac{10!}{4!6!} = 210$

20. **B** $Corr(R_A, R_B) = \dfrac{Cov(R_A, R_B)}{[\sigma(R_A)][\sigma(R_B)]}$

$$\sigma^2(R_A) = \left[\frac{Cov(R_A, R_B)}{\sigma(R_B)Corr(R_A, R_B)}\right]^2 = \left[\frac{0.0043}{(0.26)(0.5)}\right]^2 = 0.0331^2 = 0.0011$$

21. **B** Since the order of the top three finishers matters, we need to use the permutation formula.

$$_{10}P_3 = \frac{10!}{(10-3)!} = 720$$

22. **C** $\dfrac{8-5}{5} + \dfrac{8-10}{5} = \dfrac{1}{5}$, and $\dfrac{8-10}{5}$ is negative, so this satisfies neither of the requirements for a probability

distribution. The others have prob$[X_i]$ between zero and one and $\sum P[X_i] = 1$, and thus satisfy both requirements for a probability distribution.

ANSWERS – COMPREHENSIVE PROBLEMS: PROBABILITY CONCEPTS

1. **B** $E(R_{Best}) = 0.4(20\%) + 0.2(10\%) + 0.4(5\%) = 12\%$

$E(R_{Mkt}) = 0.4(15\%) + 0.2(10\%) + 0.4(0\%) = 8\%$

$$Cov(R_{Best}, R_{Mkt}) = 0.4(20\% - 12\%)(15\% - 8\%)$$
$$+ 0.2(10\% - 12\%)(10\% - 8\%)$$
$$+ 0.4(5\% - 12\%)(0\% - 8\%)$$
$$= 0.4(8)(7) + 0.2(-2)(2) + 0.4(-7)(-8) = 44$$

Remember the units of covariance (like variance) are percent squared here. We used whole number percents and got 44; if we had used decimals, we would have gotten 0.0044.

2. The portfolio weight for Dogger (W_D) is $\frac{300}{500} = 60\%$ and for Cape, the portfolio weight is $\frac{200}{500} = 40\%$. The expected return on the portfolio is $0.6(18\%) + 0.4(12\%) = 15.6\%$. The variance is $(0.6)^2(0.2)^2 + (0.4)^2(0.15)^2 + 2(0.6)(0.4)(0.6)(0.2)(0.15) = 0.02664$. The standard deviation of portfolio returns is $\sqrt{0.02664} = 16.32\%$.

3. Construct the following tree:
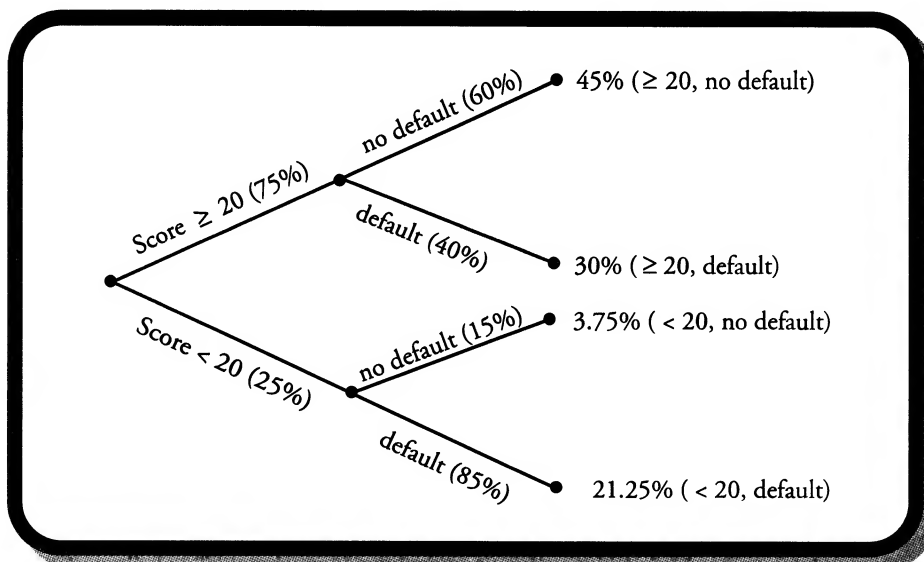
Total probability of default = 30% + 21.25% = 51.25%. Percent of defaulting bonds with score

$\geq 20 = \frac{30\%}{51.25\%} = 58.5\%$. A bond that defaults in the next five years has a 58.5% probability of having a current score greater than or equal to 20. Note that we have employed Bayes' theorem here to update the score expectation based on the additional information that a bond has defaulted.

COMMON PROBABILITY DISTRIBUTIONS

EXAM FOCUS

This topic review contains a lot of very testable material. Learn the difference between discrete and continuous probability distributions. The binomial and normal distributions are the most important here. You must learn the properties of both distributions and memorize the formulas for the mean and variance of the binomial distribution and for the calculation of the probability of a particular value when given a binomial probability distribution. Learn what shortfall risk is and how to calculate and use Roy's safety-first criterion. Candidates must know how to standardize a normally distributed random variable, use a z-table, and construct confidence intervals. These skills will be used repeatedly in the topic reviews that follow. Additionally, understand the basic features of the lognormal distribution, Monte Carlo simulation, and historical simulation. Finally, it would be a good idea to know how to get continuously compounded rates of return from holding period returns. Other than that, no problem.

LOS 10.a: Define and explain a probability distribution.

A **probability distribution** describes the probabilities of all the possible outcomes for a random variable. The probabilities of all possible outcomes must sum to 1. A simple probability distribution is that for the roll of one fair die; there are six possible outcomes and each one has a probability of 1/6, so they sum to 1. The probability distribution of all the possible returns on the S&P 500 index for the next year is a more complex version of the same idea.

LOS 10.b: Distinguish between and give examples of discrete and continuous random variables.

LOS 10.c: Describe the set of possible outcomes of a specified random variable.

A **discrete random variable** is one for which the number of possible outcomes can be counted, and for each possible outcome, there is a measurable and positive probability. An example of a discrete random variable is the number of days it rains in a given month, because there is a finite number of possible outcomes—the number of days it can rain in a month is defined by the number of days in the month.

A **continuous random variable** is one for which the number of possible outcomes is infinite, even if lower and upper bounds exist. The actual amount of daily rainfall between zero and 100 inches is an example of a continuous random variable because the actual amount of rainfall can take on an infinite number of values. Daily rainfall can be measured in inches, half inches, quarter inches, thousandths of inches, or in even smaller increments. Thus, the number of possible daily rainfall amounts between zero and 100 inches is essentially infinite.

The assignment of probabilities to the possible outcomes for discrete and continuous random variables provides us with discrete probability distributions and continuous probability distributions. The difference between these types of distributions is most apparent for the following properties:

- For a *discrete distribution,* p(x) = 0 when x cannot occur, or p(x) > 0 if it can. Recall that p(x) is read: "the probability that random variable X = x." For example, the probability of it raining on 33 days in June is zero because this cannot occur, but the probability of it raining 25 days in June has some positive value.

- For a *continuous distribution,* p(x) = 0 even though x can occur. We can only consider $P(x_1 \leq X \leq x_2)$ where x_1 and x_2 are actual numbers. For example, the probability of receiving two inches of rain in June is zero because two inches is a single point in an infinite range of possible values. On the other hand, the probability of the amount of rain being between 1.99999999 and 2.00000001 inches has some positive value. In the case of continuous distributions, it is interesting to note that $P(x_1 \leq X \leq x_2) = P(x_1 < X < x_2)$ because $p(x_1) = p(x_2) = 0$.

In finance, some discrete distributions are treated as though they are continuous because the number of possible outcomes is very large. For example, the increase or decrease in the price of a stock traded on an American exchange is recorded in dollars and cents. Yet, the probability of a change of exactly $1.33 or $1.34 or any other specific change is almost zero. It is customary, therefore, to speak in terms of the probability of a range of possible price change, say between $1.00 and $2.00. In other words p(price change = 1.33) is essentially zero, but p($1 < price change < $2) > 0.

LOS 10.d: Define a probability function, state its two key properties, and determine whether a given function satisfies those properties.

A **probability function**, denoted p(x), specifies the probability that a random variable is equal to a specific value. More formally, p(x) is the probability that random variable X takes on the value x, or p(x) = P(X = x).

The **two key properties of a probability function** are:

- $0 \leq p(x) \leq 1$.
- $\sum p(x) = 1$, the sum of the probabilities for *all* possible outcomes, x, for a random variable, X, equals 1.

    Example: Evaluating a probability function

    Consider the following function: $X = \{1, 2, 3, 4\}$, $p(x) = \dfrac{x}{10}$, else p(x) = 0

    Determine whether this function satisfies the conditions for a probability function.

    Answer:

    Note that all of the probabilities are between zero and one, and the sum of all probabilities equals one:

    $$\sum p(x) = \frac{1}{10} + \frac{2}{10} + \frac{3}{10} + \frac{4}{10} = 0.1 + 0.2 + 0.3 + 0.4 = 1$$

    Both conditions for a probability function are satisfied.

LOS 10.e: Define a probability density function.

A **probability density function** (pdf) is a function, denoted f(x), that can be used to generate the probability that outcomes of a continuous distribution lie within a particular range of outcomes. For a continuous distribution, it is the equivalent of a *probability function* for a discrete distribution. Remember, for a continuous distribution the probability of any one particular outcome (of the infinite possible outcomes) is zero. A pdf is used to calculate the probability of an outcome between two values (i.e., the probability of the outcome falling within a specified range). How that is actually done (it involves using calculus to take the integral of the function) is, thankfully, beyond the scope of the material required for the exam.

LOS 10.f: Define a cumulative distribution function and calculate and interpret probabilities for a random variable, given its cumulative distribution function.

A **cumulative distribution function** (cdf), or simply *distribution function,* defines the probability that a random variable, *X*, takes on a value equal to or less than a specific value, *x*. It represents the sum, or *cumulative value,* of the probabilities for the outcomes up to and including a specified outcome. The cumulative distribution function for random variable, *X*, may be expressed as $F(x) = P(X \leq x)$. For example, consider the probability function defined earlier for X = {1, 2, 3, 4}, p(x) = x / 10. For this distribution, F(3) = 0.6 = 0.1 + 0.2 + 0.3, and F(4) = 1 = 0.1 + 0.2 + 0.3 + 0.4. This means that F(3) is the cumulative probability that outcomes 1, 2, or 3 occur, and F(4) is the cumulative probability that one of the possible outcomes occurs.

LOS 10.g: Define a discrete uniform random variable and calculate and interpret probabilities, given a discrete uniform distribution.

A **discrete uniform random variable** is one for which the probabilities for all possible outcomes for a discrete random variable are equal. For example, consider the *discrete uniform probability distribution* defined as X = {1, 2, 3, 4, 5}, p(x) = 0.2. Here, the probability for each outcome is equal to 0.2 [i.e., p(1) = p(2) = p(3) = p(4) = p(5) = 0.2]. Also, the cumulative distribution function for the *n*th outcome, $F(x_n) = np(x)$, and the probability for a range of outcomes is p(x)k, where *k* is the number of possible outcomes in the range.

> **Example: Discrete uniform distribution**
>
> Determine p(6), F(6), and $P(2 \leq X \leq 8)$ for the discrete uniform distribution function defined as:
>
> X {2, 4, 6, 8, 10}, p(x) = 0.2
>
> **Answer:**
>
> p(6) = 0.2, since p(x) = 0.2 for all *x*. F(6) = $P(X \leq 6)$ = np(x) = 3(0.2) = 0.6. Note that n = 3 since 6 is the third outcome in the range of possible outcomes. $P(2 \leq X \leq 8)$ = 4(0.2) = 0.8. Note that k = 4, since there are four outcomes in the range $2 \leq X \leq 8$.

LOS 10.h: Define a binomial random variable and calculate and interpret probabilities, given a binomial probability distribution, and calculate and interpret the expected value and variance of a binomial random variable.

A **binomial random variable** may be defined as the number of "successes" in a given number of trials, whereby the outcome can be either "success" or "failure." The probability of success, *p*, is constant for each trial, and the trials are independent. Think of a trial as a mini-experiment. The final outcome is the number of successes in a series of *n* trials. Under these conditions, the binomial probability function defines the probability of *x* successes in *n* trials. It can be expressed using the following formula:

$$p(x) = P(X = x) = (\text{number of ways to choose } x \text{ from } n)p^x(1 - p)^{n-x}$$

where:

(number of ways to choose *x* from *n*) = $\dfrac{n!}{(n-x)!x!}$ which may also be denoted as $\binom{n}{x}$ or stated as "*n* choose *x*"

p = the probability of "success" on each trial (don't confuse it with p(x))

So the probability of exactly x successes in n trials is:

$$p(x) = \frac{n!}{(n-x)!x!}p^x(1-p)^{n-x}$$

Example: Binomial probability

Assuming a binomial distribution, **compute** the probability of drawing three black beans from a bowl of black and white beans if the probability of selecting a black bean in any given attempt is 0.6. You will draw five beans from the bowl.

Answer:

$$P(X = 3) = p(3) = \frac{5!}{2!3!}(0.6)^3(0.4)^2 = (120 / 12)(0.216)(0.160) = 0.3456$$

Some intuition about these results may help you remember the calculations. Consider that a (very large) bowl of black and white beans has 60 percent black beans and that each time you select a bean, you replace it in the bowl before drawing again. We want to know the probability of selecting exactly three black beans in five draws, as in the above problem.

One way this might happen is BBBWW. Since the draws are independent, the probability of this is easy to calculate. The probability of drawing a black bean is 60 percent and the probability of drawing a white bean is 1 − 60 percent = 40 percent. Therefore, the probability of selecting BBBWW, in order is, 0.6 × 0.6 × 0.6 × 0.4 × 0.4 = 3.456 percent. This is the $p^3(1 − p)^2$ from the formula and p is 60 percent, the probability of selecting a black bean on any single draw from the bowl.

BBBWW is not, however, the only way to choose exactly three black beans in five trials. Another possibility is BBWWB, and a third is BWWBB. Each of these will have exactly the same probability of occurring as our initial outcome, BBBWW. That's why we need to answer the question of how many ways (different orders) there are for us to choose three black beans in five draws. Using the formula, there are $\frac{5!}{3!(5-3)!} = 10$ ways; 10 × 3.456% = 34.56%, the answer we computed above.

The Expected Value and Variance of a Binomial Random Variable

For a given series of n trials, the expected number of successes or E(X) and the variance of X or Var(X) are given by the following formulas:

expected value of X = E(X) = np

variance of X = Var(X) = np(1 − p)

Example: Expected value and variance of a binomial random variable

Based on empirical data, the probability that the Dow Jones Industrial Average (DJIA) will increase on any given day has been determined to equal 0.67. Assuming that the only other outcome is that it decreases, we can state p(UP) = 0.67 and p(DOWN) = 0.33. Further, assume that movements in the DJIA are independent (i.e., an increase in one day is independent of what happened on another day).

Using the information provided, **compute** the expected value and variance of the number of up days in a 5-day period.

Answer:

Using binomial terminology, we define success as UP, so p = 0.67. Note that the definition of success is critical to any binomial problem.

expected value of X = E(X | n = 5, p = 0.67) = (5)(0.67) = 3.35

Recall that the "|" symbol means *"given."* Hence, the preceding statement is read as: the expected value of X given n = 5 and the probability of success = 67 percent.

variance of X = Var(X) = np(1 – p) = 5(0.67)(0.33) = 1.106

We should note that since the binomial distribution is a discrete distribution, the result X = 3.35 is not possible. However, if we were to record the results of many 5-day periods, the average outcome would converge to 3.35.

LOS 10.i: Construct a binomial tree to describe stock price movement.

A binomial model can be applied to stock price movements. We just need to define the two possible outcomes and the probability that each outcome will occur. Consider a stock with current price S that will, over the next period, either increase in value by 1% or decrease in value by 1% (the only two possible outcomes). The probability of an up-move (u) is p and the probability of a down-move (d) is (1 – p). For our example, the up-move factor (U) is 1.01 and the down-move factor (D) is 1/1.01. So there is a probability p that the stock price will move to S(1.01) over the next period and a probability (1 – p) that the stock price will move to S/1.01.

A binomial tree is constructed by showing all the possible combinations of up-moves and down-moves over a number of successive periods. For two periods, these combinations are UU, UD, DU, and DD. Importantly, UD and DU result in the same stock price S after two periods since S (1.01)(1/1.01) = S and the order of the moves does not change the result. Figure 1 illustrates a binomial tree for three periods.

Figure 1: A Binomial Tree
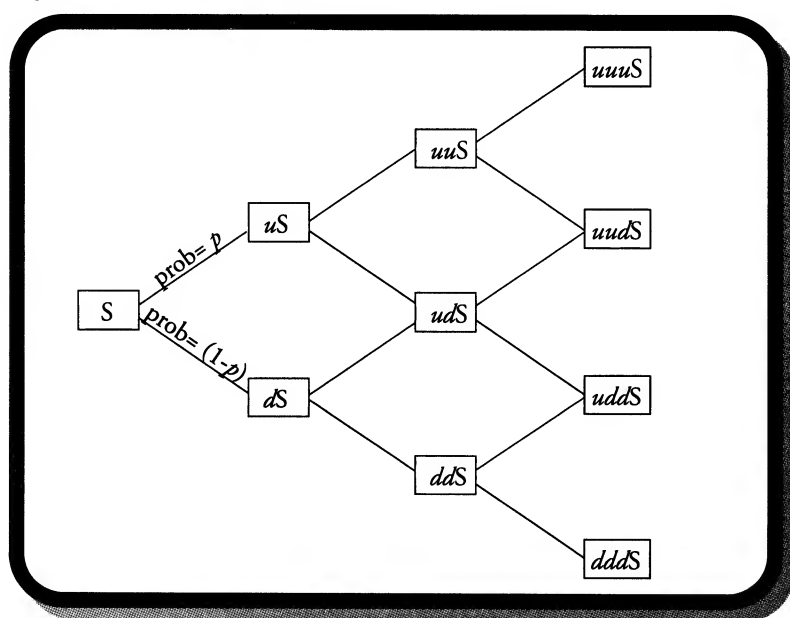
With an initial stock price S = 50, U = 1.01, D = $\frac{1}{1.01}$, and prob(u) = 0.6, we can calculate the possible stock prices after two periods as:

$$uuS = 1.01^2 \times 50 = 51.01 \text{ with probability } (0.6)^2 = 0.36$$

$$udS = 1.01\left(\frac{1}{1.01}\right) \times 50 = 50 \text{ with probability } (0.6)(0.4) = 0.24$$

$$duS = \left(\frac{1}{1.01}\right)(1.01) \times 50 = 50 \text{ with probability } (0.4)(0.6) = 0.24$$

$$ddS = \left(\frac{1}{1.01}\right)^2 \times 50 = 49.01 \text{ with probability } (0.4)^2 = 0.16$$

Since a stock price of 50 can result from either *ud* or *du* moves, the probability of a stock price of 50 after two periods (the middle value) is 2 × (0.6)(0.4) = 48%.

One of the important applications of a binomial stock price model is in pricing options. We can make a binomial tree for asset prices more realistic by shortening the length of the periods and increasing the number of periods and possible outcomes.

LOS 10.j: Describe the continuous uniform distribution and calculate and interpret probabilities, given a continuous uniform probability distribution.

The **continuous uniform distribution** is defined over a range that spans between some lower limit, *a*, and some upper limit, *b*, which serve as the parameters of the distribution. Outcomes can only occur between *a* and *b*, and since we are dealing with a continuous distribution, even if a < x < b, P(X = x) = 0. Formally, the properties of a continuous uniform distribution may be described as follows:

For all a ≤ x_1 < x_2 ≤ b, (i.e., for all x_1 and x_2 between the boundaries *a* and *b*)

P(X < a or X > b) = 0, (i.e., the probability of *X* outside the boundaries is zero)
and
P(x_1 ≤ X ≤ x_2) = (x_2 − x_1)/(b − a) (this defines the probability between x_1 and x_2)

Don't miss how simple this is just because the notation is so mathematical. For a continuous uniform distribution, the probability of outcomes in a range that is one-half the whole range is 50 percent. The probability of outcomes in a range that is one-quarter as large as the whole possible range is 25 percent.

Example: Continuous uniform distribution

X is uniformly distributed between 2 and 12. **Calculate** the probability that *X* will be between 4 and 8.

Answer:

$$\frac{8-4}{12-2} = \frac{4}{10} = 40\%$$

WARM-UP: NORMAL DISTRIBUTION

The normal distribution is important for many reasons. Besides the high probability that it will be covered on the exam, many of the random variables that are relevant to finance and other professional disciplines follow a normal distribution. In the area of investment and portfolio management, the normal distribution plays a central role in portfolio theory.

LOS 10.k: Explain the key properties of the normal distribution.

The **normal distribution** has the following key properties:

- It is completely described by its mean, μ, and variance, σ^2, stated as X ~ N(μ, σ^2). In words, this says that "*X* is normally distributed with mean μ and variance σ^2."
- Skewness = 0, meaning that the normal distribution is symmetric about its mean, so that P(X $\leq$ μ) = P(μ $\leq$ X) = 0.5, and mean = median = mode.
- Kurtosis = 3; this is a measure of how flat the distribution is. Recall that excess kurtosis is measured relative to 3, the kurtosis of the normal distribution.
- A linear combination of normally distributed random variables is also normally distributed.
- The probabilities of outcomes further above and below the mean get smaller and smaller but do not go to zero (the tails get very thin but extend infinitely).

Many of these properties are evident from examining the graph of a normal distribution's probability density function as illustrated in Figure 2.

Figure 2: Normal Distribution Probability Density Function
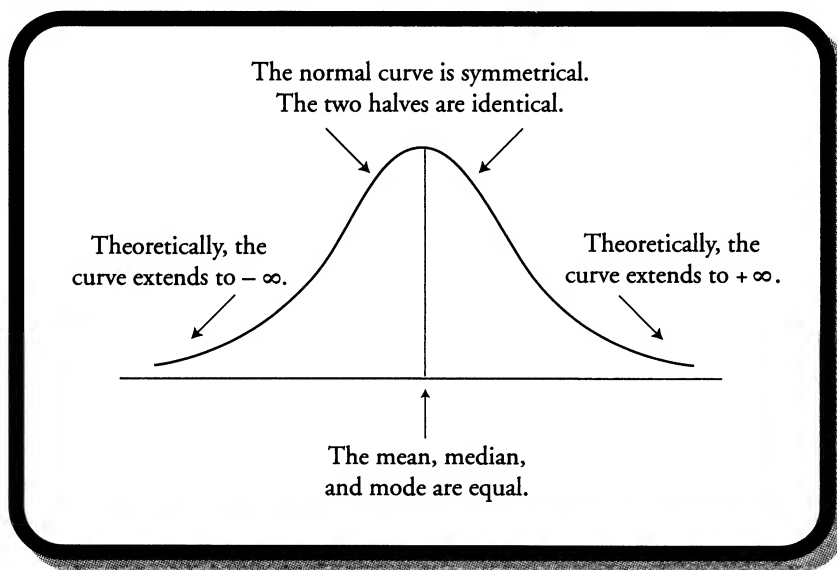
(See Exam Flashback #1.)

LOS 10.l: Distinguish between a univariate and a multivariate distribution.

Up to this point, our discussion has been strictly focused on **univariate distributions**, (i.e., the distribution of a single random variable). In practice, however, the relationships between two or more random variables are often relevant. For instance, investors and investment managers are frequently interested in the interrelationship

among the returns of one or more assets. In fact, as you will see in your study of asset pricing models and modern portfolio theory, the return on a given stock and the return on the S&P 500 or some other market index will have special significance. Regardless of the specific variables, the simultaneous analysis of two or more random variables requires an understanding of multivariate distributions.

A **multivariate distribution** specifies the probabilities associated with a group of random variables and is meaningful only when the behavior of each random variable in the group is in some way dependent upon the behavior of the others. Both discrete and continuous random variables can have multivariate distributions. Multivariate distributions between two discrete random variables are described using joint probability tables. For continuous random variables, a multivariate *normal* distribution may be used to describe them if all of the individual variables follow a normal distribution. As previously mentioned, one of the characteristics of a normal distribution is that a linear combination of normally distributed random variables is normally distributed as well. For example, if the return of each stock in a portfolio is normally distributed, the return on the portfolio will also be normally distributed.

LOS 10.m: Explain the role of correlation in the multivariate normal distribution.

Similar to a univariate normal distribution, a multivariate normal distribution can be described by the mean and variance of the individual random variables. Additionally, it is necessary to specify the correlation between the individual pairs of variables when describing a multivariate distribution. Correlation is the feature that distinguishes a multivariate distribution from a univariate normal distribution. *Correlation indicates the strength of the linear relationship between a pair of random variables.*

Using asset returns as our random variables, the multivariate normal distribution for the returns on n assets can be completely defined by the following three sets of parameters:

- n means of the n series of returns ($\mu_1, \mu_2, ..., \mu_n$).
- n variances of the n series of returns ($\sigma^2_1, \sigma^2_2, ..., \sigma^2_n$).
- $0.5n(n - 1)$ pair-wise correlations.

For example, if there are two assets, n = 2, then the multivariate returns distribution can be described with two means, two variances, and one correlation [0.5(2)(2 − 1) = 1]. If there are four assets, n = 4, the multivariate distribution can be described with four means, four variances, and six correlations [0.5(4)(4 − 1) = 6]. When building a portfolio of assets, all other things being equal, it is desirable to combine assets having low returns correlation because this will result in a portfolio with a lower variance than one composed of assets with higher correlations.

LOS 10.n: Construct and explain confidence intervals for a normally distributed random variable.

A **confidence interval** is a range of values around the expected outcome within which we expect the actual outcome to be some specified percentage of the time. A 95 percent confidence interval is a range that we expect the random variable to be in 95 percent of the time. For a normal distribution, this interval is based on the expected value (sometimes called a point estimate) of the random variable and on its variability, which we measure with standard deviation.

Confidence intervals for a normal distribution are illustrated in Figure 3. For any normally distributed random variable, 68 percent of the outcomes are within one standard deviation of the expected value (mean) and approximately 95 percent of the outcomes are within two standard deviations of the expected value.

Figure 3: Confidence Intervals for a Normal Distribution
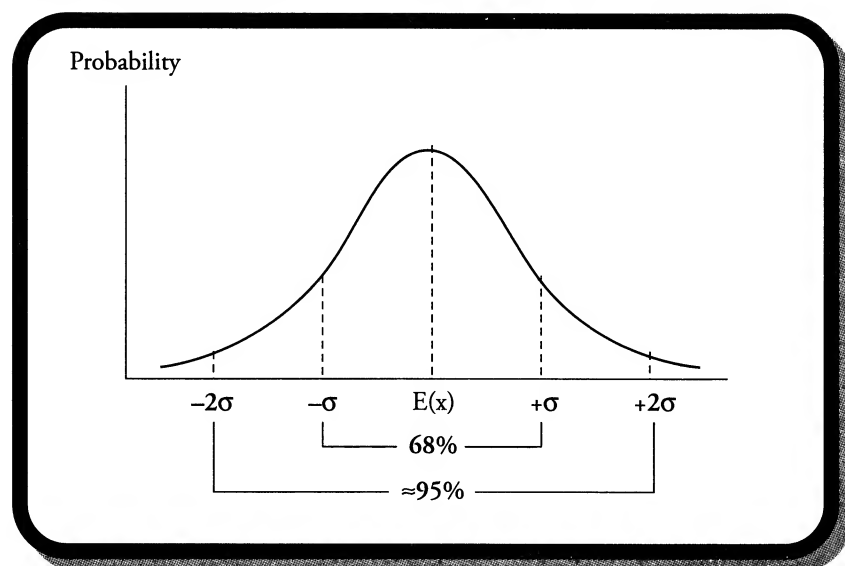
In practice we will not know the actual values for the mean and standard deviation of the distribution, but will have estimated them as $\overline{X}$ and s. The three confidence intervals of most interest are given by:

- The *90 percent confidence interval* for X is $\overline{X} - 1.65s$ to $\overline{X} + 1.65s$.
- The *95 percent confidence interval* for X is $\overline{X} - 1.96s$ to $\overline{X} + 1.96s$.
- The *99 percent confidence interval* for X is $\overline{X} - 2.58s$ to $\overline{X} + 2.58s$.

Example: Confidence intervals

Using a 20-year sample, the average return of a mutual fund has been 10.5 percent per year with a standard deviation of 18 percent. What is the 95 percent confidence interval for the mutual fund return next year?

Answer:

Here the estimates of μ and σ are 10.5 percent and 18 percent, respectively. Thus, the 95 percent confidence interval for the return, R, is:

$$10.5 \pm 1.96(18) = -24.78\% \text{ to } 45.78\%$$

Symbolically, this result can be expressed as:

$$P(-24.78 < R < 45.78) = 0.95 \text{ or } 95\%$$

LOS 10.o: Define the standard normal distribution, explain how to standardize a random variable, and calculate and interpret probabilities using the standard normal distribution.

The **standard normal distribution** is a normal distribution that has been standardized so that it has a mean of zero and a standard deviation of 1 [i.e., $N \sim (0,1)$]. To standardize an observation from a given normal distribution, the *z-value* of the observation must be calculated. The z-value represents the number of standard deviations a given observation is from the population mean. *Standardization* is the process of converting an observed value for a random variable to its z-value. The following formula is used to *standardize a random variable*:

$$z = \frac{\text{observation} - \text{population mean}}{\text{standard deviation}} = \frac{x - \mu}{\sigma}$$

Professor's Note: The term z-value *will be used for a standardized observation in this document. The terms* z-score *and* z-statistic *are also commonly used.*

Example: Standardizing a random variable (z-values)

Assume that the annual earnings per share (EPS) for a large sample of firms are normally distributed with a mean of $6.00 and a standard deviation of $2.00.

What is the approximate probability of an observed EPS value falling between $2.00 and $8.00?

Answer:

> If EPS = x = $8, then z = $(x - \mu)/\sigma$ = ($8 − $6) / $2 = +1
> If EPS = x = $2, then z = $(x - \mu)/\sigma$ = ($2 − $6) / $2 = −2

Here, z = +1 indicates that an EPS of $8 is one standard deviation above the mean, and z = −2 means that an EPS of $2 is two standard deviations below the mean.

Owing to the symmetry property of a normal distribution, we know that 68 percent of all observations will fall within ± one standard deviation of the mean and that 95 percent will fall within ± two standard deviations of the mean. Since the mean of the standard normal distribution is zero, we can approximate the probability of the EPS falling between $2 and $8 as 0.68 / 2 + 0.95 / 2 = 0.815, or P(2 ≤ EPS ≤ 8) = P(−2 ≤ Z ≤ 1) = 81.5 percent.

(See Exam Flashbacks #2 and #3.)

Calculating standard normal probabilities. In the preceding example, we approximated the probability of a range of values for a random variable. Now we will show how to use standardized values and a table of probabilities for Z to determine the exact probability of a normally distributed random variable falling between any two values. A portion of a table of the cumulative distribution function for Z is shown in Figure 4. We will refer to this table as the z-table, as it contains values generated using the cumulative density function for Z, denoted by F(Z). Thus, the values in the z-table are the probabilities of observing a z-value that is less than a given value, z (i.e., P(Z < z)). The numbers in the first column are z-values that have only one decimal place. The columns to the right supply probabilities for z-values with two decimal places.

Note that the z-table in Figure 4 only provides probabilities for positive z-values. This is not a problem because we know from the symmetry of the standard normal distribution that F(−Z) = 1 − F(Z). The tables in the back of many texts actually provide probabilities for negative z-values, but we will work with only the positive portion of the table because this may be all you get on the exam. In Figure 4 we can find the probability that a standard normal random variable will be less than 1.66, for example. The table value is 95.15 percent.

Figure 4: Cumulative Probabilities for a Standard Normal Distribution

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
				Cdf Values for the Standard Normal Distribution: The z-table						
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.5	.6915	Please note that several of the rows have been deleted to save space.*								
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990

* A complete normal table is shown at the back of this book.

Example: Using a standard normal probability table (z-table)

What is the probability of a normally distributed random variable taking on a z-value that is between +2 and −2? That is, what is $P(-2 \leq Z \leq +2)$?

Answer:

From the z-table, we see that $F(2) = 0.9772$, meaning that the cumulative probability of an observed value falling below +2 is 0.9772, or 97.72 percent. Because of the distribution's symmetry, we also know that $F(-2) = 1 - 0.9772 = 0.0228$. This means that the probability of an observed value falling below −2 is 0.0228. These results indicate that 2.28 percent of all observations fall below z = −2 and an equal amount falls above z = +2. Thus:

$P(-2 \leq Z \leq 2) = (1 - 0.0228) - 0.0228 = 0.9544$

Another way to determine this probability is as follows:

$P(-2 \leq Z \leq 2) = F(2) - F(-2) = 0.9772 - 0.0228 = 0.9544$

Note how close this probability comes to our approximation rule that states that approximately 95 percent of all observations fall in the interval $\mu \pm 2\sigma$.

We can also use the z-table to measure confidence intervals for normally distributed random variables. For example, the 95 percent confidence interval for Z is the range of z-values such that 2.5 percent of the z-distribution falls above the upper value and 2.5 percent falls below the lower limit. That is, $P(z_1 \leq Z \leq z_2) = 0.95$. This corresponds to the z-value in the z-table for which the probability (the area under the curve to the left of the value) is 0.975, or $F(Z) = 0.975$. Why 0.975? Because we want a z-value such that $F(Z) - F(-Z) = 0.95$, or

0.975 – (1 – 0.975) = 0.95 (i.e., we want to split the extra 5 percent evenly at each end of the curve). The probability 0.9750 in the table in Figure 4 corresponds to z = 1.96. Thus, F(1.96) – F(–1.96) = 0.975 – (1 – 0.975) = 0.95 or 95 percent. This confirms our previously stated confidence interval where we said that P(X will be within $\overline{X}$ ± 1.96s) = 0.95 or 95 percent.

Example: Using the z-Table

Using the distribution of EPS (μ = $6.00, σ = $2.00) again, what is the area under the standard normal distribution curve between $3.50 and $9.34? That is, what is P(3.5 ≤ EPS ≤ 9.34)?

Answer:

The z-values for the corresponding EPS values are:

EPS = $3.50: z_1 = (3.50 – 6) / 2 = –1.25
EPS = $9.34: z_2 = (9.34 – 6) / 2 = +1.67

Using the z-table, and referencing the distribution shown in Figure 5, the area under the curve between these z-values is:

$$P(3.5 \leq EPS \leq 9.34) = P(-1.25 \leq Z \leq 1.67) = F(1.67) - F(-1.25)$$
$$= 0.9525 - [1 - 0.8944]$$
$$= 0.8469, \text{ or } 84.69\%$$

Figure 5: Using the z-Table
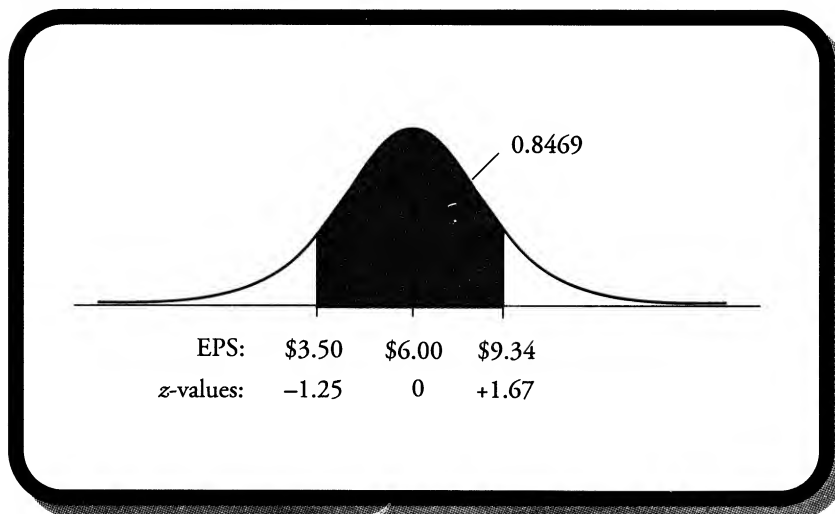
Example: Using the z-table

Returning again to the EPS figures (μ = $6, σ = $2), what percent of the EPS values are $9.70 or more?

Answer:

Here we want to know P(EPS > $9.70), which is the area under the curve to the right of the z-value corresponding to EPS = $9.70 (see Figure 6).

The z-value for EPS = $9.70 is:

$$z = \frac{(x-\mu)}{\sigma} = \frac{(9.70-6)}{2} = 1.85$$

From the table we have F(1.85) = 0.9678, but this is P(EPS ≤ 9.70). We want P(EPS > 9.70), which is determined as:

P(EPS > 9.70) = 1 – P(EPS ≤ 9.70)
                = 1 – P(Z ≤ 1.85) = 1 – F(1.85)
                = 1 – 0.9678 = 0.0322, or 3.2%

Figure 6: P(EPS>$9.70)
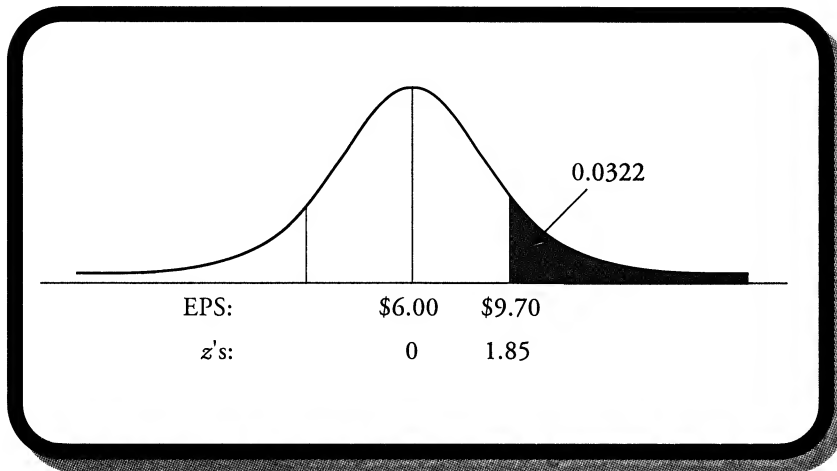
Example: Using the z-table

Using the distribution of EPS (μ = $6.00, σ = $2.00) again, what percent of the observed EPS values are likely to be less than $4.10?

Answer:

As shown graphically in Figure 7, we want to know P(EPS < $4.10). This requires a two-step approach like the one taken in the preceding example.

First, the corresponding z-value must be determined as follows:

$$z = \frac{(\$4.10 - \$6)}{2} = -0.95$$

Now, from the z-table *in the back of this book*, we find that F(0.95) = 0.8289, but this is P(Z ≤ +0.95) and we want P(Z ≤ –0.95). The probability that EPS will fall short of $4.10 is determined as:

P(EPS < 4.10) = P(Z < –0.95) = P(Z > 0.95)
                = 1 – F(0.95) = 1 – 0.8289
                = 0.1711, or 17.11%

Figure 7: Finding a Left-Tail Probability
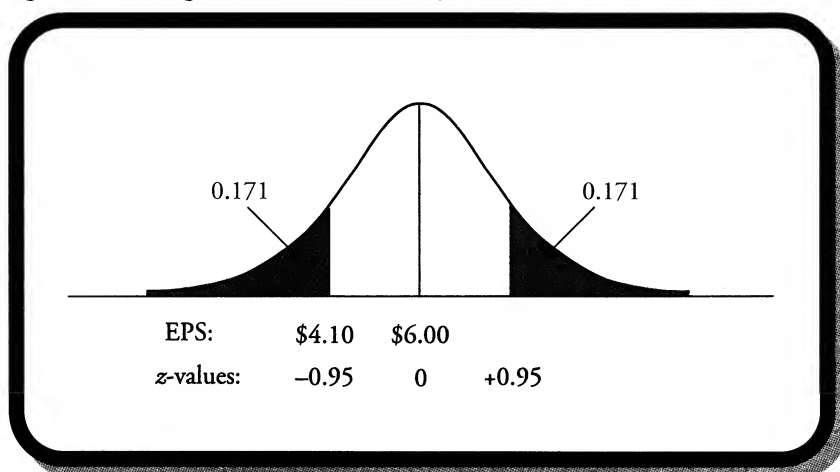
Here, we have used the fact that the probability of being more than 0.95 standard deviations above the mean is equal to the probability of being 0.95 standard deviations below the mean. The z-table gave us the probability that the outcome will be less than 0.95 standard deviations above the mean. We subtract this probability from 1 to get the probability of being more than 0.95 standard deviations above the mean of $6.00, which is the same as the probability of outcomes more than 0.95 standard deviations *below* the mean.

Example: Using the z-table

Continuing with the EPS values, **determine** the probability that EPS will exceed $3.64.

Answer:

The z-value associated with EPS = $3.64 is $z = (3.64 − 6) / 2 = −1.18$. $3.64 is 1.18 standard deviations below $6.00.

As shown in Figure 8, we are interested in P(EPS > $3.64) = P(Z > −1.18) which is determined as follows:

$$
\begin{aligned}
P(\text{EPS} > \$3.64) &= P(Z > -1.18) \\
&= 1 - F(-1.18) \\
&= 1 - [1 - F(1.18)] = F(1.18) \\
&= 0.8810 \text{ or } 88.10\%
\end{aligned}
$$

Figure 8: P(EPS > $3.64)
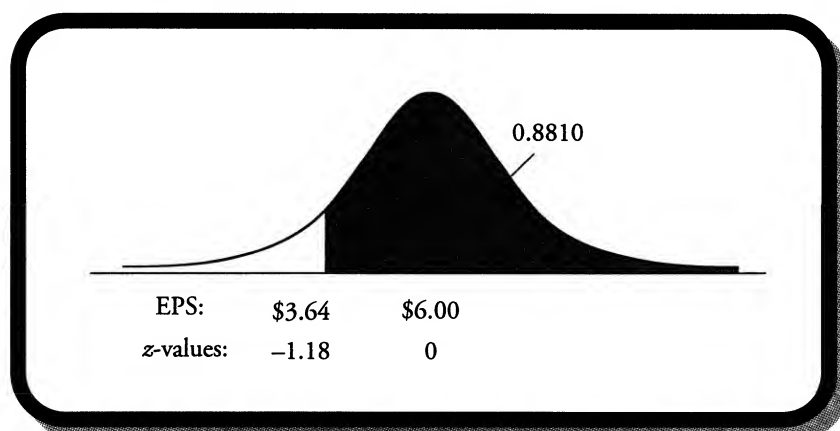
Note: Refer to the z-table at the back of this book to get F(1.18).

(See Exam Flashbacks #4 and #5.)

LOS 10.p: Define shortfall risk, calculate the safety-first ratio and select an optimal portfolio using Roy's safety-first criterion.

Shortfall risk is the probability that a portfolio value or return will fall below a particular (target) value or return over a given time period.

Roy's safety-first criterion states that the optimal portfolio minimizes the probability that the return of the portfolio falls below some minimum acceptable level. This minimum acceptable level is called the "threshold" level. Symbolically, Roy's safety-first criterion can be stated as:

minimize $P(R_p < R_L)$

where:
R_p = portfolio return
R_L = threshold level return

If portfolio returns are normally distributed, then Roy's safety-first criterion can be stated as:

maximize the SFRatio where SFRatio = $\dfrac{\left[E\left(R_p\right)-R_L\right]}{\sigma_p}$

Professors's Note: Notice the similarity to the Sharpe ratio: Sharpe = $\dfrac{\left[E\left(R_p\right)-R_f\right]}{\sigma_p}$. The only difference is that the SFRatio utilizes the excess return over the threshold return, R_L, where the Sharpe ratio uses the excess return over the risk-free rate, R_f.

The reasoning behind the safety-first criterion is illustrated in Figure 9. Assume an investor is choosing between two portfolios: Portfolio A with expected return of 12 percent and standard deviation of returns of 18 percent, and Portfolio B with expected return of 10 percent and standard deviation of returns of 12 percent. The investor has stated that he wants to minimize the probability of losing money (negative returns). Assuming that returns are normally distributed, the portfolio with the larger SFR using zero percent as the threshold return (R_L) will be the one with the lower probability of negative returns.

Figure 9: The Safety-First Criterion and Shortfall Risk

A. Normally Distributed Returns

Portfolio A – $\sigma_A = 18\%$ Portfolio B – $\sigma_B = 12\%$

Probability of returns < 0%
– i.e. short fall risk

0%   12% = $E(R_A)$ 0%   10% = $E(R_B)$

$$SFR_A = \frac{12-0}{18} = 0.667 \qquad SFR_B = \frac{10-0}{12} = 0.833$$

B. Standard Normal

25.14%   20.13%

– 0.67 0   – 0.83 0
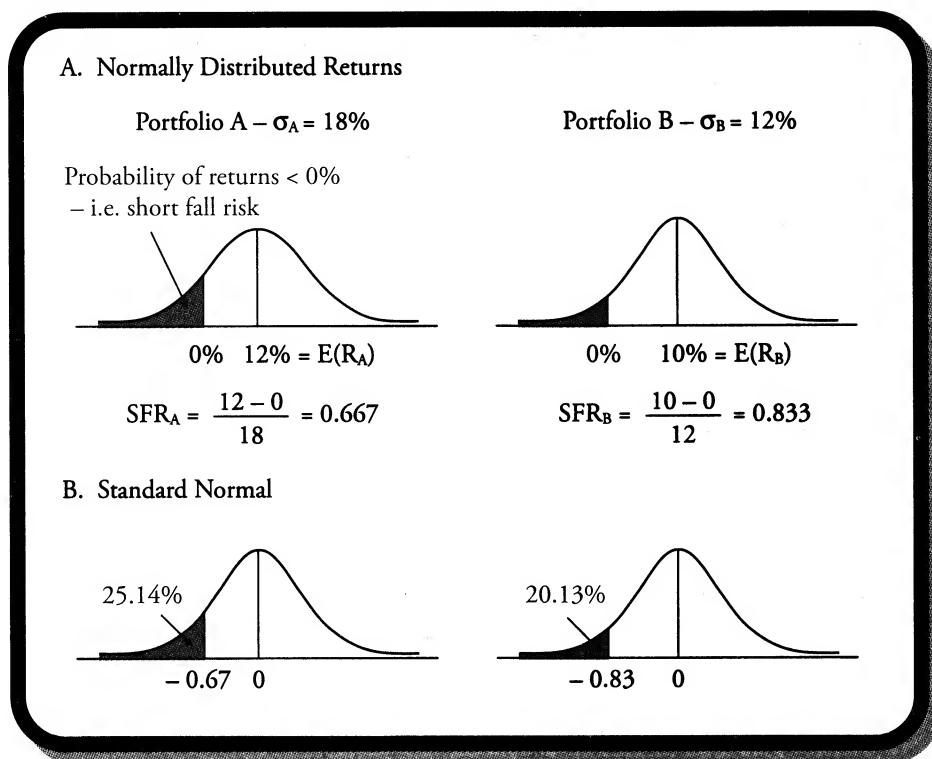
Panel B of Figure 9 relates the SFRatio to the standard normal distribution. Note that the SFR is the number of standard deviations *below* the mean. Thus, the portfolio with the larger SFR has the lower probability of returns below the threshold return, zero in our example. Using the standard normal distribution tables, we can find the probabilities in the left-hand tails as indicated. These probabilities (25 percent for Portfolio A and 20 percent for Portfolio B) are also the shortfall risk for a target return of zero percent. Portfolio B has the higher SFR which means it has the lower probability of negative returns.

In summary, when choosing among portfolios with normally distributed returns using Roy's safety-first criterion, there are two steps:

Step 1: Calculate the SFRatio = $\dfrac{\left[E\left(R_p\right) - R_L\right]}{\sigma_p}$.

Step 2: Choose the portfolio that has the *largest* SFRatio.

Example: Roy's safety-first criterion

For the next year, the managers of a $120 million college endowment plan, have set a minimum acceptable end-of-year portfolio value of $123.6 million. Three portfolios are being considered which have the expected returns and standard deviation shown in the first two rows of Figure 10. **Determine** which of these portfolios is the most desirable using Roy's safety-first criterion.

Answer:

The threshold return is $R_L = (123.6 - 120) / 120 = 0.030 = 3\%$. The SFRs are shown in Figure 10. As indicated, the best choice is Portfolio A because it has the largest SFR.

Figure 10: Roy's Safety-First Ratios

Portfolio	Portfolio A	Portfolio B	Portfolio C
$E(R_p)$	9%	11%	6.6%
σ_p	12%	20%	8.2%
SFRatio	$0.5 = \dfrac{(9-3)}{12}$	$0.4 = \dfrac{(11-3)}{20}$	$0.44 = \dfrac{(6.6-3)}{8.2}$

LOS 10.q: Explain the relationship between the lognormal and normal distributions and explain and interpret the use of the lognormal distribution in modeling asset prices.

The **lognormal distribution** is generated by the function e^x, where x is normally distributed. Since the natural logarithm, ln, of e^x is x, the logarithms of lognormally distributed random variables are normally distributed, thus the name.

Figure 11 illustrates the differences between a normal distribution and a lognormal distribution.

Figure 11: Normal vs. Lognormal Distributions
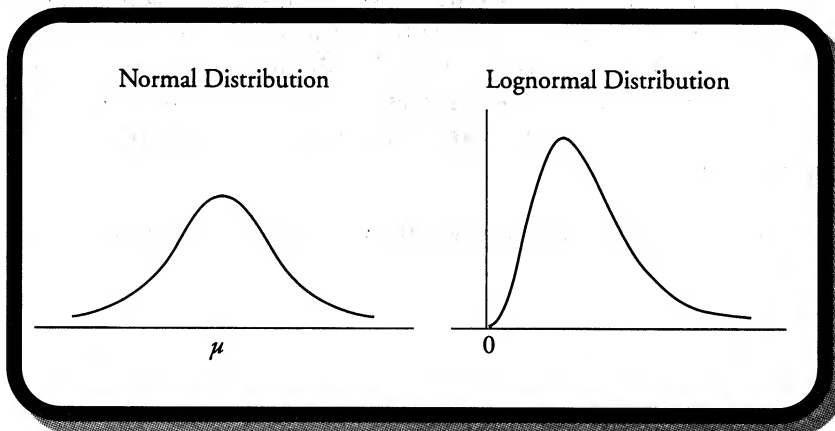
In Figure 11, we can see that:

- The lognormal distribution is skewed to the right.
- The lognormal distribution is bounded from below by zero so that it is useful for modeling asset prices which never take negative values.

If we used a normal distribution of returns to model asset prices over time, we would admit the possibility of returns less than –100%, which would admit the possibility of asset prices less than zero. Using a lognormal distribution to model *price relatives* avoids this problem. A price relative is just the end-of-period price of the asset over the beginning price (S_1/S_0) and is equal to (1 + the holding period return). To get the end-of-period asset price, we can simply multiply the price relative times the beginning-of-period asset price. Since a lognormal distribution takes a minimum value of zero, end-of-period asset prices cannot be less than zero. A price relative of

zero corresponds to a holding period return of –100% (i.e., the asset price has gone to zero). Recall that we used price relatives as the up-move and down-move (multiplier) terms in constructing a binomial tree for stock price changes over a number of periods.

LOS 10.r: Distinguish between discretely and continuously compounded rates of return, and calculate and interpret the continuously compounded rate of return, given a specific holding period return.

Discretely compounded returns are just the compound returns we are familiar with, given some discrete compounding period, such as semiannual or quarterly. Recall that the more frequent the compounding period, the greater the effective annual return. For a stated rate of 10 percent, semiannual compounding results in an effective yield of $\left(1+\dfrac{0.10}{2}\right)^2 -1=10.25\%$ and monthly compounding results in an effective yield of $\left(1+\dfrac{0.10}{12}\right)^{12} -1=10.47\%$. Daily or even hourly compounding will produce still larger effective yields. The limit of this exercise, as the compounding period gets shorter and shorter, is called **continuous compounding**. The effective annual rate, based on continuous compounding for a stated annual rate of i, can be calculated from the formula:

effective annual rate = $e^i - 1$

Based on a stated rate of 10 percent, the effective rate with continuous compounding is $e^{0.10} -1 = 10.5171$ percent. Please verify this by entering 0.1 in your calculator and finding the e^x function.

Since the natural log, ln, of e^x is x, we can get the continuously compounded rate from an effective annual rate by using the ln calculator function. Using our previous example, ln (1 + 10.517 percent) = ln 1.105171 = 10 percent. Verify this by entering 1.105171 in your calculator and then entering the ln key. (Using the HP calculator, the keystrokes are 1.1 [ENTER] [g] [ln].)

We can use this method to find the continuously compounded rate that will generate a particular holding period return. If we are given a holding period return of 12.5 percent for the year, the equivalent continuously compounded rate is ln 1.125 = 11.778 percent. Since the calculation is based on 1 plus the holding period return, we can also do the calculation directly from the *price relative*. The price relative is just the end-of-period value divided by the beginning of period value. The continuously compounded rate of return is:

$$\ln\left(\frac{S_1}{S_0}\right) = \ln\left(1+\text{HPR}\right)$$

Example: Calculating continuously compounded returns

A stock was purchased for $100 and sold one year later for $120. **Calculate** the investor's annual rate of return on a continuously compounded basis.

Answer:

$$\ln\left(\frac{120}{100}\right) = 18.232\%$$

If we had been given the return (20 percent) instead, the calculation is:

$$\ln(1+0.20)=18.232\%$$

LOS 10.s: Explain Monte Carlo simulation and historical simulation and describe their major applications and limitations.

Monte Carlo simulation is a technique based on the repeated generation of one or more risk factors that affect security values, in order to generate a distribution of security values. For each of the risk factors, the analyst must specify the parameters of the probability distribution that the risk factor is assumed to follow. A computer is then used to generate random values for each risk factor based on its assumed probability distributions. Each set of randomly generated risk factors is used with a pricing model to value the security. This procedure is repeated many times (100s, 1,000s, or 10,000s) and the distribution of simulated asset values is used to draw inferences about the expected (mean) value of the security and possibly the variance of security values about the mean as well.

As an example, consider the valuation of stock options that can only be exercised on a particular date. The main risk factor is the value of the stock itself, but interest rates could affect the valuation as well. The simulation procedure would be to:

1. Specify the probability distributions of stock prices and of the relevant interest rate, as well as the parameters (mean, variance, possibly skewness) of the distributions.

2. Randomly generate values for both stock prices and interest rates.

3. Value the options for each pair of risk factor values.

4. After many iterations, calculate the mean option value and use that as your estimate of the option's value.

Monte Carlo simulation is used to:

- Value complex securities.
- Simulate the profits/losses from a trading strategy.
- Calculate estimates of value at risk (VAR) to determine the riskiness of a portfolio of assets and liabilities.
- Simulate pension fund assets and liabilities over time to examine the variability of the difference between the two.
- Value portfolios of assets that have non-normal returns distributions.

The **limitations of Monte Carlo simulation** are that it is fairly complex and will provide answers that are no better than the assumptions about the distributions of the risk factors and the pricing/valuation model that is used. Also, simulation is not an analytic method but a statistical one, and cannot provide the insights that analytic methods can.

Historical simulation is based on actual changes in value or actual changes in risk factors over some prior period. Rather than model the distribution of risk factors, as in Monte Carlo simulation, the set of all changes in the relevant risk factors over some prior period is used. Each iteration of the simulation involves randomly selecting one of these past changes for each risk factor and calculating the value of the asset or portfolio in question, based on those changes in risk factors.

Historical simulation has the advantage of using the actual distribution of risk factors so that the distribution of changes in the risk factors does not have to be estimated. It suffers from the fact that past changes in risk factors may not be a good indication of future changes. Events that occur infrequently may not be reflected in historical simulation results unless the events occurred during the period from which the values for risk factors are drawn.

An additional limitation of historical simulation is that it cannot address the sort of "what if" questions that Monte Carlo simulation can. With Monte Carlo simulation we can investigate the effect on the distribution of security/portfolio values of increasing the variance of one of the risk factors by 20 percent; with historical simulation we cannot do this.

KEY CONCEPTS

1. A probability distribution lists all the possible outcomes of an experiment along with their associated probabilities.
2. A probability function specifies the probability that a random variable is equal to a specific value; $P(X = x) = p(x)$.
3. A probability density function (pdf) is the expression for probability function for a continuous random variable.
4. The two key properties of a probability function are: (i) $0 \leq p(x) \leq 1$, and (ii) $\sum p(x) = 1$.
5. A cumulative distribution function (cdf) gives the probability of the random variable being equal to or less than each specific value. It is the area under the probability distribution to the left of a specified value.
6. A discrete random variable has positive probabilities associated with specific single number outcomes.
7. A continuous random variable has positive probabilities associated with a range of outcome values—the probability of it equaling any single value is zero.
8. The binomial distribution is a probability distribution for a binomial (discrete) random variable, X, that has one of two possible outcomes: success or failure, where the probability of success is p. The probability of a specific number of successes in n independent trials is:

    $$p(x) \quad = P(X = x) = \frac{n!}{(n-x)!x!} \, p^{x}(1-p)^{n-x}$$

    $$E(X) \quad = np = \text{expected value of } X$$

    $$\sigma^{2}(X) = np(1-p) = \text{variance of } X$$

9. A discrete uniform distribution is one where there are n equally spaced outcomes, and for each outcome $p(x) = 1/n$.
10. A continuous uniform distribution is one where the probability of X occurring in a possible range is the length of the range relative to the total of all possible values. Letting a and b be the lower and upper limit of

    the uniform distribution, respectively, then for $a \leq x_1 < x_2 \leq b$, $P\left(x_1 \leq X \leq x_2\right) = \dfrac{\left(x_2 - x_1\right)}{\left(b - a\right)}$.

11. The normal probability distribution and normal curve have the following characteristics:
    *   The normal curve is symmetrical and bell-shaped with a single peak at the exact center of the distribution.
    *   Mean = median = mode, and all are in the exact center of the distribution.
    *   The normal distribution can be completely defined by its mean and standard deviation.
12. A confidence interval is a range within which we have a given level of confidence of finding a point estimate

    (e.g., the 90 percent confidence interval for X is $\overline{X} - 1.65s$ to $\overline{X} + 1.65s$).
13. The standard normal probability distribution has a mean of zero and a standard deviation of 1.

    A normally distributed random variable X can be normalized by $Z = \dfrac{(x - \mu)}{\sigma}$.

    A table that gives the cumulative probabilities for Z, the z-table, is used to find the probability that X falls within certain regions.

    $$P(X < x) = F(x) = F\left[\frac{(x-\mu)}{\sigma}\right] = F(z), \text{ which is found in the standard normal probability table.}$$

    $$P(X > x) = 1 - P(X < x) = 1 - F(z)$$

14. Multivariate distributions describe groups of random variables. A normal multivariate distribution with n individual random variables is completely described by the n means, n variances, and the $n(n-1)/2$ correlations (or covariances).

15. Correlation is a measure of the strength of the linear relationship between two random variables.

16. Safety-first rules focus on the probability of a portfolio return, R_p, falling below a certain lower threshold return, R_L. The goal is to minimize $P(R_p < R_L)$ or equivalently, maximize:

$$\text{SFRatio} = \frac{\left[E\left(R_p \right) - R_L \right]}{\sigma_p}$$

17. Shortfall risk is the probability that a portfolio's value will fall below a specified minimum value over a specified period of time.

18. A lognormal distribution exists for random variable Y, when $Y = e^X$, and X is normally distributed.
    - $Y > 0$ and it is skewed right.
    - An increase in the mean or variance of X will increase both the mean and variance of Y.

19. As we decrease the length of discrete compounding periods (e.g., from quarterly to monthly) the EAY increases. As the length of the compounding period in discrete compounding gets shorter and shorter, the compounding becomes continuous where the effective annual rate $= e^i - 1$.

20. For a holding period return (HPR) over any period t, the equivalent continuously compounded rate over the period is $\ln(1 + \text{HPR})$.

21. Monte Carlo simulation uses randomly generated values for risk factors, based on their assumed distributions, to produce a distribution of security values. Historical simulation uses randomly selected past changes in these risk factors to generate a distribution of security values.

EXAM FLASHBACKS

Required CFA Institute disclaimer: Due to CFA curriculum changes from year to year, published sample exam questions and guideline answers prior to the current year may not reflect the current curriculum.

Exam Flashback # 1
Source: Question #30 from '00–'03 sample exams.

A normal distribution would *least likely* be described as:
A. asymptotic.
B. a discrete probability distribution.
C. a symmetrical or bell-shaped distribution.
D. a curve that theoretically extends from negative infinity to positive infinity.

Exam Flashback # 2
Source: Question #26 from '03 sample exam.

What are the mean and standard deviation of a standard normal distribution?

	Mean	Standard Deviation
A.	0	0
B.	0	1
C.	1	0
D.	1	1

Exam Flashback # 3
Source: Question #32 from '99–'03 sample exams.

Based on a normal distribution with a mean of 500 and a standard deviation of 150, the *z*-value for an observation of 200 is *closest* to:
A. −2.00.
B. −1.75.
C. 1.75.
D. 2.00.

Exam Flashback # 4
Source: Question #31 from '99–'03 sample exams.

An investment strategy has an expected return of 12 percent and a standard deviation of 10 percent. If investment returns are normally distributed, the probability of earning a return less than 2 percent is *closest* to:
A. 10%.
B. 16%.
C. 32%.
D. 34%.

Exam Flashback # 5
Source: Question #38 from '98 sample exam.

The mean and variance of price-to-earnings ratios (P/Es) of all companies in a given industry are 18 and 9, respectively. If the P/Es are normally distributed, about 68 percent of the P/Es will fall between:
A. 9.0 and 27.0.
B. 12.0 and 24.0.
C. 13.5 and 22.5.
D. 15.0 and 21.0.

CONCEPT CHECKERS: COMMON PROBABILITY DISTRIBUTIONS

1. Which of the following is **NOT** an example of a discrete random variable?
   A. The number of stocks a person owns.
   B. The time spent by a portfolio manager with a client.
   C. The number of days it rains in a month in Iowa City.
   D. The number of people holding Microsoft in their portfolios.

2. For a continuous random variable X, the probability of any single value of X is:
   A. one.
   B. zero.
   C. determined by the cdf.
   D. determined by the pdf.

Use the following table to answer Questions 3 through 7.

Probability distribution of a discrete random variable X								
X	0	1	2	3	4	5	6	7
P(X)	0.04	0.11	0.18	0.24	0.14	0.17	0.09	0.03

3. The probability that X = 3 is:
   A. 0.18.
   B. 0.24.
   C. 0.43.
   D. 0.70.

4. The cdf of 5, or F(5) is:
   A. 0.14.
   B. 0.17.
   C. 0.71.
   D. 0.88.

5. The probability that X is *greater* than 3 is:
   A. 0.24.
   B. 0.43.
   C. 0.57.
   D. 0.67.

6. What is $P(2 \leq X \leq 5)$?
   A. 0.12.
   B. 0.17.
   C. 0.38.
   D. 0.73.

7. The expected value of the random variable X is:
   A. 1.89.
   B. 3.35.
   C. 3.70.
   D. 5.47.

8. Which of the following is **NOT** a condition of a binomial experiment?
    A. There are only two trials.
    B. The trials are independent.
    C. Each trial has two and only two possible outcomes.
    D. If p is the probability of success, and q is the probability of failure, then p + q = 1.

9. A recent study indicated that 60 percent of all businesses have a fax machine. From the binomial probability distribution table, the probability that exactly four businesses will have a fax machine in a random selection of six businesses is:
    A. 0.138.
    B. 0.276.
    C. 0.311.
    D. 0.324.

10. Ten percent of all college graduates hired stay with the same company for more than five years. In a random sample of six recently hired college graduates, the probability that exactly two will stay with the same company for more than five years is *closest* to:
    A. 0.015.
    B. 0.098.
    C. 0.114.
    D. 0.185.

11. Assume that 40 percent of candidates who sit for the CFA® examination pass it the first time. Of a random sample of 15 candidates who are sitting for the exam for the first time, what is the expected number of candidates that will pass?
    A. 0.375.
    B. 4.000.
    C. 6.000.
    D. 6.667.

12. For the standard normal distribution, the *z*-value gives the distance between the mean and a point in terms of the:
    A. mean.
    B. variance.
    C. standard deviation.
    D. center of the curve.

13. For a standard normal distribution, F(0) is:
    A. 0.00.
    B. 0.10.
    C. 0.50.
    D. 1.00.

14. For the standard normal distribution, $P(0 \leq Z \leq 1.96)$ is:
    A. 0.4713.
    B. 0.4761.
    C. 0.4745.
    D. 0.4750.

15. For the standard normal distribution, $P(-2.05 \leq Z \leq 0.00)$ is:
    A. 0.4798.
    B. 0.4803.
    C. 0.4938.
    D. 0.9586.

Use the following data to answer Questions 16 through 19.

A study of hedge fund investors found that their annual household incomes are normally distributed with a mean of $175,000 and a standard deviation of $25,000.

16. What percent of hedge fund investors have incomes *less* than $100,000?
    A. 0.05%.
    B. 0.10%.
    C. 0.13%.
    D. 0.25%.

17. Approximately what percent of hedge fund investors have incomes between $150,000 and $200,000?
    A. 50%.
    B. 68%.
    C. 75%.
    D. 95%.

18. What percent of hedge fund investors have incomes *greater* than $225,000?
    A. 0.50%.
    B. 1.10%.
    C. 2.28%.
    D. 3.46%.

19. What percent of hedge fund investors have incomes *greater* than $150,000?
    A. 15.87%.
    B. 34.13%.
    C. 68.26%.
    D. 84.13%.

Use the following table to answer Questions 20 and 21.

Portfolio	Portfolio A	Portfolio B	Portfolio C	Portfolio D
$E(R_p)$	5%	11%	14%	18%
σ_p	8%	21%	34%	40%

20. Given a threshold level of return of 4 percent, use Roy's safety-first criterion to choose the optimal portfolio. Portfolio:
    A. A.
    B. B.
    C. C.
    D. D.

21. Given a threshold level of return of zero percent, use Roy's safety-first criterion to choose the optimal portfolio. Portfolio:
    A. A.
    B. B.
    C. C.
    D. D.

22. If a stock's initial price is $20 and its year-end price is $23, then its continuously compounded annual rate of return is:
    A. 9.86%.
    B. 13.64%.
    C. 13.98%.
    D. 15.00%.

23. For a lognormal distribution, the:
    A. mean equals the median.
    B. standard deviation equals 1.
    C. probability of a negative outcome is zero.
    D. probability of a positive outcome is 50 percent.

24. Using hypothesized parameter values and a random number generator to study the behavior of certain asset returns is part of:
    A. historical analysis.
    B. normalizing a random variable.
    C. Monte Carlo simulation.
    D. standardizing a random variable.

25. A continuous uniform distribution has the parameters a = 4 and b = 10. The F(20) is:
    A. 0.25.
    B. 0.50.
    C. 1.00.
    D. 2.00.

26. All of the following statements accurately describe the binomial distribution EXCEPT:
    A. the trials are independent.
    B. it is a discrete distribution.
    C. the probability of an outcome of zero is zero.
    D. the combination formula is used in computing probabilities.

27. Approximately 50 percent of all observations for a normally distributed random variable fall in the interval:
    A. $\mu \pm 0.67\sigma$.
    B. $\mu \pm \sigma$.
    C. $\mu \pm 2\sigma$.
    D. $\mu \pm 3\sigma$.

28. The probability that a normally distributed random variable will be more than two standard deviations above its mean is:
    A 0.0217.
    B. 0.0228.
    C. 0.4772.
    D. 0.9772.

29. A stock doubled in value last year. Its continuously compounded return over the period was *closest* to:
   A. 18.2%.
   B. 69.3%.
   C. 100%.
   D. 200%.

30. Portfolio A has a safety-first ratio of 1.3 with a threshold return of 2 percent. What is the shortfall risk for a target return of 2 percent?
   A. 90.3%.
   B. 40.3%.
   C. 9.68%.
   D. 49.68%.

31. Of all the bonds currently rated B, 20 percent will default over an investor's horizon. The expected number of defaults and the variance of the number of defaults in a randomly selected 40-bond portfolio over the investor's horizon is:

	Expected Defaults	Variance
A.	8	32
B.	32	6.4
C.	8	6.4
D.	32	32

ANSWERS – EXAM FLASHBACKS

1. **B** The normal distribution is a continuous distribution, not discrete. It takes the form of a symmetrical, bell-shaped curve with tails that asymptotically approaches infinity.

2. **B** By definition, the standard normal distribution, z-distribution, has a mean of zero and standard deviation of 1.

3. **A** The standardized value for a normally distributed random variable is its z-value, which is determined as: $Z = (X - \mu) / \sigma$. In this case, we have $Z = (200 - 500) / 150 = -2.0$.

4. **B** For a normal distribution, about 68% of the distribution falls within ± 1 standard deviations of the mean. So, with μ = 12% and a standard deviation of 10%, about 68% of the distribution will fall in the range from 12% ± 10%, or 2% to 22%. This leaves approximately 16% of the total area of the normal curve to the left of 2% (and 16% to the right of 22%). The 16% of the area under the curve to the left of 2% represents the probability of the return being less than 2%.

5. **D** For a normal distribution, about 68% of the distribution falls within one standard deviation of the mean. So, with μ = 18% and a σ = 3%, about 68% of the distribution will fall in the range from 18% ± 3%, or 15% to 21%.

   Professor's Note: Watch out for problems like this where CFA Institute gives you a variance and expects you to use the standard deviation, the square root of the variance.

ANSWERS – CONCEPT CHECKERS: COMMON PROBABILITY DISTRIBUTIONS

1. **B** Time is usually a continuous random variable; all the others are discrete.

2. **B** For a continuous distribution p(x) = 0 for all X; only ranges of value of X have positive probabilities.

3. **B** From the table.

4. **D** (0.04 + 0.11 + 0.18 + 0.24 + 0.14 + 0.17) = 0.88

5. **B** (0.14 + 0.17 + 0.09 + 0.03) = 0.43

6. **D** (0.18 + 0.24 + 0.14 + 0.17) = 0.73

7. **B** 0 + 1(0.11) + 2(0.18) + 3(0.24) + 4(0.14) + 5(0.17) + 6(0.09) + 7(0.03) = 3.35

8. **A** There may be any number of independent trials, each with only two possible outcomes.

9. **C** Success = having a fax machine. $[6! / 4!(6 - 4)!](0.6)^4(0.4)^{6-4} = 15(0.1296)(0.16) = 0.311$.

10. **B** Success = staying for five years. $[6! / 2!(6 - 2)!](0.10)^2(0.90)^{6-2} = 15(0.01)(0.656) = 0.0984$.

11. **C** Success = passing the exam. Then, E(success) = np = 15 × 0.4 = 6.

12. **C** This is true by the formula for z.

13. **C** By the symmetry of the z-distribution and F(0) = 0.5. Half the distribution lies on each side of the mean.

14. **D** From the table F(1.96) = 0.9750, thus the answer is 0.9750 − 0.5 = 0.4750. Knowing that 95% lie between −1.96 and +1.96, and that 0 is the midpoint, we can say that $\dfrac{95\%}{2}$ = 47.5% lie between 0 and +1.96.

15. **A** From the table, and via symmetry, F(2.05) = 0.9798, thus the answer is 0.9798 − 0.5 = 0.4798.

16. **C** z = −3 = (100 − 175) / 25, F(−3) = 1 − 0.9987 = 0.0013

17. **B** This is $\pm 1\sigma$ from the mean. For a normal distribution, 68% of observations are between $\pm 1\sigma$ from the mean.

18. **C** $1 - F(2)$, where $F(2)$ equals 0.9772. Hence, $1 - 0.9772 = 0.0228$.

19. **D** $1 - F(-1) = F(1) = 0.8413$

20. **D** SFR = $(18 - 4) / 40$ is the largest value.

21. **A** SFR = $(5 - 0) / 8$ is the largest value.

22. **C** $\ln(23 / 20) = 0.1398$

23. **C** A lognormally distributed variable is never negative.

24. **C** Monte Carlo simulation.

25. **C** $F(x) = 1$ for all $x > b$. Remember $F(x)$ is the cumulative probability, $P(x < 20)$ here.

26. **C** With only two possible outcomes, there must be some positive probability for each. It does not matter if one of the possible outcomes happens to be zero. If this were not the case, the variable in question would not be a random variable, and a probability distribution would be meaningless.

27. **A** $\mu \pm 0.67\sigma$

28. **B** $1 - F(2) = 1 - 0.9772 = 0.0228$.

29. **B** $\ln(2) = 0.6931$.

30. **C** Using the tables, the cdf for -1.3 is 9.68%, which is the probability of returns less than 2%.

31. **C** Treating the number of defaults as a binomial random variable, x, $E(x) = 0.2(40) = 8$ and the Var$(x) = (0.2)(0.8)40 = 6.4$.

SAMPLING AND ESTIMATION

EXAM FOCUS

This topic review covers random samples and inferences about population means from sample data. It is essential that you know the central limit theorem, for it allows us to use sampling statistics to construct confidence intervals for point estimates of population means. Make sure you can calculate confidence intervals for population means given sample parameter estimates and a level of significance, and know when it is appropriate to use the z-statistic versus the t-statistic. You should also understand the basic procedures for creating random samples, and recognize the warning signs of various sampling biases from nonrandom samples.

WARM-UP: APPLIED STATISTICS

In many real-world statistics applications, it is impractical (or impossible) to study an entire population. When this is the case, a subgroup of the population, called a sample, can be evaluated. Based upon this sample, the parameters of the underlying population can be estimated.

For example, rather than attempting to measure the performance of the U.S. stock market by observing the performance of all 10,000 or so stocks trading in the United States at any one time, the performance of the subgroup of 500 stocks in the S&P 500 can be measured. The results of the statistical analysis of this sample can then be used to draw conclusions about the entire population of U.S. stocks.

LOS 11.a: Define simple random sampling, define and interpret sampling error, and define a sampling distribution, and interpret sampling error.

Simple random sampling is a method of selecting a sample in such a way that each item or person in the population being studied has the same likelihood of being included in the sample. As an example of simple random sampling, assume that you want to draw a sample of five items out of a group of 50 items. This can be accomplished by numbering each of the 50 items, placing them in a hat, and shaking the hat. Next, one number can be drawn randomly from the hat. Repeating this process (experiment) four more times results in a set of five numbers. The five drawn numbers (items) comprise a simple random sample from the population. In applications like this one, a random-number table or a computer random-number generator is often used to create the sample.

Sampling error is the difference between a sample statistic (the mean, variance, or standard deviation of the sample) and its corresponding population parameter (the true mean, variance, or standard deviation of the population). For example, the sampling error for the mean is:

sampling error of the mean = sample mean − population mean = $\bar{x} - \mu$

A Sampling Distribution

It is important to recognize that the sample statistic itself is a random variable and, therefore, has a probability distribution. The **sampling distribution** of the sample statistic is a probability distribution of all possible sample

statistics computed from a set of equal size samples that were randomly drawn from the same population. Think of it as the probability distribution of a statistic from many samples.

For example, suppose a random sample of 100 bonds is selected from a population of a major municipal bond index consisting of 1,000 bonds, and then the mean return of the 100-bond sample is calculated. Repeating this process many times will result in many different estimates of the population mean return, (i.e., one for each sample). The distribution of these estimates of the mean is the *sampling distribution of the mean.*

It is important to note that this sampling distribution is distinct from the distribution of the actual prices of the 1,000 bonds in the underlying population and will have different parameters.

LOS 11.b: Distinguish between simple random and stratified random sampling.

Stratified random sampling uses a classification system to separate the population into smaller groups based on one or more distinguishing characteristics. From each subgroup, or stratum, a random sample is taken and the results are pooled. The size of the samples from each stratum is based on the size of the stratum relative to the population.

Stratified sampling is often used in bond indexing because of the difficulty and cost of completely replicating the entire population of bonds. In this case, bonds in a population are categorized (stratified) according to major bond risk factors such as duration, maturity, coupon rate, etc. Then samples are drawn from each separate category and combined to form a final sample.

To see how this works, suppose you want to construct a bond portfolio that is indexed to the major municipal bond index using a stratified random sampling approach. First, the entire population of 1,000 municipal bonds in the index can be classified on the basis of maturity and coupon rate. Then, cells (stratum) can be created for different maturity/coupon combinations, and random samples can be drawn from each of the maturity/coupon cells. To sample from a cell containing 50 bonds with 2- to 4-year maturities and coupon rates less than 5 percent, we would select 5 bonds. The number of bonds drawn from a given cell corresponds to the cell's weight relative to the population (index), or $(50/1000) \times (100) = 5$ bonds. This process is repeated for all of the maturity/coupon cells, and the individual samples are combined to form the portfolio.

By using stratified sampling, we guarantee that we sample five bonds from this cell. If we had used simple random sampling, there would be no guarantee that we would sample any of the bonds in the cell. Or, we may have selected more than five bonds.

LOS 11.c: Distinguish between time-series and cross-sectional data.

Time-series data consists of observations taken *over a period of time* at specific and equally spaced time intervals. The set of monthly returns on Microsoft stock from January 1994 to January 2004 is an example of a time series data sample.

Cross-sectional data are a sample of observations taken *at a single point in time*. The sample of reported earnings per share of all NASDAQ companies as of December 31, 2004, is an example of a cross-sectional data sample.

LOS 11.d: State the central limit theorem and describe its importance.

The **central limit theorem** states that for simple random samples of size n from a *population* with a mean μ and a finite variance σ^2, the sampling distribution of the sample mean $\bar{x}$ approaches a normal probability distribution with mean μ and a variance equal to $\dfrac{\sigma^2}{n}$ as the sample size becomes large.

The central limit theorem is extremely useful because the normal distribution is relatively easy to apply to hypothesis testing and to the construction of confidence intervals. Specific inferences about the population mean can be made from the sample mean, *regardless of the population's distribution*, as long as the sample size is "sufficiently large," which usually means n ≥ 30.

Important properties of the central limit theorem include:

- If the sample size *n* is sufficiently large (n ≥ 30), the sampling distribution of the sample means will be approximately normal. Remember what's going on here, random samples of size *n* are repeatedly being taken from an overall larger population. Each of these random samples has its own mean, which is itself a random variable, and this set of sample means has a distribution that is approximately normal.
- The mean of the population, *μ*, and the mean of the distribution of all possible sample means are equal.
- The variance of the distribution of sample means is $\dfrac{\sigma^2}{n}$, the population variance divided by the sample size.

LOS 11.e: Calculate and interpret the standard error of the sample mean.

The **standard error of the sample mean** is the standard deviation of the distribution of the sample means.

When the standard deviation of the population, *σ*, is *known*, the standard error of the sample mean is calculated as:

$$\sigma_{\overline{x}} = \frac{\sigma}{\sqrt{n}}$$

where:
$\sigma_{\overline{x}}$ = standard error of the sample mean
σ = standard deviation of the population
n = size of the sample

Example: Standard error of sample mean (known population variance)

The mean hourly wage for Iowa farm workers is $13.50 with a *population standard deviation* of $2.90. Letting $\overline{x}$ represent the mean hourly wage for a single random sample of Iowa farm workers, **calculate** and **interpret** the standard error of the sample mean for a sample size of 30.

Answer:

Because the population standard deviation, *σ*, *is known*, the standard error of the sample mean is expressed as:

$$\sigma_{\overline{x}} = \frac{\sigma}{\sqrt{n}} = \frac{\$2.90}{\sqrt{30}} = \$0.53$$

Professor's Calculator Tip: On the TI BAII Plus, the use of the square root key is obvious. On the HP 12C, the square root of 30 is computed as: [30] [ENTER] [g] [$\sqrt{x}$].

This means that if we were to take all possible samples of size 30 from the Iowa farm worker population and prepare a sampling distribution of the sample means, we will obtain a mean of $13.50 and standard error of $0.53.

Practically speaking, the *population's standard deviation is almost never known*. Instead, the standard error of the sample mean must be estimated by dividing the standard deviation of *the sample* mean by $\sqrt{n}$:

$$s_{\overline{x}} = \frac{s}{\sqrt{n}}$$

Note: Use this when the population variance is unknown.

where:

$s_{\overline{x}}$ = standard error of the sample mean

s = standard deviation of the sample = $\sqrt{\dfrac{\sum\limits_{i=1}^{n}(x_i - \overline{x})^2}{n-1}}$

n = size of the sample

Example: Standard error of sample mean (unknown population variance)

Suppose a sample contains the past 30 monthly returns for McCreary, Inc. The mean return is 2 percent and the *sample* standard deviation is 20 percent. **Calculate** and **interpret** the standard error of the sample mean.

Answer:

Since σ is unknown, the standard error of the sample mean is:

$$s_{\overline{x}} = \frac{s}{\sqrt{n}} = \frac{20\%}{\sqrt{30}} = 3.6\%$$

This implies that if we took all possible samples of size 30 from McCreary's monthly returns and prepared a sampling distribution of the sample means, the mean would be 2 percent with a standard error of 3.6 percent.

Example: Standard error of sample mean (unknown population variance)

Continuing with our example, suppose that instead of a sample size of 30, we take a sample of the past 200 monthly returns for McCreary, Inc. In order to highlight the effect of sample size on the sample standard error, let's assume that the mean return and standard deviation of this larger sample remain at 2 percent and 20 percent, respectively. Now, **calculate** the standard error of the sample mean for the 200-return sample.

Answer:

The standard error of the sample mean is computed as:

$$s_{\overline{x}} = \frac{s}{\sqrt{n}} = \frac{20\%}{\sqrt{200}} = 1.4\%$$

The result of the preceding two examples illustrates an important property of sampling distributions. Notice that the value of the standard error of the sample mean decreased from 3.6 percent to 1.4 percent as the sample size increased from 30 to 200. This is because as the sample size increases it gets closer to the size of the population, and the distribution of the sample means about the population mean gets smaller and smaller, which causes the standard error of the sample mean to decrease.

(See Exam Flashback #1.)

LOS 11.f: Distinguish between a point estimate and a confidence interval estimate of a population parameter.

LOS 11.h: Explain the construction of confidence intervals.

Point estimates are single (sample) values used to estimate population parameters. The formula used to compute the point estimate is called the estimator. For example, the sample mean, $\overline{x}$, is an estimator of the population mean μ and is computed using the familiar formula:

$$\overline{x} = \frac{\sum x}{n}$$

The value generated with this calculation for a given sample is called the *point estimate* of the mean.

Confidence interval estimates result in a range of values within which the actual value of a parameter will lie, given the probability of $1 - \alpha$. Here, alpha, α, is called the *level of significance* for the confidence interval, and the probability $1 - \alpha$ is referred to as the *degree of confidence*. For example, we might estimate that the population mean of random variables will range from 15 to 25 with a 95 percent degree of confidence, or at the 5 percent level of significance.

Confidence intervals are usually constructed by adding or subtracting an appropriate value from the point estimate. In general, confidence intervals take on the following form:

point estimate ± (reliability factor × standard error)

where:
point estimate   =   value of a sample statistic of the population parameter
reliability factor =   number that depends on the sampling distribution of the point estimate and the probability that the point estimate falls in the confidence interval, $(1 - \alpha)$
standard error   =   standard error of the point estimate

LOS 11.g: Identify and describe the desirable properties of an estimator.

Regardless of whether we are concerned with point estimates or confidence intervals, there are certain statistical properties that make some estimates more desirable than others. These **desirable properties of an estimator** are *unbiasedness, efficiency, and consistency.*

- An *unbiased* estimator is one for which the expected value of the estimator is equal to the parameter you are trying to estimate. For example, because the expected value of the sample mean is equal to the population mean [$E(\overline{x}) = \mu$], the sample mean is an unbiased estimator of the population mean.
- An unbiased estimator is also *efficient* if the variance of its sampling distribution is smaller than all the other unbiased estimators of the parameter you are trying to estimate. The sample mean, for example, is an unbiased and efficient estimator of the population mean.
- A *consistent* estimator is one for which the accuracy of the parameter estimate increases as the sample size increases. As the sample size increases, the standard error of the sample mean falls, and the sampling distribution bunches more closely around the population mean. In fact, as the sample size approaches infinity, the standard error approaches zero.

LOS 11.i: Describe the properties of Student's t-distribution.

LOS 11.j: Calculate, explain, and interpret degrees of freedom.

Student's *t*-distribution, or simply, the *t*-distribution, is a bell-shaped probability distribution that is symmetrical about its mean. It is the appropriate distribution to use when constructing confidence intervals based on *small samples* (n < 30) from populations with *unknown variance* and a normal, or approximately normal, distribution. It may also be appropriate to use the *t*-distribution when the population variance is unknown and the sample size is large enough that the central limit theorem will assure that the sampling distribution is approximately normal.

Student's *t*-distribution has the following properties:

- It is symmetrical.
- It is defined by a single parameter, the **degrees of freedom** (df), where the degrees of freedom are equal to the number of sample observations minus 1, n − 1, for sample means.
- It is less peaked than a normal distribution, with more probability in the tails ("fatter tails").
- As the degrees of freedom (the sample size) gets larger, the shape of the *t*-distribution more closely approaches a standard normal distribution.

When *compared to the normal distribution*, the *t*-distribution is flatter with more area under the tails (i.e., it has fatter tails). As the degrees of freedom, *df*, for the *t*-distribution increases, however, its shape approaches that of the normal distribution.

The degrees of freedom for sample means is n − 1 because, given the mean, only n − 1 observations can be unique.

LOS 11.k: Calculate and interpret a confidence interval for a population mean when sampling from a normal distribution with 1) a known population variance, 2) an unknown population variance, or 3) when sampling from a population with an unknown variance and the sample size is large.

If the population has a *normal distribution with a known variance*, a **confidence interval for the population mean** can be calculated as:

$$\overline{x} \pm z_{\alpha/2} \frac{\sigma}{\sqrt{n}}$$

where:

$\overline{x}$ =   *point estimate* of the population mean (sample mean).

$z_{\alpha/2}$ =   *reliability factor*, a standard normal random variable for which the probability in the right-hand tail of the distribution is $\alpha/2$. In other words, this is the *z*-score that leaves $\alpha/2$ of probability in the upper tail.

$\dfrac{\sigma}{\sqrt{n}}$ =   the *standard error* of the sample mean where σ is the known standard deviation of the population, and *n* is the sample size.

The most commonly used standard normal distribution reliability factors are:

$z_{\alpha/2}$ = 1.645 for 90% confidence intervals (the significance level is 10%, 5% in each tail)

$z_{\alpha/2}$ = 1.960 for 95% confidence intervals (the significance level is 5%, 2.5% in each tail)

$z_{\alpha/2}$ = 2.575 for 99% confidence intervals (the significance level is 1%, 0.5% in each tail)

Do these numbers look familiar? They should! In our review of common probability distributions, we found the probability under the standard normal curve between z = −1.96 and z = +1.96 to be 0.95, or 95 percent. Owing to symmetry, this leaves a probability of 0.025 under each tail of the curve beyond z = −1.96 of z = +1.96, for a total of 0.05, or 5 percent—just what we need for a significance level of 0.05, or 5 percent.

Example: Confidence interval

Consider a practice exam that was administered to 100 Level 1 candidates. The mean score on this practice exam was 80 for all 36 of the candidates in the sample who studied at least 10 hours a week in preparation for the exam. Assuming a population standard deviation equal to 15, **construct** and **interpret** a 99 percent confidence interval for the mean score on the practice exam for 36 candidates who study at least 10 hours a week. *Note that in this example the population standard deviation is known, so we don't have to estimate it.*

Answer:

At a confidence level of 99 percent, $z_{\alpha/2} = z_{0.005} = 2.575$. So, the 99 percent confidence interval is calculated as follows:

$$\overline{x} \pm z_{\alpha/2}\frac{\sigma}{\sqrt{n}} = 80 \pm 2.575\frac{15}{\sqrt{36}} = 80 \pm 6.4$$

Thus, the 99 percent confidence interval ranges from 73.6 to 86.4.

Confidence intervals can be interpreted from a probabilistic perspective or a practical perspective. With regard to the outcome of the CFA® practice exam example, these two perspectives can be described as follows:

- *Probabilistic interpretation.* After repeatedly taking samples of CFA candidates who studied 10 hours or more per week, administering the practice exam, and constructing confidence intervals for each sample's mean, 99 percent of the resulting confidence intervals will, in the long run, include the population mean.
- *Practical interpretation.* We are 99 percent confident that the population mean score is between 73.6 and 86.4 for candidates who study more than 10 hours per week.

Confidence Intervals for the Population Mean: Normal With Unknown Variance

If the distribution of the *population is normal with unknown variance*, we can use the *t*-distribution to construct a confidence interval:

$$\overline{x} \pm t_{\alpha/2}\frac{s}{\sqrt{n}}$$

where:

$\overline{x}$	=	the point estimate of the population mean
$t_{\alpha/2}$	=	the t-reliability factor (a.k.a., t-statistic or critical t-value) corresponding to a t-distributed random variable with n−1 degrees of freedom, where n is the sample size. The area under the tail of the t-distribution to the right of $t_{\alpha/2}$ is $\alpha/2$.
$\dfrac{s}{\sqrt{n}}$	=	standard error of the sample mean
s	=	sample standard deviation

Unlike the standard normal distribution, the reliability factors for the *t*-distribution depend on the sample size, so we can't rely on a commonly used set of reliability factors. Instead, reliability factors for the *t*-distribution have to be looked up in a table of Student's *t*-distribution, like the one at the back of this book.

Owing to the relatively fatter tails of the t-distribution, confidence intervals constructed using t-reliability factors ($t_{\alpha/2}$) will be more conservative (wider) than those constructed using z reliability factors ($z_{\alpha/2}$).

Example: Confidence intervals

Let's return to the McCreary, Inc. example. Recall that we took a sample of the past 30 monthly stock returns for McCreary, Inc. and determined that the mean return was 2 percent and the sample standard deviation was 20 percent. Since the population variance is unknown, the standard error of the sample was estimated to be:

$$s_{\overline{x}} = \frac{s}{\sqrt{n}} = \frac{20\%}{\sqrt{30}} = 3.6\%$$

Now, let's **construct** a 95 percent confidence interval for the mean monthly return.

Answer:

Here we will use the t-reliability factor because the population variance is unknown. Since there are 30 observations, the degrees of freedom are $29 = 30 - 1$. Remember, because this is a two-tailed test at the 95 percent confidence level, the probability under each tail must be $\alpha/2 = 2.5$ percent, for a total of 5 percent. So, referencing the 1-tailed probabilities for Student's t-distribution at the back of this book, we find the critical t-value (reliability factor) for $\alpha/2 = 0.025$ and df = 29 to be $t_{29,\,2.5} = 2.045$. Thus, the 95 percent confidence interval for the population mean is:

$$2\% \pm 2.045\left(\frac{20\%}{\sqrt{30}}\right) = 2\% \pm 2.045\left(3.6\%\right) = 2\% \pm 7.4\%$$

Thus, the 95 percent confidence has a lower limit of –5.4 percent and an upper limit of +9.4 percent.

We can interpret this confidence interval by saying that we are 95 percent confident that the population mean monthly return for McCreary stock is between –5.4 percent and +9.4 percent.

Professor's Note: You should practice looking up reliability factors (a.k.a. critical t-values or t-statistics) in a t-table. The first step is always to compute the degrees of freedom, which is n − 1. The second step is to find the appropriate level of alpha or significance. This depends on whether the test you're concerned with is one-tailed (use α) or two-tailed (use α/2). In this review, our tests will always be two-tailed because confidence intervals are designed to compute an upper and lower limit. Thus, we will use α/2. To look up t_{29, 2.5}, find the 29 df row and match it with the 0.025 column; t = 2.045 is the result. We'll do more of this in our study of hypothesis testing and regression analysis.

Confidence Interval for a Population Mean When the Population Variance Is Unknown Given a Large Sample From any Type of Distribution

We now know that the z-statistic should be used to construct confidence intervals when the population distribution is normal and the variance is known, and the t-statistic should be used when the distribution is normal but the variance is unknown. But what do we do when the *distribution* is *nonnormal?*

As it turns out, the size of the sample influences whether or not we can construct the appropriate confidence interval for the sample mean.

- If the *distribution is nonnormal*, but the *population variance is known*, the *z*-statistic can be used as long as the sample size is large (n ≥ 30). We can do this because the central limit theorem assures us that the distribution of the sample mean is approximately normal when the sample is large.
- If the *distribution is nonnormal* and the *population variance is unknown*, the *t*-statistic can be used as long as the sample size is large (n ≥ 30). It is also acceptable to use the *z*-statistic, although use of the *t*-statistic is more conservative.

What this means is that if we are sampling from a nonnormal distribution (which is sometimes the case in finance), *we cannot create a confidence interval if the sample size is less than 30*. So, all else equal, make sure you have a sample of at least 30, and the larger, the better.

The table in Figure 1 summarizes this discussion.

Professor's Note: You should commit the criteria in this table to memory.

Figure 1: Criteria for Selecting the Appropriate Test Statistic

When sampling from a:	Test Statistic	
	Small Sample (n < 30)	Large Sample (n ≥ 30)
Normal distribution with *known* variance	*z*-statistic	*z*-statistic
Normal distribution with *unknown* variance	*t*-statistic	*t*-statistic*
Nonnormal distribution with *known* variance	not available	*z*-statistic
Nonnormal distribution with *unknown* variance	not available	*t*-statistic*

* The *z*-statistic is theoretically acceptable here, but use of the *t*-statistic is more conservative.

All of the preceding analysis depends on the sample we draw from the population being random. If the sample isn't random, the central limit theorem doesn't apply, our estimates won't have the desirable properties, and we can't form unbiased confidence intervals. Surprisingly, creating a *random sample* is not as easy as one might believe. There are a number of potential mistakes in sampling methods that can bias the results. These biases are particularly problematic in financial research, where available historical data are plentiful, but the creation of new sample data by experimentation is restricted.

LOS 11.l: Discuss the issues regarding selection of the appropriate sample size.

We have seen so far that a larger sample reduces the sampling error and the standard deviation of the sample statistic around its true (population) value. Confidence intervals are narrower (more precise) when samples are larger and the standard errors of the point estimates of population parameters are less.

There are two limitations on this idea of "larger is better" when it comes to selecting an appropriate sample size. One is that larger samples may contain observations from a different population (distribution). If we include observations which come from a different population, that has a different population parameter, we will not necessarily improve the precision of our population parameter estimates. The other consideration is cost. The costs of using a larger sample must be weighed against the value of the increase in precision from the increase in sample size. Both of these factors suggest that the largest possible sample size is not always the most appropriate choice.

LOS 11.m: Define and discuss data-mining bias, sample selection bias, survivorship bias, look-ahead bias, and time-period bias.

Data mining occurs when analysts repeatedly use the same database to search for patterns or trading rules until one that "works" is discovered. For example, empirical research has provided evidence that value stocks appear to outperform growth stocks. Some researchers argue that this anomaly is actually the product of data mining. Because the data set of historical stock returns is quite limited, it is difficult to know for sure whether the difference between value and growth stock returns is a true economic phenomenon, or simply a chance pattern that was stumbled upon after repeatedly looking for any identifiable pattern in the data.

Data-mining bias refers to results where the statistical significance of the pattern is overestimated because the results were found through data mining.

When reading research findings that suggest a profitable trading strategy, make sure you heed the following warning signs of data mining:

- Evidence that many different variables were tested, most of which are unreported, until significant ones were found.
- The lack of any economic theory that is consistent with the empirical results.

The best way to avoid data mining is to test a potentially profitable trading rule on a data set different from the one you used to develop the rule (i.e., use out-of-sample data).

- **Sample selection bias** occurs when some data is systematically excluded from the analysis, usually because of the lack of availability. This practice renders the observed sample to be nonrandom, and any conclusions drawn from this sample can't be applied to the population because the observed sample and the portion of the population that was not observed are different.

- **Survivorship bias** is the most common form of sample selection bias. A good example of the existence of survivorship bias in investments is the study of mutual fund performance. Most mutual fund databases, like Morningstar®'s, only include funds currently in existence—the "survivors." They do not include funds that have ceased to exist due to closure or merger.

  This would not be a problem if the characteristics of the surviving funds and the missing funds were the same; then the sample of survivor funds would still be a random sample drawn from the population of mutual funds. As one would expect, however, and as evidence has shown, the funds that are dropped from the sample have lower returns relative to the surviving funds. Thus, the surviving sample is biased toward the better funds (i.e., it is not random). The analysis of a mutual fund sample with survivorship bias will yield results that overestimate the average mutual fund return because the database only includes the better-performing funds. The solution to survivorship bias is to use a sample of funds that all started at the same time and not drop funds that have been dropped from the sample.

- **Look-ahead bias** occurs when a study tests a relationship using sample data that was not available on the test date. For example, consider the test of a trading rule that is based on the price-to-book ratio at the end of the fiscal year. Stock prices are available for all companies at the same point in time, while end-of-year book values may not be available until 30 to 60 days after the fiscal year ends. In order to account for this bias, a study that uses price-to-book value ratios to test trading strategies might estimate the book value as reported at fiscal year end and the market value two months later.

- **Time-period bias** can result if the time period over which the data is gathered is either too short or too long. If the time period is too short, research results may reflect phenomena specific to that time period, or perhaps even data mining. If the time period is too long, the fundamental economic relationships that underlie the results may have changed.

For example, research findings may indicate that small stocks outperformed large stocks during the 1980–1985 time period. This may well be the result of time-period bias—in this case, using too short a time period. It's not clear whether this relationship will continue in the future or if it is just an isolated occurrence.

On the other hand, a study that quantifies the relationship between inflation and unemployment (the Phillips Curve) during the period from 1940–2000 will also result in time-period bias—because this period is too long, and it covers a fundamental change in the relationship between inflation and unemployment that occurred in the 1980s. In this case, the data should be divided into two subsamples that span the period before and after the change.

KEY CONCEPTS

1. Simple random sampling is a method of selecting a sample in such a way that each item or person in the population being studied has the same likelihood of being included in the sample.
2. Sampling error is the difference between a sample statistic and its corresponding population parameter (e.g., the sample mean minus the population mean).
3. A sampling distribution is the distribution of all values that a sample statistic can take on when computed from samples of identical size randomly drawn from the same population.
4. Stratified random sampling involves randomly selecting samples proportionally from subgroups that are formed based on one or more distinguishing characteristics.
5. A time-series sample consists of observations taken at specific and equally spaced points in time, while a cross-sectional data sample consists of observations taken at a single point in time.
6. The central limit theorem states that for a population with a mean μ and a finite variance σ^2, the sampling distribution of the sample mean of all possible samples of size n will be approximately normally distributed with a mean equal to μ and a variance equal to σ^2/n.
7. The standard error of the sample mean is the standard deviation of the distribution of the sample means and is calculated as:
   - $\sigma_{\overline{x}} = \dfrac{\sigma}{\sqrt{n}}$, where σ, the population standard deviation, is known.
   - $s_{\overline{x}} = \dfrac{s}{\sqrt{n}}$, where s is the sample standard deviation and the population standard deviation is unknown.
8. Point estimates are single value estimates of population parameters, and a confidence interval is a range of estimated values within which the actual value of the parameter will lie with a given probability.
9. Desirable statistical properties of an estimator include: unbiasedness, efficiency, and consistency.
10. The t-distribution is similar, but not identical, to the normal distribution in shape—it is defined by the degrees of freedom, has a lower peak, and has fatter tails.
11. Degrees of freedom for the t-distribution is equal to $n - 1$; Student's t-distribution is closer to the normal distribution when df is greater, and confidence intervals are narrower when df is greater.
12. The t-distribution is used to construct confidence intervals for the population mean when the population variance is not known. The $(1-\alpha)$ confidence interval for the population mean, μ, is: $\overline{x} \pm t_{\alpha/2} \dfrac{s}{\sqrt{n}}$
13. Use the t-distribution if:
    - Population distribution is normal with an unknown variance.
    - Population distribution is normal, or approximately normal, with unknown variance and the sample size is small ($n < 30$).
    - Population distribution is nonnormal with unknown variance, but the sample is large ($n > 30$).

14. The standard normal distribution (z-distribution) is used to construct confidence intervals for the population mean when the population variance is known. The $(1 - \alpha)$ confidence interval for the population mean, μ, is: $\bar{x} \pm z_{\alpha/2} \dfrac{\sigma}{\sqrt{n}}$

15. Use the z-distribution if:
    - Population distribution is normal with known variance.
    - Population distribution is nonnormal and the sample is large ($n \geq 30$).

16. There are a number of potential mistakes in the sampling method that can bias results. These biases include data mining, sample selection bias, look-ahead bias, survivorship bias, and time-period bias.

EXAM FLASHBACKS

Required CFA Institute disclaimer: Due to CFA curriculum changes from year to year, published sample exam questions and guideline answers prior to the current year may not reflect the current curriculum.

Exam Flashback # 1
Source: Question #34 from '99–'03 sample exams.

If the standard deviation of a population is 100 and a sample size taken from that population is 64, the standard error of the sample means is *closest* to:
A. 0.08.
B. 1.56.
C. 6.40.
D. 12.50.

CONCEPT CHECKERS: SAMPLING AND ESTIMATION

1. Which of the following *most accurately* defines a simple random sample? It is a sample:
    A. that includes every tenth element of an arranged population.
    B. drawn in such a way that each member of the population has some chance of being selected in the sample.
    C. drawn in such a way that each member of the population has an equal chance of being included in the sample.
    D. drawn in such a way that each member of the population has a 1 percent chance of being included in the sample.

2. Sampling error is defined as:
    A. an error that occurs when a sample of more than 30 elements is drawn.
    B. an error that occurs when a sample of less than 30 elements is drawn.
    C. an error that occurs during collection, recording, and tabulation of data.
    D. the difference between the value of a sample statistic and the value of the corresponding population parameter.

3. The mean age of all CFA candidates is 28 years. The mean age of a random sample of 100 candidates is found to be 26.5 years. The difference, $28 - 26.5 = 1.5$, is called the:
    A. random error.
    B. sampling error.
    C. population error.
    D. probability error.

4. If *n* is large and the population standard deviation is unknown, the standard error of the sampling distribution of the sample mean is *equal* to the:
    A. sample standard deviation divided by the sample size.
    B. population standard deviation multiplied by the sample size.
    C. sample standard deviation divided by the square root of the sample size.
    D. population standard deviation divided by the sample size.

5. The standard error of the sampling distribution of the sample mean for a sample size of *n* drawn from a population with a mean of μ and a standard deviation of σ is:
    A. sample standard deviation divided by the sample size.
    B. population standard deviation multiplied by the square root of the sample size.
    C. sample standard deviation divided by the square root of the sample size.
    D. population standard deviation divided by the square root of the sample size.

6. To apply the central limit theorem to the sampling distribution of the sample mean, the sample is usually considered to be large if *n* is *greater* than:
   A. 15.
   B. 20.
   C. 25.
   D. 30.

7. Assume that a population has a mean of 14 with a standard deviation of 2. If a random sample of 49 observations is drawn from this population, the standard error of the sample mean is *closest* to:
   A. 0.04.
   B. 0.29.
   C. 2.00.
   D. 7.00.

8. The population's mean is 30 and the mean of a sample of size 100 is 28.5. The variance of the sample is 25. The standard error of the sample mean is *closest* to:
   A. 0.05.
   B. 0.25.
   C. 0.50.
   D. 2.50.

9. A random sample of 100 computer store customers spent an average of $75 at the store. Assuming the distribution is normal and the population standard deviation is $20, the 95 percent confidence interval for the population mean is *closest* to:
   A. $69.84 to $80.16.
   B. $71.08 to $78.92.
   C. $73.89 to $80.11.
   D. $74.56 to $79.44.

10. Best Computers, Inc., sells computers and computer parts by mail. A sample of 25 recent orders showed the mean time taken to ship out these orders was 70 hours with a sample standard deviation of 14 hours. Assuming the population is normally distributed, the 99 percent confidence interval for the population mean is:
    A. 25 ± 6.98 hours.
    B. 70 ± 2.80 hours.
    C. 70 ± 6.98 hours.
    D. 70 ± 7.83 hours.

11. The sampling distribution of a statistic is the probability distribution made up of all possible:
    A. observations from the underlying population.
    B. confidence intervals from sample sizes greater than 30.
    C. sample statistics computed from samples of varying sizes drawn from the same population.
    D. sample statistics computed from samples of the same size drawn from the same population.

12. The sample of debt/equity ratios of 25 publicly traded U.S. banks as of fiscal year-end 2003 is an example of:
    A. a point estimate.
    B. time-series data.
    C. cross-sectional data.
    D. a stratified random sample.

13. Which of the following is **NOT** a desirable property of an estimate?
    A. Reliability.
    B. Efficiency.
    C. Consistency.
    D. Unbiasedness.

14. If the variance of the sampling distribution of an estimator is smaller than all other unbiased estimators of the parameter of interest, the estimator is:
    A. reliable.
    B. efficient.
    C. unbiased.
    D. consistent.

15. Which of the following is **NOT** a property of Student's *t*-distribution?
    A. It is symmetrical.
    B. As the degrees of freedom get larger, the variance approaches zero.
    C. It is defined by a single parameter, the degrees of freedom, which is equal to n − 1.
    D. It has more probability in the tails and less at the peak than a standard normal distribution.

16. An analyst who uses historical data that was not publicly available at the time period being studied will have a sample with:
    A. look-ahead bias.
    B. time-period bias.
    C. survivorship bias.
    D. sample selection bias.

17. The 95 percent confidence interval of the sample mean of employee age for a major corporation is 19 years to 44 years based on a *z*-statistic. The population of employees is over 5,000 and the sample size of this test is 100. Assuming the population is normally distributed, the standard error of mean employee age is *closest* to:
    A. 1.96.
    B. 2.58.
    C. 6.38.
    D. 12.50.

18. Which of the following is *most closely* associated with survivorship bias?
    A. Price-to-book studies.
    B. Stratified bond sampling studies.
    C. Equity-index-linked note studies.
    D. Mutual fund performance studies.

19. What is the *most appropriate* test statistic for constructing confidence intervals for the population mean when the population is normally distributed, but the variance is unknown.
    A. The *z*-statistic at α with *n* degrees of freedom.
    B. The *z*-statistic with n − 1 degrees of freedom.
    C. The *t*-statistic at $\alpha/2$ with *n* degrees of freedom.
    D. The *t*-statistic at $\alpha/2$ with n − 1 degrees of freedom.

20. The *acceptable* test statistic for constructing confidence intervals for the population mean of a nonnormal distribution when the population variance is unknown and the sample size is large (n > 30) is the:
    A. z-statistic or the t-statistic.
    B. z-statistic at α with n degrees of freedom.
    C. t-statistic at α with 29 degrees of freedom.
    D. t-statistic at $\alpha/2$ with n degrees of freedom.

21. Jenny Fox evaluates managers who have a cross-sectional population standard deviation of returns of 8%. If returns are independent across managers, how large of a sample does Fox need so that the standard deviation of sample means is 1.265%?
    A. 6.
    B. 7.
    C. 30.
    D. 40.

22. Annual returns on small stocks have a population mean of 12 percent and a population standard deviation of 20 percent. If the returns are normally distributed, a 90 percent confidence interval on mean returns over a 5-year period is:
    A. 5.4% to 18.6%.
    B. –2.75% to 26.75%.
    C. –5.52% to 29.52%.
    D. 4.16% to 19.84%.

ANSWERS – EXAM FLASHBACKS

1. **D** Standard error of the sample mean = $\sigma_{\bar{x}} = \dfrac{\sigma}{\sqrt{n}} = \dfrac{100}{\sqrt{64}} = 12.50$

   Professor's Note: Recall that this is the standard deviation of the distribution of sample means of samples of size 64.

ANSWERS – CONCEPT CHECKERS: SAMPLING AND ESTIMATION

1. **C** In a simple random sample, each element of the population has an equal probability of being selected. Choice D allows for an equal chance, but only if there are 100 elements in the population from which the random sample is drawn.

2. **D** An example might be the difference between a particular sample mean and the average value of the overall population.

3. **B** The sampling error is the difference between the population parameter and the sample statistic.

4. **C** The formula for the standard error when the population standard deviation is unknown is: $s_{\bar{x}} = \dfrac{s}{\sqrt{n}}$.

5. **D** The formula for the standard error when the population standard deviation is known is: $\sigma_{\bar{x}} = \dfrac{\sigma}{\sqrt{n}}$.

6. **D** By definition.

7. **B** $s_{\bar{x}} = \dfrac{s}{\sqrt{n}}$. Given $s = 2$, $s_{\bar{x}} = \dfrac{2}{\sqrt{49}} = \dfrac{2}{7} = 0.2857$.

8. **C** $s_{\bar{x}} = \dfrac{\sigma}{\sqrt{n}}$. Given $\sigma^2 = 25$, $s_{\bar{x}} = \dfrac{5}{\sqrt{100}} = \dfrac{5}{10} = 0.5$.

9. **B** Since the population variance is known and $n \geq 30$, the confidence interval is determined as: $\bar{x} \pm z_{\alpha/2}(\sigma/\sqrt{n})$. $z_{\alpha/2} = z_{0.025} = 1.96$. So, the confidence interval is: $75 \pm 1.96(20/10) = 75 \pm 3.92 = 71.08$ to 78.92.

10. **D** Since the population variance is unknown and $n < 30$, the confidence interval is determined as: $\bar{x} \pm t_{\alpha/2}(s/\sqrt{n})$. Look up $t_{\alpha/2}$ and df = $n - 1$ to get critical t. $t_{0.01/2}$ and df = 24 is 2.797. So, the confidence interval is: $70 \pm 2.797(14/5) = 70 \pm 7.83$.

11. **D** Suppose you have a population of 10,000 employees. If you take 100 samples of 50 employees each, the distribution of the 100 sample means is the sampling distribution.

12. **C** Cross-sectional data is a set of data that are all collected as of the same point in time.

13. **A** Efficiency, consistency, and unbiasedness are all desirable properties of an estimate.

14. **B** By definition. Efficiency is a desirable property of an estimator.

15. **B** As the degrees of freedom get larger, the *t*-distribution approaches the normal distribution. As the degrees of freedom fall, the peak of the *t*-distribution flattens and its tails get fatter (more probability in the tails—that's why, all else the same, the critical *t* increases as the *df* decreases).

16. **A** The primary example of look-ahead bias is using year-end financial information in conjunction with market pricing data to compute ratios like the price/earnings, P/E. The E in the denominator is typically not available for 30-60 days after the end of the period. Hence, data that was available on the test date (P) is mixed with information that was not available (E). That is, the P is "ahead" of the E.

17. **C** At the 95% level of significance, with sample size n = 100 and mean 31.5 years, the appropriate test statistic is $z_{\alpha/2}$ = 1.96. *Note: The mean of 31.5 is calculated as the midpoint of the interval, or (19 + 44) / 2.* Thus, the confidence interval is 31.5 ±1.96$s_{\overline{X}}$, where $s_{\overline{X}}$ is the standard error of the sample mean. If we take the upper bound, we know that 31.5 + 1.96$s_{\overline{X}}$ = 44, or 1.96$s_{\overline{X}}$ = 12.5, or $s_{\overline{X}}$ = 6.38 years.

18. **D** Mutual fund performance studies are most closely associated with survivorship bias because only the better-performing funds remain in the sample over time.

19. **D** Use the *t*-statistic at $\alpha/2$ and n − 1 degrees of freedom when the population variance is unknown. While the *z*-statistic is acceptable when the sample size is large, sample size is not given here, and the *t*-statistic is always appropriate under these conditions.

20. **A** When the sample size is large, and the central limit theorem can be relied upon to assure a sampling distribution that is normal, either the *t*-statistic or the *z*-statistic is acceptable for constructing confidence intervals for the population mean. The *t*-statistic, however, will provide a more conservative range (wider) at a given level of significance.

21. **D** $1.265 = \dfrac{8}{\sqrt{N}}, N = \left(\dfrac{8}{1.265}\right)^2 \approx 40$

22. **B** With a known population standard deviation of returns and a normally distributed population, we can use the *z*-distribution. The sample mean for a sample of 5 years will have a standard deviation of $\dfrac{20}{\sqrt{5}} = 8.94\%$. A 90% confidence interval around the mean return of 12% is $12\% \pm 1.65(8.94\%) = -2.75\%$ to 26.75%.

HYPOTHESIS TESTING

EXAM FOCUS

This review addresses common hypothesis testing procedures. These procedures are used to conduct tests of population means, population variances, differences in means, differences in variances, and mean differences. Specific tests reviewed include the z-test, t-test, chi-square test, and F-test. You should know when and how to apply each of these. A standard hypothesis testing procedure is utilized in this review. Know it! You should be able to perform a hypothesis test on the value of the mean without being given any formulas. Confidence intervals, levels of significance, the power of a test, and types of hypothesis testing errors are also discussed. These are concepts you are likely to see on the exam. Don't worry about memorizing the messy formulas on testing for the equalities and differences in means and variances at the end of this review, but be able to interpret these statistics.

WARM-UP: HYPOTHESIS TESTING

Hypothesis testing is the statistical assessment of a statement or idea regarding a population. For instance, a statement could be as follows: "The mean return for the U.S. equity market is greater than zero." Given the relevant returns data, hypothesis testing procedures can be employed to test the validity of this statement at a given significance level.

To illustrate the hypothesis testing concepts and procedures presented in this topic review, an ongoing example pertaining to stock option returns will be used. The background for this example is as follows.

Stock Option Returns: The Common Example

There is an investor who believes that call options should have a mean daily return greater than zero. To empirically assess this belief, she has gathered data on the daily return of a very large portfolio of call options. The mean daily return for the sample portfolio over a period of 250 days is 0.001, or 0.1 percent, and the sample standard deviation of returns is 0.0025, or 0.25 percent.

LOS 12.a: Define a hypothesis, describe the steps of hypothesis testing; define and interpret the null hypothesis and alternative hypothesis, and distinguish between one-tailed and two-tailed tests of hypothesis.

A **hypothesis** is a statement about the value of a population parameter developed for the purpose of testing a theory or belief. Hypotheses are stated in terms of the population parameter to be tested, like the population mean, μ. For example, a researcher may be interested in the mean daily return on stock options. Hence, the hypothesis may be that the mean daily return on a portfolio of stock options is positive.

Hypothesis testing procedures, based on sample statistics and probability theory, are used to determine whether a hypothesis is a reasonable statement and should not be rejected or if it is an unreasonable statement and should be rejected. The process of hypothesis testing consists of a series of steps shown in Figure 1.

Figure 1: Hypothesis Testing Procedure

State the hypothesis
↓
Select the appropriate test statistic
↓
Specify the level of significance
↓
State the decision rule regarding the hypothesis
↓
Collect the sample and calculate the sample statistics
↓
Make a decision regarding the hypothesis
↓
Make a decision based on the results of the test
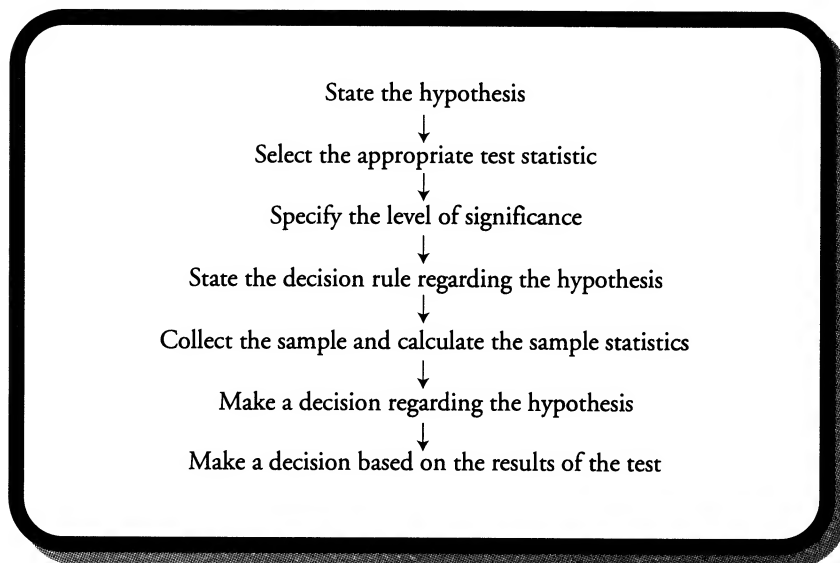
* *(Source:* Wayne W. Daniel and James C. Terrell, *Business Statistics,*
Basic Concepts and Methodology, Houghton Mifflin, Boston, 1997.)

Professor's Note: You should know this process!

The Null Hypothesis and Alternative Hypothesis

The **null hypothesis**, designated H_0, is the hypothesis that the researcher wants to reject. It is the hypothesis that is actually tested and is the basis for the selection of the test statistics. The null is generally stated as a simple statement about a population parameter. Typical statements of the null hypothesis for the population mean include $H_0: \mu = \mu_0$, $H_0: \mu \leq \mu_0$, and $H_0: \mu \geq \mu_0$, where μ is the population mean and μ_0 is the hypothesized value of the population mean. The null hypothesis always includes the = sign.

The **alternative hypothesis**, designated H_a, is what is concluded if there is sufficient evidence to reject the null hypothesis. It is usually the alternative hypothesis that you are really trying to assess. Why? Since you can never really prove anything with statistics, when the null hypothesis is discredited, the implication is that the alternative hypothesis is valid.

One-Tailed and Two-Tailed Tests of Hypotheses

The alternative hypothesis can be one-sided or two-sided. A one-sided test is referred to as a **one-tailed test**, and a two-sided test is referred to as a **two-tailed test**. Whether the test is one- or two-sided depends on the proposition being tested. If a researcher wants to test whether the return on stock options is greater than zero, a one-tailed test should be used. However, a two-tailed test should be used if the research question is whether the return on options is simply different from zero. Two-sided tests allow for deviation on both sides of the hypothesized value (zero). In practice, most hypothesis tests are constructed as two-tailed tests.

A **two-tailed** test for the population mean may be structured as:

$H_0: \mu = \mu_0$ versus $H_a: \mu \neq \mu_0$.

Since the alternative hypothesis allows for values above and below the hypothesized parameter, a two-tailed test uses two critical values.

The *general decision rule for a two-tailed test* is:

Reject H_0 if: test statistic > upper critical value or
test statistic < lower critical value

Let's look at the development of the decision rule for a two-tailed test using a z-distributed test statistic (a z-test) at a 5 percent level of significance, $\alpha = 0.05$.

- At $\alpha = 0.05$, the computed test statistic is compared with the critical z-values of ±1.96. The values of ±1.96 correspond to $\pm z_{\alpha/2} = \pm z_{0.025}$, which is the range of z-values within which 95 percent of the probability lies. These values are obtained from the cumulative probability table for the standard normal distribution (z-table), which is included at the back of this book.
- If the computed test statistic falls outside the range of critical z-values (i.e., test statistic > 1.96, or test statistic < −1.96), we reject the null and conclude that the sample statistic is sufficiently different from the hypothesized value.
- If the computed test statistic falls within the range ±1.96, we conclude that the sample statistic is not sufficiently different from the hypothesized value ($\mu = \mu_0$ in this case), and we fail to reject the null hypothesis.

The *decision rule* (rejection rule) *for a two-tailed z-test* at $\alpha = 0.05$ can be stated as:

Reject H_0 if test statistic < −1.96 or if test statistic > 1.96

Figure 2 shows the standard normal distribution for a two-tailed hypothesis test using the z-distribution. Notice that the significance level of 0.05 means that there is $0.05 / 2 = 0.025$ probability (area) under each tail of the distribution beyond ±1.96.

Figure 2: Two-Tailed Hypothesis Test Using the Standard Normal (z) Distribution
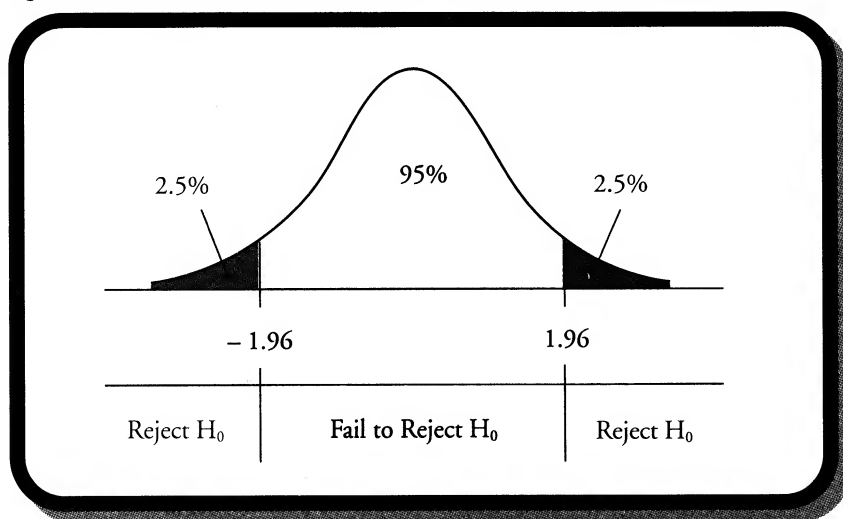
Professor's Note: The next two examples are extremely important. Don't move on until you understand them!

Example: Two-tailed test

Referencing our option return data, test the hypothesis that the mean return for options is not zero at the 5 percent level of significance.

Answer:

Finally, we can perform a hypothesis test for our option return data. Let's start by specifying the null and alternative hypotheses using a two-tailed structure as follows:

H_0: $\mu_0 = 0$ versus H_a: $\mu_0 \neq 0$

At a 5 percent level of significance, the critical z-values for a two-tailed test are ± 1.96, so the decision rule can be stated as:

Reject H_0 if $+1.96 <$ test statistic < -1.96

Our test statistic is $\dfrac{0.001}{\left(\dfrac{0.0025}{\sqrt{250}}\right)} = \dfrac{0.001}{0.000158} = 6.33.$

Since 6.33 > 1.96, we reject the null hypothesis that the mean daily option return is equal to zero. Note that when we reject the null, we conclude that the sample value is significantly different from the hypothesized value. We are saying that the two values are different from one another *after considering the variation in the sample*. That is, the sample mean of 0.001 is statistically different from zero given the sample's standard deviation and size.

For a **one-tailed hypothesis test** of the population mean, the null and alternative hypotheses are either:

Upper tail: H_0: $\mu \leq \mu_0$ versus H_a: $\mu > \mu_0$, or
Lower tail: H_0: $\mu \geq \mu_0$ versus H_a: $\mu < \mu_0$

The appropriate set of hypotheses depends on whether we believe the population mean, μ, to be greater than (upper tail) or less than (lower tail) the hypothesized value, μ_0. Using a z-test at the 5 percent level of significance, the computed test statistic is compared with the critical values of 1.645 for the upper tail tests (i.e., H_a: $\mu > \mu_0$) or -1.645 for lower tail tests (i.e., H_a: $\mu < \mu_0$). These critical values are obtained from a z-table, where $-z_{0.05} = -1.645$ corresponds to a cumulative probability equal to 5 percent, and the $z_{0.05} = 1.645$ corresponds to a cumulative probability of 95 percent (1 − 0.05).

Let's use the upper tail test structure where H_0: $\mu \leq \mu_0$ and H_a: $\mu > \mu_0$.

- If the calculated test statistic is greater than 1.645, we conclude that the sample statistic is sufficiently greater than the hypothesized value. In other words, we reject the null hypothesis.
- If the calculated test statistic is less than 1.645, we conclude that the sample statistic is not sufficiently different from the hypothesized value, and we fail to reject the null hypothesis.

Figure 3 shows the standard normal distribution and the rejection region for a one-tailed test (upper tail) at the 5 percent level of significance.

Figure 3: One-Tailed Hypothesis Test Using the Standard Normal (z) Distribution
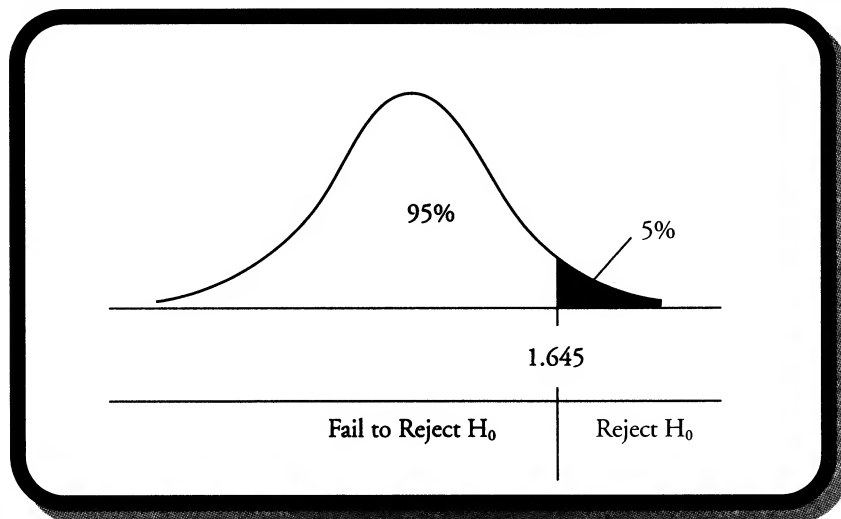
Example: One-tailed test

Perform a z-test on our option data to test the proposition that option returns are positive.

Answer:

In this case, we use a one-tailed test with the following structure:

H_0: $\mu \leq 0$ versus H_a: $\mu > 0$

Recalling that $z_{0.05} = 1.645$, the appropriate decision rule for our one-tailed z-test at a significance level of 5 percent is:

Reject H_0 if test statistic > 1.645

The test statistic is computed the same way regardless of whether we are using a one-tailed or two-tailed test. From the previous example, we know that the test statistic for the option return sample is 6.33. Since 6.33 > 1.645, we reject the null hypothesis and conclude that mean returns are statistically greater than zero at a 5 percent level of significance. *(Exam Flashback #1.)*

LOS 12.b: Discuss the choice of the null and alternative hypotheses.

The most common null hypothesis will be an "equal to" hypothesis. Combined with a "not equal to" alternative, this will require a two-tailed test. The alternative is often the hoped-for hypothesis. When the null is that a coefficient is equal to zero, we hope to reject it and show the significance of the relationship.

When the null is less than or equal to, the (mutually exclusive) alternative is framed as greater than, and a one-tail test is appropriate. If we are trying to demonstrate that a return is greater than the risk-free rate, this would be the correct formulation. We will have set up the null and alternative hypothesis so that rejection of the null will lead to acceptance of the alternative, our goal in performing the test.

LOS 12.c: Define and interpret a test statistic, a Type I and a Type II error, and a significance level, and explain how significance levels are used in hypothesis testing.

Hypothesis testing involves two statistics: the test statistic calculated from the sample data and the *critical value* of the test statistic. The value of the computed test statistic relative to the critical value is a key step in assessing the validity of a hypothesis.

A test statistic is calculated by comparing the point estimate of the population parameter with the hypothesized value of the parameter (i.e., the value specified in the null hypothesis). With reference to our option return example, this means we are concerned with the difference between the mean return of the sample (i.e., $\overline{x}$ = 0.001) and the hypothesized mean return (i.e., μ_0 = 0). As indicated in the following expression, the **test statistic** is the difference between the sample statistic and the hypothesized value, scaled by the standard error of the sample statistic.

$$\text{test statistic} = \frac{\text{sample statistic} - \text{hypothesized value}}{\text{standard error of the sample statistic}}$$

The standard error of the sample statistic is the adjusted standard deviation of the sample. When the sample statistic is the sample mean, $\overline{x}$, the standard error of the sample statistic for sample size n, is calculated as:

$$\sigma_{\overline{x}} = \sigma \big/ \sqrt{n}$$

when the population standard deviation, σ, *is known*, or

$$s_{\overline{x}} = s \big/ \sqrt{n}$$

when the population standard deviation, σ, *is not known*. In this case, it is estimated using the standard deviation of the sample, s.

Professor's Note: Don't be confused by the notation here. A lot of the literature you will encounter in your studies simply uses the term $\sigma_{\overline{x}}$ for the standard error of the test statistic, regardless of whether the population standard deviation was actually used in its computation.

Example: Test statistic

Compute the test statistic for our option returns example.

Answer:

To compute the test statistic, it is first necessary to calculate the standard error of the sample statistic.

$$s_{\overline{x}} = s \big/ \sqrt{n} = 0.0025 \big/ \sqrt{250} = 0.000158$$

Now, the test statistic can be computed as follows:

$$\text{test statistic} = \frac{\text{sample statistic} - \text{hypothesized value}}{\text{standard error of the sample statistic}}$$

$$= \frac{0.001 - 0}{0.000158} = 6.33$$

As you will soon see, a test statistic is a random variable that may follow one of several distributions, depending on the characteristics of the sample and the population. We will look at four distributions for test statistics: the t-distribution, the z-distribution (standard normal distribution), the chi-square distribution, and the F-distribution. The critical value for the appropriate test statistic—the value against which the computed test statistic is compared—is a function of its distribution.

Type I and Type II Errors

Keep in mind that hypothesis testing is used to make inferences about the parameters of a given population on the basis of statistics computed for a sample that is drawn from that population. We must be aware that there is some probability that the sample, in some way, does not represent the population, and any conclusion based on the sample about the population may be made in error.

When drawing inferences from a hypothesis test, there are two types of errors:

- **Type I error**: the rejection of the null hypothesis when it is actually true.
- **Type II error**: the failure to reject the null hypothesis when it is actually false.

(Exam Flashback #2.)

The **significance level** is the probability of making a Type I error (rejecting the null when it is true) and is designated by the Greek letter alpha (α). For instance, a significance level of 5 percent ($\alpha = 0.05$) means that there is a 5 percent chance of rejecting a true null hypothesis. When conducting hypothesis tests, a significance level must be specified when selecting the critical values against which test statistics are compared.

LOS 12.d: Define and interpret a decision rule and the power of a test.

The decision for a hypothesis test is to either reject the null hypothesis or fail to reject the null hypothesis. Note that it is statistically incorrect to say "accept" the null hypothesis, it can only be supported or rejected. The **decision rule** for rejecting or failing to reject the null hypothesis is based on the distribution of the test statistic. For example, if the test statistic follows a normal distribution, the decision rule is based on critical values determined from the standard normal distribution (z-distribution). Regardless of the appropriate distribution, it must be determined if a one-tailed or two-tailed hypothesis test is appropriate before a decision rule (rejection rule) can be determined.

A decision rule is specific and quantitative. Once we have determined whether a one- or two-tailed test is appropriate, the significance level we require, and the distribution of the test statistic, we can calculate the exact critical value for the test statistic. Then we have a decision rule of the following form: if the test statistic is (greater, less than) the value X, reject the null.

The Power of a Test

While the significance level of a test is the probability of rejecting the null hypothesis when it is true, the **power of a test** is the probability of correctly rejecting the null hypothesis when it is false. The power of a test is actually one minus the probability of making a Type II error, or 1 – P(Type II error). In other words, the probability of

rejecting the null when it is false (power of the test) equals one minus the probability of *not* rejecting the null when it is false (Type II error). When more than one test statistic may be used, the power of the test for the competing test statistics may be useful in deciding which test statistic to use. Ordinarily, we wish to use the test statistic that provides the most powerful test among all possible tests.

Figure 4 shows the relationship between the level of significance, the power of a test, and the two types of errors.

Figure 4: Type I and Type II Errors in Hypothesis Testing

Decision	*True Condition*	
	H_0 is true	H_0 is false
Do not reject H_0	Correct Decision	Incorrect Decision **Type II Error**
Reject H_0	Incorrect Decision **Type I Error** Significance level, α, = P(Type I Error)	Correct Decision Power of the test = 1 – P(Type II Error)

Note that decreasing the probability of making a Type I error (i.e., decreasing the level of significance of the test), makes it more difficult to reject the null when it is true. All else equal, however, the decrease in the chance of making a Type I error comes at the cost of increasing the probability of making a Type II error because the null is rejected less frequently, even when it is actually false. In addition, as the probability of a Type II error increases, the power of the test declines because it is defined as one minus the probability of a Type II error.

LOS 12.e: Explain the relation between confidence intervals and hypothesis tests.

A confidence interval is a range of values within which the researcher believes the true population parameter may lie.

A confidence interval is determined as:

$$\left\{ \left[\begin{array}{c} \text{sample} \\ \text{statistic} \end{array} - \left(\begin{array}{c} \text{critical} \\ \text{value} \end{array} \right) \left(\begin{array}{c} \text{standard} \\ \text{error} \end{array} \right) \right] < \begin{array}{c} \text{population} \\ \text{parameter} \end{array} < \left[\begin{array}{c} \text{sample} \\ \text{statistic} \end{array} + \left(\begin{array}{c} \text{critical} \\ \text{value} \end{array} \right) \left(\begin{array}{c} \text{standard} \\ \text{error} \end{array} \right) \right] \right\}$$

The interpretation of a confidence interval is that for a level of confidence of, say, 95 percent, there is a 95 percent probability that the true population parameter is contained in the interval.

From the expression above, we see that a confidence interval and a hypothesis test are linked by the critical value. For example, a 95 percent confidence interval uses a critical value associated with a given distribution at the 5 percent level of significance. Similarly, a hypothesis test would compare a test statistic to a critical value at the 5 percent level of significance. To see this relationship more clearly, the expression for the confidence interval can be manipulated and restated as:

–critical value < test statistic < +critical value

This is the range within which we fail to reject the null for a two-tailed hypothesis test at a given level of significance.

Example: Confidence interval

Using our option example, **construct** a 95 percent confidence interval for the population mean. Use a z-distribution. **Decide** if the hypothesis $\mu = 0$ should be rejected.

Answer:

Given a sample size of 250 with a standard deviation of 0.25 percent, the standard error can be computed as

$$s_{\bar{x}} = \frac{s}{\sqrt{n}} = 0.25/\sqrt{250} = 0.0158\%.$$

At the 5 percent level of significance, the critical z-values for the confidence interval are $z_{0.025} = 1.96$ and $-z_{0.025} = -1.96$. Thus, given a sample mean equal to 0.1 percent, the 95 percent confidence interval for the population mean is:

$$0.1 - 1.96(0.0158) < \mu < 0.1 + 1.96(0.0158), \text{ or}$$

$$0.069\% < \mu < 0.1310\%$$

Since there is a 95 percent probability that the true mean is within this confidence interval, we can reject the hypothesis $\mu = 0$ because 0 is not within the confidence interval. Alternatively, the z-statistic is

$$\frac{0.1}{0.0158} = 6.33, \text{ so we reject } \mu = 0.$$

The *p-value* is the probability of obtaining a critical value that is the same as the computed test statistic, assuming the null hypothesis is true. It is the smallest level of significance for which the null hypothesis can be rejected. For one-tailed tests, the *p*-value is the probability that lies above the computed test statistic for upper tail tests or below the computed test statistic for lower tail tests. For two-tailed tests, the *p*-value is the probability that lies above the positive value of the computed test statistic *plus* the probability that lies below the negative value of the computed test statistic.

LOS 12.f: Distinguish between a statistical decision and an economic decision.

A **statistical decision** is based solely on the sample information, the test statistic, and the hypotheses. The decision to reject the null hypothesis is a statistical decision. On the other hand, an **economic decision** considers the relevance of the statistical decision after considering factors such as transaction costs, taxes, risk, and other factors that don't play into the statistical decision. For example, it is possible for an investment strategy to produce returns that are statistically significant, but once the effects of trading costs are considered, the strategy is deemed not to be economically significant. In our option portfolio we may find that the return is statistically significant and positive. However, it may be that the transaction costs necessary to maintain the portfolio are so high that the optimal economic decision is to not buy the portfolio.

WARM-UP: THE t-DISTRIBUTION

The *t*-distribution is a symmetrical distribution that is centered about zero. The shape of the *t*-distribution is dependent on the number of degrees of freedom, and degrees of freedom are based on the number of sample observations. The *t*-distribution is flatter and has thicker tails than the standard normal distribution. As the number of observations increases (i.e., the degrees of freedom increase), the *t*-distribution becomes more spiked and its tails become thinner. As the number of degrees of freedom increases without bound, the *t*-distribution converges to the standard normal distribution (z-distribution). The thickness of the tails relative to those of the

z-distribution is important in hypothesis testing because thicker tails mean more observations away from the center of the distribution (i.e., more "outliers"). Hence, hypothesis testing using the t-distribution makes it more difficult to reject the null relative to hypothesis testing using the z-distribution.

The table in Figure 5 contains one-tailed critical values for the t-distribution at the 0.05 and 0.025 levels of significance with various degrees of freedom (df). Note that, unlike the z-table, the t-values are contained within the table, and the probabilities are located at the column headings. Also note that the level of significance of a t-test corresponds to the *one-tailed probabilities*, p, that head the columns in the t-table.

Figure 6 portrays the different shapes of the t-distribution associated with different degrees of freedom and levels of significance. Illustrated in Figure 6 is the tendency for the t-distribution to become more peaked and begin to look more and more like the normal distribution as the degrees of freedom increases.

Figure 5: Table of Critical t-Values

df	*One-Tailed Probabilities, p*	
	p = 0.05	p = 0.025
5	2.015	2.571
10	1.812	2.228
15	1.753	2.131
20	1.725	2.086
25	1.708	2.060
30	1.697	2.042
40	1.684	2.021
50	1.676	2.009
60	1.671	2.000
70	1.667	1.994
80	1.664	1.990
90	1.662	1.987
100	1.660	1.984
120	1.658	1.980
∞	1.645	1.960

Figure 6: *t*-Distributions for Different Degrees of Freedom (df)
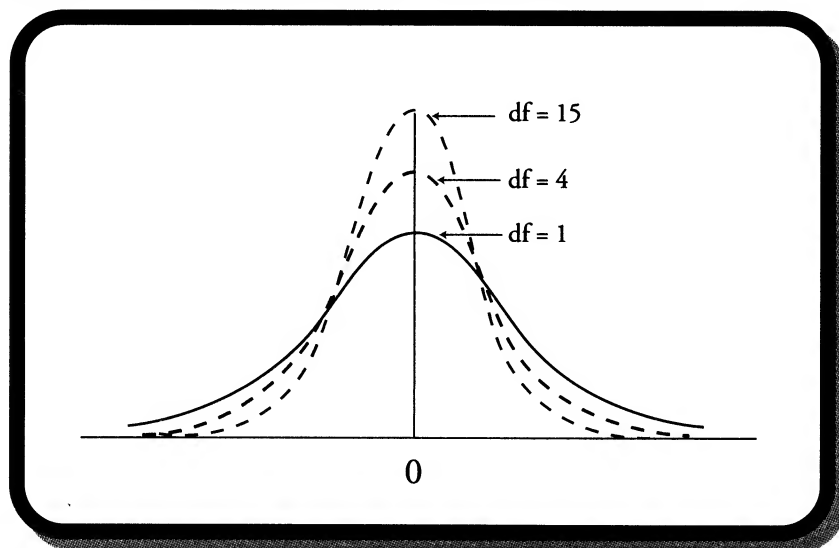
LOS 12.g: Identify the appropriate test statistic and interpret the results for a hypothesis test concerning the population mean of a normally distributed population with 1) known or 2) unknown variance.

When hypothesis testing, the choice between using a critical value based on the *t*-distribution or the *z*-distribution depends on sample size, the distribution of the population, and whether or not the variance of the population is known.

The *t*-Test

The *t*-test is a widely used hypothesis test that employs a test statistic that is distributed according to a *t*-distribution. Following are the rules for when it is appropriate to use the *t*-test for hypothesis tests of the population mean.

Use the t-test if the population variance is unknown and either of the following conditions exist:

- The sample is large (n ≥ 30).
- The sample is small (less than 30), but the distribution of the population is normal or approximately normal.

The computed value for the test statistic based on the *t*-distribution is referred to as the *t*-statistic. For hypothesis tests of a population mean, a *t*-statistic with n − 1 degrees of freedom is computed as:

$$t_{n-1} = \frac{\overline{x} - \mu_0}{s/\sqrt{n}}$$

where:
$\overline{x}$ = sample mean
μ_0 = hypothesized population mean (i.e., the null)
s = standard deviation of the sample
n = sample size

Professor's Note: This computation is not new. It is the same test statistic computation that we have been performing all along. Note the use of the sample standard deviation, s, in the standard error term in the denominator.

To conduct a *t*-test, the *t*-statistic is compared to a critical *t*-value at the desired level of significance with the appropriate degrees of freedom.

In the real world, the underlying variance of the population is rarely known, so the *t*-test enjoys widespread application.

The *z*-Test

The *z*-test is the appropriate hypothesis test of the population mean when the *population is normally distributed with known variance*. The computed test statistic used with the *z*-test is referred to as the *z*-statistic. The *z*-statistic for a hypothesis test for a population mean is computed as follows:

$$z\text{-statistic} = \frac{\overline{x} - \mu_0}{\sigma / \sqrt{n}}$$

where:
$\overline{x}$ = sample mean
μ_0 = hypothesized population mean
σ = standard deviation of the *population*
n = sample size

To test a hypothesis, the *z*-statistic is compared to the critical *z*-value corresponding to the significance of the test. Critical *z*-values for the most common levels of significance are displayed in Figure 7. You should have these memorized by now.

Figure 7: Critical *z*-Values

Level of Significance	Two-Tailed Test	One-Tailed Test
0.10 = 10%	±1.65	+1.28 or −1.28
0.05 = 5%	±1.96	+1.65 or −1.65
0.01 = 1%	±2.58	+2.33 or −2.33

When the *sample size is large* and the *population variance is unknown*, the *z*-statistic is:

$$z\text{-statistic} = \frac{\overline{x} - \mu_0}{s / \sqrt{n}}$$

where:
$\overline{x}$ = sample mean
μ_0 = hypothesized population mean
s = standard deviation of the *sample*
n = sample size

Note the use of the sample standard deviation, *s*, versus the population standard deviation, σ. Remember, this is acceptable if the sample size is large, although the *t*-statistic is the more conservative measure when the population variance is unknown.

Example: *z*-test or *t*-test?

Referring to our option return problem once more, **explain** which test statistic (*z* or *t*) should be used and the difference in the likelihood of rejecting the null with each distribution.

Answer:

The population variance for our sample of returns is unknown. Hence, the *t*-distribution is appropriate. With 250 observations, however, the sample is considered to be large, so the *z*-distribution would also be acceptable. This is a trick question—either distribution, *t* or *z*, is appropriate. With regard to the difference in the likelihood of rejecting the null, since our sample is so large, the critical values for the *t* and *z* are almost identical. Hence, there is almost no difference in the likelihood of rejecting the null.

Example: The *z*-test

When your company's gizmo machine is working properly, the mean length of gizmos is 2.5 inches. However, from time to time the machine gets out of alignment and produces gizmos that are either too long or too short. When this happens, production is stopped and the machine is adjusted. To check the machine, the quality control department takes a gizmo sample each day. Today a random sample of 49 gizmos showed a mean length of 2.49 inches. The population standard deviation is known to be 0.021 inches. Using a 5 percent significance level, **determine** if the machine should be shut down and adjusted.

Answer:

Let μ be the mean length of all gizmos made by this machine, and let $\bar{x}$ be the corresponding mean for the sample.

Let's follow the hypothesis testing procedure presented earlier in Figure 1. Again, you should know this process!

Statement of hypothesis. For the information provided, the null and alternative hypotheses are appropriately structured as:

H_0: $\mu = 2.5$ (The machine does not need an adjustment.)
H_a: $\mu \neq 2.5$ (The machine needs an adjustment.)

Note that since this is a two-tailed test, H_a allows for values above and below 2.5.

Select the appropriate test statistic. Since the population variance is known and the sample size is > 30, the *z*-statistic is the appropriate test statistic. The *z*-statistic is computed as:

$$z = \frac{\bar{x} - \mu_0}{\sigma / \sqrt{n}}$$

Specify the level of significance. The level of significance is given at 5 percent, implying that we are willing to accept a 5 percent probability of rejecting the null when the null is true.

State the decision rule regarding the hypothesis. The $\neq$ sign in the alternative hypothesis indicates that the test is two-tailed with two rejection regions, one in each tail of the standard normal distribution curve. Because the total area of both rejection regions combined is 0.05 (the significance level), the area of the rejection region in each tail is 0.025. You should know that the critical *z*-values for $\pm z_{0.025}$ are ± 1.96. This means that the null hypothesis should not be rejected if the computed *z*-statistic lies between -1.96 and $+1.96$ and should be rejected if it lies outside of these critical values. The decision rule can be stated as:

Reject H_0 if $-z_{0.025}$ > *z*-statistic > $z_{0.025}$, or equivalently,

Reject H_0 if: -1.96 > *z*-statistic > $+1.96$

Collect the sample and calculate the test statistic. The value of $\overline{x}$ from the sample is 2.49. Since σ is given as 0.021, we calculate the z-statistic using σ as follows:

$$z = \frac{\overline{x} - \mu_0}{\sigma/\sqrt{n}} = \frac{2.49 - 2.5}{0.021/\sqrt{49}} = \frac{-0.01}{0.003} = -3.33$$

Make a decision regarding the hypothesis. The calculated value of the z-statistic is $= -3.33$. Since this value is less than the critical value, $-z_{0.025} = -1.96$, it falls in the rejection region in the left tail of the z-distribution. Hence, there is sufficient evidence to reject H_0.

Make a decision based on the results of the test. Based on the sample information and the results of the test, it is concluded that the machine is out of adjustment and should be shut down for repair.

WARM-UP: TESTS OF DIFFERENCES BETWEEN MEANS

Up to this point, we have been concerned with tests of a single population mean. In practice, we frequently want to know if there is a difference between the means of two populations. There are two t-tests that are used to test differences between the means of two populations. Application of either of these tests requires that we are reasonably certain that our samples are independent and that they are taken from two normally distributed populations. Both of these t-tests are used when the population variance is unknown. In one case, the population variances are assumed to be equal, and the sample observations are pooled. In the other case, however, no assumption is made regarding the equality between the two population variances, and the t-test uses an approximated value for the degrees of freedom.

When testing differences between the mean of population 1, μ_1, and mean of population 2, μ_2, we may be interested in knowing if the two means are equal (i.e., $\mu_1 = \mu_2$), if the mean of population 1 is greater than that of population 2 (i.e., $\mu_1 > \mu_2$), or if the mean of population 2 exceeds that of population 1 (i.e., $\mu_2 > \mu_1$). These three sets of hypotheses are structured as:

H_0: $\mu_1 - \mu_2 = 0$ versus H_a: $\mu_1 - \mu_2 \neq 0$ (a two-tail test)
H_0: $\mu_1 - \mu_2 \leq 0$ versus H_a: $\mu_1 - \mu_2 > 0$ (a one-tail test)
H_0: $\mu_1 - \mu_2 \geq 0$ versus H_a: $\mu_1 - \mu_2 < 0$ (a one-tail test)

Note that it is also possible to structure other hypotheses, such as H_0: $\mu_1 - \mu_2 = 50$ versus H_a: $\mu_1 - \mu_2 \neq 50$. Regardless of the specific structure, the hypothesis testing procedure is the same.

LOS 12.h: Identify the appropriate test statistic and interpret the results for a hypothesis test concerning the equality of the population means of two normally distributed populations, based on independent random samples with 1) equal or 2) unequal assumed variances.

A pooled variance is used with the t-test for testing differences between the means of normally distributed populations with **unknown variances that are assumed to be equal**. Assuming independent samples, the t-statistic in this case is computed as:

$$t = \frac{(\overline{x}_1 - \overline{x}_2) - (\mu_1 - \mu_2)}{\left(\dfrac{s_p^2}{n_1} + \dfrac{s_p^2}{n_2}\right)^{1/2}}$$

where:

$$s_p^2 = \frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}$$

s_1^2 = variance of the first sample

s_2^2 = variance of the second sample

n_1 = number of observations in the first sample

n_2 = number of observations in the second sample

Note: The degrees of freedom, df, is $(n_1 + n_2 - 2)$, and for a test of equality of means, $\mu_1 - \mu_2 = 0$.

When testing the hypothesis of equality, $\mu_1 - \mu_2 = 0$ so that the numerator is just the difference between the sample means, $\overline{x}_1 - \overline{x}_2$. Since we assume that the variances are equal, we just add the variances of the two sample means in order to calculate the standard error in the denominator.

The *t*-test for differences between population means when the populations are normally distributed having **variances that are unknown and assumed to be unequal** uses the sample variances for both populations. Assuming independent samples, the *t*-statistic in this case is computed as follows:

$$t = \frac{(\overline{x}_1 - \overline{x}_2) - (\mu_1 - \mu_2)}{\left(\dfrac{s_1^2}{n_1} + \dfrac{s_2^2}{n_2}\right)^{1/2}}$$

where:

$$\text{degrees of freedom} = \frac{\left(\dfrac{s_1^2}{n_1} + \dfrac{s_2^2}{n_2}\right)^2}{\dfrac{\left(s_1^2/n_1\right)^2}{n_1} + \dfrac{\left(s_2^2/n_2\right)^2}{n_2}}$$

and where:

s_1^2 = variance of the first sample

s_2^2 = variance of the second sample

n_1 = number of observations in the first sample

n_2 = number of observations in the second sample

Again, a test of equality of means will have only the difference in sample means in the numerator. However, with no assumption of equal variances, the denominator (standard error) is based on the individual sample variances of the means for each sample. You do not need to memorize these two formulas but should understand the numerator, the fact that these are *t*-statistics, and that the variance of the pooled sample is used when the sample variances are assumed to be equal.

Example: Difference between means – equal variances

Sue Smith is investigating whether the abnormal returns that occur in acquiring firms during merger announcement periods differ for horizontal and vertical mergers. She estimated the abnormal returns for a sample of acquiring firms associated with horizontal mergers and a sample of acquiring firms involved in vertical mergers. Her sample findings are reported in Figure 8.

Figure 8: Abnormal Returns During Merger Announcement Periods

	Abnormal Returns Horizontal Mergers	*Abnormal Returns Vertical Mergers*
Mean	1.0%	2.5%
Standard deviation	1.0%	2.0%
Sample size (n)	64	81

Assuming the samples are independent, the population means are normally distributed, and the population variances are equal, **determine** if there is a statistically significant difference in the announcement period abnormal returns for these two types of mergers.

Answer:

State the hypothesis. Since this is a two-sided test, the structure of the hypotheses takes the following form:

H_0: $\mu_1 - \mu_2 = 0$ versus H_a: $\mu_1 - \mu_2 \neq 0$

where:
μ_1 = the mean of the abnormal returns for the horizontal mergers
μ_2 = the mean of the abnormal returns for the vertical mergers

Select the appropriate test statistic. Since we are assuming equal variances, the test statistic is computed using the following formula:

$$t = \frac{(\overline{x}_1 - \overline{x}_2) - (\mu_1 - \mu_2)}{\left(\dfrac{s_p^2}{n_1} + \dfrac{s_p^2}{n_2}\right)^{1/2}}$$

where:

$$s_p^2 = \frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}$$

Specify the level of significance. We will use the common significance level of 5 percent ($\alpha = 0.05$). In order to look up the critical t-value, we also need the degrees of freedom, which in this case is $n_1 + n_2 - 2$, or df = 64 + 81 − 2 = 143.

State the decision rule regarding the hypothesis. We must identify the critical *t*-value for a 5 percent level of significance and the *closest* degrees of freedom specified in a *t*-table. As you should verify with the partial *t*-table contained in Figure 9, the closest entry for df = 143 is df = 120. At $\alpha/2$ = p = 0.025 with df = 120, the critical *t*-value = 1.980.

Figure 9: Partial *t*-Table

df	*p = 0.10*	*p = 0.05*	*p = 0.025*
110	1.289	1.659	1.982
120	1.289	1.658	**1.980**
200	1.286	1.653	1.972

One-Tailed Probabilities (p) (spanning header over the three probability columns)

Thus, the decision rule can be stated as:

Reject H_0 if *t*-statistic < −1.980 or *t*-statistic > 1.980

The rejection region for this test is illustrated in Figure 10.

Figure 10: Decision Rule for Two-Tailed *t*-Test
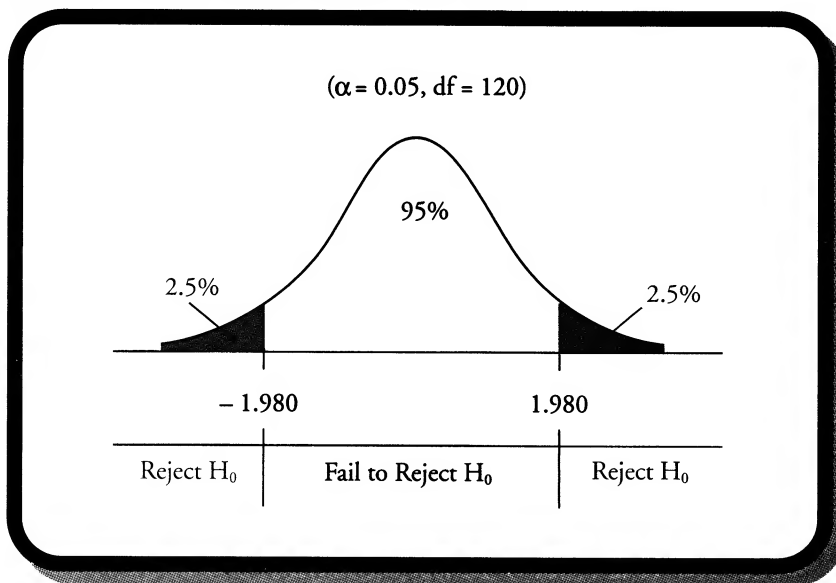
(α = 0.05, df = 120)

95%

2.5%   2.5%

− 1.980   1.980

Reject H_0 | Fail to Reject H_0 | Reject H_0

Collect the sample and calculate the sample statistics. Using the information provided, the *t*-statistic can be computed as follows (note that the −0.015 in the numerator equals 0.01 − 0.025, which represents the difference in means) since the hypothesized difference in means $(\mu_1 - \mu_2)$ is zero.

$$t = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\left(s_p^2/n_1 + s_p^2/n_2\right)^{1/2}} = \frac{-0.015}{0.00274} = -5.474$$

where:

$$s_p^2 = \frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2} = \frac{(63)(0.0001) + (80)(0.0004)}{143} = 0.000268$$

Make a decision regarding the hypothesis. Since the calculated test statistic falls to the left of the lowest critical *t*-value, we reject the null hypothesis and conclude that the announcement period abnormal returns are different for horizontal and vertical mergers.

LOS 12.i: Identify the appropriate test statistic and interpret the results for a hypothesis test concerning the mean difference of two normally distributed populations (paired comparisons test).

While the tests considered in the previous section were of the difference between the means of two independent samples, sometimes our samples may be dependent. If the observations in the two samples both depend on some other factor, we can construct a "paired comparisons" test of whether the means of the differences between observations for the two samples are different. Dependence may result from an event that affects both sets of observations for a number of companies or because observations for two firms over time are both influenced by market returns or economic conditions.

For an example of a paired comparisons test, consider a test of whether the returns on two steel firms were equal over a 5-year period. We can't use the difference in means test because we have reason to believe that the samples are not independent. Both will depend to some extent on the returns on the overall market (market risk) and the conditions in the steel industry (industry specific risk). In this case our pairs will be the returns on each firm over the same time periods, so we use the differences in monthly returns for the two companies. The paired comparisons test is just a test of whether the average difference between monthly returns is significantly different from zero, based on the standard error of the average difference estimated from the sample data.

Remember, the paired comparisons test also requires that the sample data be normally distributed. Although we frequently just want to test the hypothesis that the mean of the differences in the pairs is zero ($\mu_{dz} = 0$), the general form of the test for any hypothesized mean difference, μ_{dz}, is as follows:

H_0: $\mu_d = \mu_{dz}$ versus H_a: $\mu_d \neq \mu_{dz}$

where:
μ_d = mean of the population of paired differences
μ_{dz} = hypothesized mean of paired differences, which is commonly zero

For one-sided tests, the hypotheses are structured as either:

H_0: $\mu_d \leq \mu_{dz}$ versus H_a: $\mu_d > \mu_{dz}$, or H_0: $\mu_d \geq \mu_{dz}$ versus H_a: $\mu_d < \mu_{dz}$

For the paired comparisons test, the *t*-statistic with n − 1 degrees of freedom is computed as:

$$t = \frac{\overline{d} - \mu_{dz}}{s_{\overline{d}}}$$

where:

$\overline{d}$ = sample mean difference $= \dfrac{1}{n}\displaystyle\sum_{i=1}^{n} d_i$

d_i = difference between the *i*th pair of observations

$s_{\overline{d}}$ = standard error of the mean difference $= \dfrac{s_d}{\sqrt{n}}$

s_d = sample standard deviation $= \left(\dfrac{\displaystyle\sum_{i=1}^{n}\left(d_i - \overline{d}\right)^2}{n-1} \right)^{\!\! 1/2}$

n = the number of paired observations

Example: Paired comparisons test

Joe Andrews is examining changes in estimated betas for the common stock of companies in the telecommunications industry before and after deregulation. Andrews believes that the betas may decline because of deregulation since companies are no longer subject to the uncertainties of rate regulation or that they may increase because there is more uncertainty regarding competition in the industry. The sample information he gathered is reported in Figure 11. **Determine** whether there is a change in betas.

Figure 11: Beta Differences After Merger Announcement

Mean of differences in betas (before minus after)	0.23
Sample standard deviation of differences	0.14
Sample size	39

Answer:

Once again, we follow our hypothesis testing procedure.

State the hypothesis. There is reason to believe that the mean differences may be positive or negative, so a two-sided alternative hypothesis is in order here. Thus, the hypotheses are structured as:

$H_0: \mu_d = 0$ versus $H_a: \mu_d \neq 0$

Select the appropriate test statistic. As described above, the test statistic for a paired comparisons test is:

$$t = \frac{\overline{d} - \mu_{dz}}{s_{\overline{d}}}$$

Specify the level of significance. Let's use a 5 percent level of significance.

State the decision rule regarding the hypothesis. There are 39 − 1 = 38 degrees of freedom. Using the *t*-distribution, the two-tailed critical *t*-values for a 5 percent level of significance with df = 38 is ±2.024. As indicated in the table in Figure 12, the critical *t*-value of 2.024 is located at the intersection of the *p* = 0.025 column and the df = 38 row. The one-tailed probability of 0.025 is used because we need 2.5 percent in each tail for 5 percent significance with a two-tailed test.

Figure 12: Partial *t*-Table

df	*One-Tailed Probabilities (p)*		
	p = 0.10	*P = 0.05*	*p = 0.025*
38	1.304	1.686	**2.024**
39	1.304	1.685	2.023
40	1.303	1.684	2.021

Thus, the decision rule becomes:

Reject H_0 if *t*-statistic < −2.024, or *t*-statistic > 2.024

This decision rule is illustrated in Figure 13.

Figure 13: Decision Rule for a Two-Tailed Paired Comparisons Test
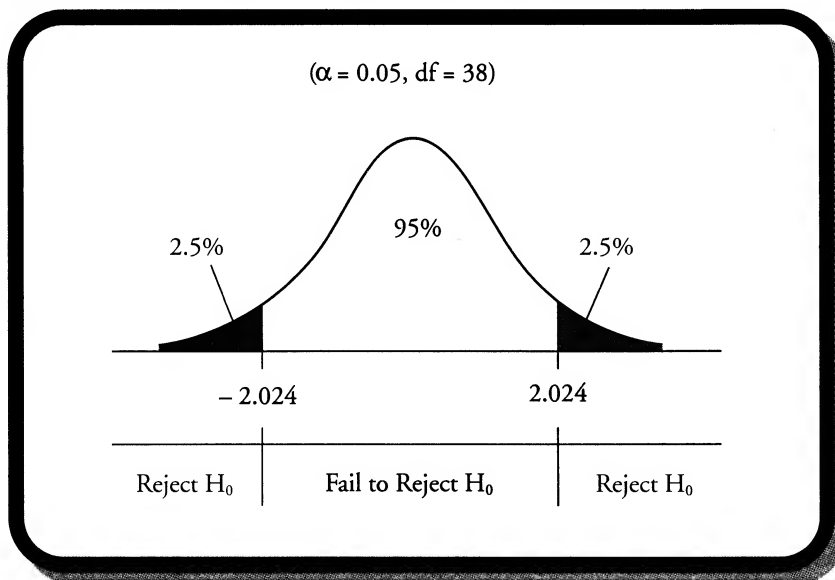
Collect the sample and calculate the sample statistics. Using the sample data provided, the test statistic is computed as follows:

$$t = \frac{\overline{d} - \mu_{dz}}{s_{\overline{d}}} = \frac{0.23}{0.14/\sqrt{39}} = \frac{0.23}{0.022418} = 10.2596$$

Make a decision regarding the hypotheses. The computed test statistic, 10.2596, is greater than the critical *t*-value, 2.024—it falls in the rejection region to the right of 2.024 in Figure 13. Thus we reject the null hypothesis of no difference, concluding that there *is* a statistically significant difference in betas from before to after deregulation.

Make a decision based on the results of the test. We have support for the hypothesis that betas are lower as a result of deregulation, providing support for the proposition that deregulation resulted in decreased risk.

Keep in mind that we have been describing two distinct hypothesis tests: differences between the means of two populations versus the mean of the paired differences from two normal populations. Here are rules for when these tests may be applied:

- The test of the differences in means is used when there are two *independent samples.*
- The test of the mean of the difference is used when the samples are *not independent* but in fact allow *paired comparisons.*

Professor's Note: The LOS here say "Identify the appropriate test statistic and interpret the results..." I can't believe candidates are expected to memorize these formulas (or that you would be a better analyst if you did). The CFA exam is not known for requiring the use of complicated formulas from memory. You should instead focus on the fact that both of these tests involve t-statistics and depend on the degrees of freedom. Also note that when samples are independent you can use the difference in means test and when they are dependent, the statistic is the average difference in (paired) observations divided by the standard error of the average difference.

LOS 12.j: Identify the appropriate test statistic and interpret the results for a hypothesis test concerning the variance of a normally distributed population.

The *chi-square test* is used for **hypothesis tests concerning the variance of a normally distributed population.**

Letting σ^2 represent the true population variance and σ_0^2 represent the hypothesized variance, the hypotheses for a two-tailed test of a single population variance are structured as:

H_0: $\sigma^2 = \sigma_0^2$ versus H_a: $\sigma^2 \neq \sigma_0^2$

The hypotheses for one-tailed tests are structured as:

H_0: $\sigma^2 \leq \sigma_0^2$ versus H_a: $\sigma^2 > \sigma_0^2$, or
H_0: $\sigma^2 \geq \sigma_0^2$ versus H_a: $\sigma^2 < \sigma_0^2$

Hypothesis testing of the population variance requires the use of a chi-square distributed test statistic, denoted χ^2. The chi-square distribution is asymmetrical and approaches the normal distribution in shape as the degrees of freedom increase.

To illustrate the chi-square distribution, consider a two-tailed test with a 5 percent level of significance and 30 degrees of freedom. As displayed in Figure 14, the critical chi-square values are 16.791 and 46.979 for the lower and upper bounds, respectively. These values are obtained from a chi-square table, which is used in the same manner as a *t*-table. A portion of a chi-square table is presented in Figure 15.

Note that the chi-square values in the table in Figure 15 correspond to the probabilities in the right tail of the distribution. As such, the 16.791 in Figure 14 is from the column headed *0.975* because 95% + 2.5% of the probability is to the right of it. The 46.979 is from the column headed *0.025* because only 2.5 percent probability is to the right of it. Similarly, at a 5 percent level of significance with 10 degrees of freedom, Figure 15 shows that the critical chi-square values for a two-tailed test are 3.247 and 20.483.

Figure 14: Decision Rule for a Two-Tailed Chi-square Test
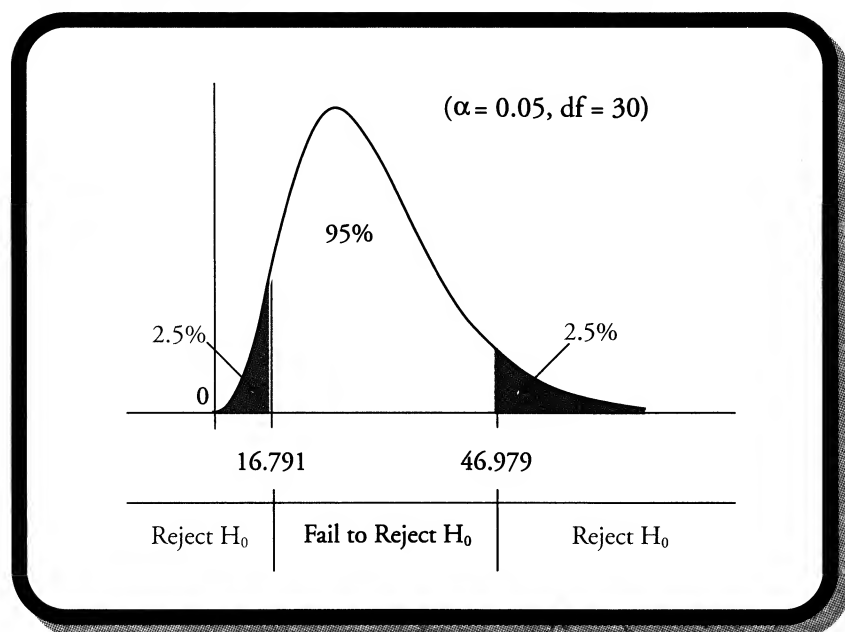
Figure 15: Chi-Square Table

Degrees of Freedom	Probability in Right Tail			
	0.975	0.1	0.05	0.025
9	2.700	14.684	16.919	19.023
10	3.247	15.987	18.307	20.483
11	3.816	17.275	19.675	21.920
30	16.791	40.256	43.773	46.979

The chi-square test statistic, χ^2, with n − 1 degrees of freedom, is computed as:

$$\chi^2_{n-1} = \frac{(n-1)s^2}{\sigma_0^2}$$

where:
n   = sample size
s^2 = sample variance
σ_0^2 = hypothesized value for the population variance.

Similar to other hypothesis tests, the chi-square test compares the test statistic, χ^2_{n-1}, to a critical chi-square value at a given level of significance and n − 1 degrees of freedom. Note that since the chi-square distribution is bounded below by zero, chi-square values cannot be negative.

Example: Chi-square test for a single population variance

Historically, High-Return Equity Fund has advertised that its monthly returns have a standard deviation equal to 4 percent. This was based on estimates from the 1990-1998 period. High-Return wants to verify whether this claim still adequately describes the standard deviation of the fund's returns. High-Return

collected monthly returns for the 24-month period between 1998 and 2000 and measured a standard deviation of monthly returns of 3.8 percent. **Determine** if the more recent standard deviation is different from the advertised standard deviation.

Answer:

State the hypothesis. The null hypothesis is that the variance of monthly returns for the population is $(0.04)^2 = 0.0016$ percent squared. Since High-Return simply wants to test whether the standard deviation has changed, up or down, a two-sided test should be used. The hypothesis test structure takes the form:

$$H_0: \sigma_0^2 = 0.0016 \text{ versus } H_a: \sigma^2 \neq 0.0016$$

Select the appropriate test statistic. The appropriate test statistic for tests of variance using the chi-square distribution is computed as follows:

$$\chi^2 = \frac{(n-1)s^2}{\sigma_0^2}$$

Specify the level of significance. Let's use a 5 percent level of significance, meaning there will be 2.5 percent probability in each tail of the chi-square distribution.

State the decision rule regarding the hypothesis. With a 24-month sample, there are 23 degrees of freedom. Using the table of chi-square values at the back of this book, for 23 degrees of freedom and probabilities of 0.975 and 0.025, we find two critical values, 11.689 and 38.076. Thus, the decision rule is:

Reject H_0 if $\chi^2 < 11.689$, or $\chi^2 > 38.076$

This decision rule is illustrated in Figure 16.

Figure 16: Decision Rule for a Two-Tailed Chi-Square Test of a Single Population Variance
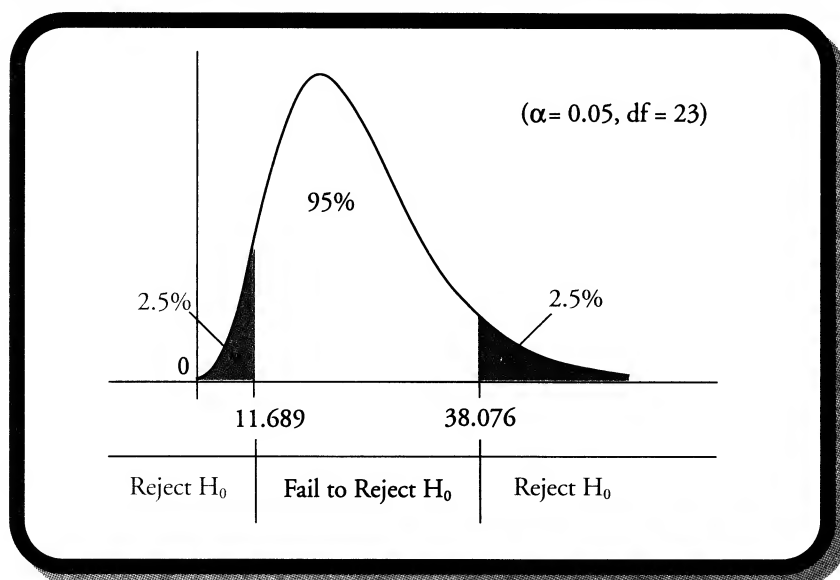
Collect the sample and calculate the sample statistics. Using the information provided, the test statistic is computed as:

$$\chi^2 = \frac{(n-1)s^2}{\sigma_0^2} = \frac{(23)(0.001444)}{0.0016} = \frac{0.033212}{0.0016} = 20.7575$$

Make a decision regarding the hypothesis. Since the computed test statistic, χ^2, falls between the two critical values, we fail to reject the null hypothesis that the variance is equal to 4 percent.

Make a decision based on the results of the test. It can be concluded that the recently measured standard deviation is close enough to the advertised standard deviation that we cannot say that it is different, at a 5 percent level of significance.

LOS 12.k: Identify the appropriate test statistic and interpret the results for a hypothesis test concerning the equality of the variances of two normally distributed populations, based on two independent random samples.

The hypotheses concerned with the equality of the variances of two populations are tested with an F-distributed test statistic. Hypothesis testing using a test statistic that follows an F-distribution is referred to as the F-test. The F-test is used under the assumption that the populations from which samples are drawn are normally distributed and that the samples are independent.

If we let σ_1^2 and σ_2^2 represent the variances of normal population 1 and population 2, respectively, the hypotheses for the two-tailed F-test of differences in the variances can be structured as:

$$H_0: \sigma_1^2 = \sigma_2^2 \text{ versus } H_a: \sigma_1^2 \neq \sigma_2^2$$

and the one-sided test structures can be specified as:

$$H_0: \sigma_1^2 \leq \sigma_2^2 \text{ versus } H_a: \sigma_1^2 > \sigma_2^2, \text{ or } H_0: \sigma_1^2 \geq \sigma_2^2 \text{ versus } H_a: \sigma_1^2 < \sigma_2^2$$

The test statistic for the F-test is the ratio of the sample variances. The F-statistic is computed as:

$$F = \frac{s_1^2}{s_2^2}$$

where:

s_1^2 = variance of the sample of n_1 observations drawn from population 1

s_2^2 = variance of the sample of n_2 observations drawn from population 2

Note that $n_1 - 1$ and $n_2 - 1$ are the degrees of freedom used when computing s_1^2 and s_2^2, respectively.

Professor's Note: Always put the larger variance in the numerator (s_1^2).

An F-distribution is presented in Figure 17. As indicated, the F-distribution is right-skewed and is truncated at zero on the left-hand side. The shape of the F-distribution is determined by *two separate degrees of freedom*, the numerator degrees of freedom, df_1, and the denominator degrees of freedom, df_2. Also shown in Figure 17 is that the *rejection region is in the right-side tail* of the distribution. This will always be the case as long as the F-statistic is computed with the largest sample variance in the numerator. The labeling of 1 and 2 is arbitrary anyway.

Figure 17: *F*-Distribution
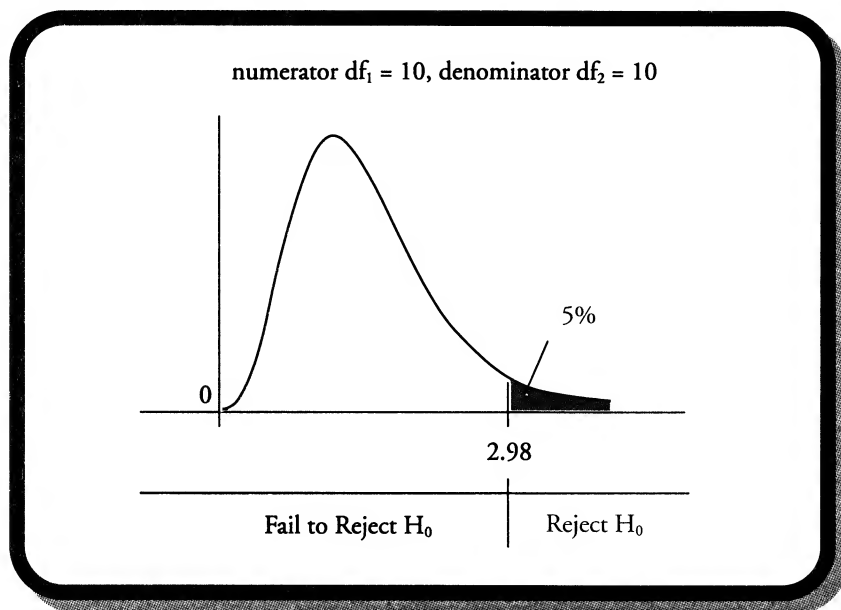
Example: *F*-test for equal variances

Annie Cower is examining the earnings for two different industries. Cower suspects that the earnings of the textile industry are more divergent than those of the paper industry. To confirm this suspicion, Cower has looked at a sample of 31 textile manufacturers and a sample of 41 paper companies. She measured the sample standard deviation of earnings across the textile industry to be $4.30 and that of the paper industry companies to be $3.80. **Determine** if the earnings of the textile industry are more divergent than those of the paper industry.

Answer:

State the hypothesis. In this example, we are concerned with whether the variance of the earnings of the textile industry is greater (more divergent) than the variance of the earnings of the paper industry. As such, the test hypotheses can be appropriately structured as:

$$H_0: \sigma_1^2 \leq \sigma_2^2 \text{ versus } H_a: \sigma_1^2 > \sigma_2^2$$

where:

σ_1^2 = variance of earnings for the textile industry

σ_2^2 = variance of earnings for the paper industry

Note: $\sigma_1^2 > \sigma_2^2$

Select the appropriate test statistic. For tests of difference between variances, the appropriate test statistic is:

$$F = \frac{s_1^2}{s_2^2}$$

Specify the level of significance. Let's conduct our hypothesis test at the 5 percent level of significance.

State the decision rule regarding the hypothesis. Using the sample sizes for the two industries, the critical *F*-value for our test is found to be 1.74. This value is obtained from the table of the *F*-distribution at the 5 percent level of significance with $df_1 = 30$ and $df_2 = 40$. Thus, if the computed *F*-statistic is greater than the critical value of 1.74, the null hypothesis is rejected. The decision rule, illustrated in Figure 18 below, can be stated as:

Reject H_0 if F > 1.74

Figure 18: Decision Rule for *F*-Test
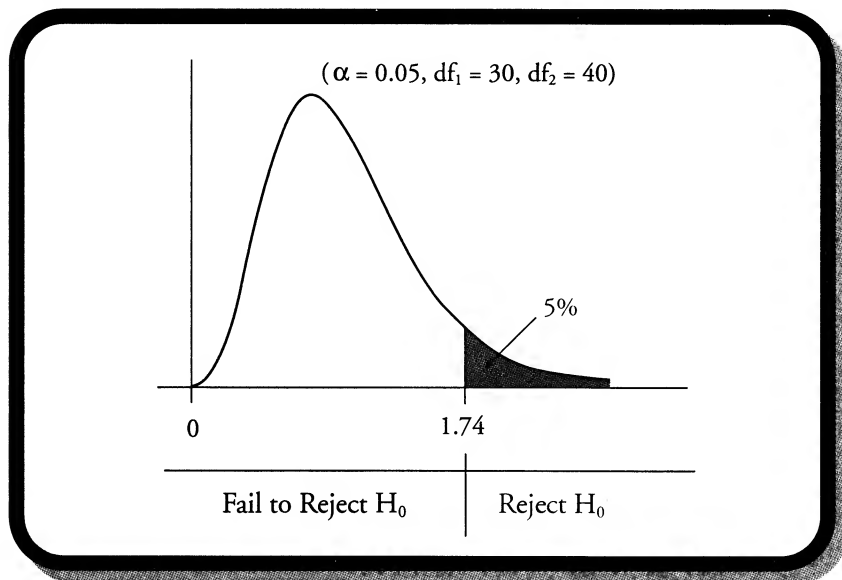
Collect the sample and calculate the sample statistics. Using the information provided, the *F*-statistic can be computed as:

$$F = \frac{s_1^2}{s_2^2} = \frac{\$4.30^2}{\$3.80^2} = \frac{\$18.49}{\$14.44} = 1.2805$$

Professor's Note: Remember to square the standard deviations to get the variances.

Make a decision regarding the hypothesis. Since the calculated *F*-statistic of 1.2805 is less than the critical *F*-statistic of 1.74, we fail to reject the null hypothesis.

Make a decision based on the results of the test. Based on the results of the hypothesis test, Cower should conclude that the earnings variances of the industries are not statistically significantly different from one another at a 5 percent level of significance. More pointedly, the earnings of the textile industry are not more divergent than those of the paper industry.

LOS 12.l: Distinguish between parametric and nonparametric tests and describe the situations in which the use of nonparametric tests may be appropriate.

Parametric tests rely on assumptions regarding the distribution of the population and are specific to population parameters. For example, the *z*-test relies upon a mean and a standard deviation to define the normal distribution. The *z*-test also requires that either the sample is large, relying on the central limit theorem to assure a normal sampling distribution, or that the population is normally distributed.

Nonparametric tests either do not consider a particular population parameter or have few assumptions about the population that is sampled. Nonparametric tests are used when there is concern about quantities other than the parameters of a distribution or when the assumptions of parametric tests can't be supported. They are also used when the data are not suitable for parametric tests (e.g., ranked observations). Nonparametric tests are often used along with parametric tests. In this way, the nonparametric test is a backup in case the assumptions underlying the parametric test do not hold.

KEY CONCEPTS

1. The hypothesis testing process requires a statement of a null and an alternative hypothesis, the selection of the appropriate test statistic, specification of the significance level, a decision rule, the calculation of a sample statistic, a decision regarding the hypotheses based on the test, and a decision based on the test results.

2. The null hypothesis is what the researcher wants to reject. The alternative hypothesis is what the researcher wants to prove, and it is accepted when the null hypothesis is rejected.

3. A two-tailed test results from a two-sided alternative hypothesis (e.g., H_a: $\mu \neq \mu_0$). A one-tailed test results from a one-sided alternative hypothesis (e.g., H_a: $\mu > \mu_0$, or H_a: $\mu < \mu_0$).

4. The decision rule depends on the alternative hypothesis and the distribution of the test statistic.

5. A Type I error is the rejection of the null hypothesis when it is actually true, while a Type II error is the failure to reject the null hypothesis when it is actually false.

6. The significance level can be interpreted as the probability that a test statistic will reject the null hypothesis by chance when it is actually true (i.e., the probability of a Type I error.)

7. The power of a test is the probability of rejecting the null when it is false. The power of a test = 1 – P(Type II error).

8. Hypothesis testing compares a computed test statistic to a critical value at a stated level of significance, which is the decision rule for the test.

9. A hypothesis about a population parameter is rejected when the sample statistic lies outside a confidence interval around the hypothesized value for the chosen level of significance.

10. Statistical decisions are based on hypothesis testing and statistical significance, whereas economic decisions consider the real world relevance of the decision.

11. With unknown population variance, the t-statistic is used for tests of the mean of a normally distributed population: $t_{n-1} = \dfrac{\overline{x} - \mu_0}{s/\sqrt{n}}$. If the population variance is known, the appropriate test statistic is $z = \dfrac{\overline{x} - \mu_0}{\sigma/\sqrt{n}}$ for tests of the mean of a population.

12. For two independent samples from two normally distributed populations, the difference in means can be tested with a *t*-statistic. When the two population variances are assumed to be equal, the denominator is based on the variance of the pooled samples, but when sample variances are assumed to be unequal, the denominator is based on a combination of the two samples variances.

13. A paired comparisons test is concerned with the mean of the differences between the paired observations of two dependent, normally distributed samples. A *t*-statistic is used: $t = \dfrac{\overline{d} - \mu_{dz}}{s_{\overline{d}}}$, where $s_{\overline{d}} = \dfrac{s_d}{\sqrt{n}}$, and $\overline{d}$ is the average difference of the *n* paired observations.

14. The test of a hypothesis about the population variance for a normally distributed population uses a chi-square test statistic: $\chi^2 = \dfrac{(n-1)s^2}{\sigma_0^2}$, where *n* is the sample size, s^2 is the sample variance, and σ_0^2 is the hypothesized value for the population variance. Degrees of freedom is N – 1.

15. The test comparing two variances based on independent samples from two normally distributed populations uses an F-distributed test statistic: $F = \dfrac{s_1^2}{s_2^2}$, where s_1^2 is the variance of the first sample and s_2^2 is the (smaller) variance of the second sample.

16. Parametric tests, like the t-test, F-test, and chi-square tests, make assumptions regarding the distribution of the population from which samples are drawn, while nonparametric tests either do not consider a particular population parameter or have few assumptions about the sampled population.

EXAM FLASHBACKS

Required CFA Institute disclaimer: Due to CFA curriculum changes from year to year, published sample exam questions and guideline answers prior to the current year may not reflect the current curriculum.

Exam Flashback # 1
Source: Question #104 from '90 actual exam.

HYPOTHESIS STATEMENT #1
Null hypothesis H_0: Mean return = 0
Alternative hypothesis H_a: Mean return > 0
HYPOTHESIS STATEMENT #2
Null hypothesis H_0: Mean return = 0
Alternative hypothesis H_a: Mean return ≠ 0

For statements 1 and 2, respectively, the hypothesis tests are:
A. one-tailed and one-tailed.
B. one-tailed and two-tailed.
C. two-tailed and two-tailed.
D. two-tailed and one-tailed.

Exam Flashback # 2
Source: Question #35 from the '99–'03 sample exams.

In hypothesis testing, a Type II error is:
A. rejecting the null hypothesis when it is true.
B. rejecting the null hypothesis when it is false.
C. accepting the null hypothesis when it is true.
D. accepting the null hypothesis when it is false.

CONCEPT CHECKERS: HYPOTHESIS TESTING

1. Which of the following statements about hypothesis testing is **TRUE**?
   A. A Type II error is rejecting the null when it is actually true.
   B. The significance level equals one minus the probability of a Type I error.
   C. If the alternative hypothesis is H_a: $\mu > \mu_0$, the test is a two-tailed test.
   D. A two-tailed test with a significance level of 5 percent has z-critical values of ±1.96.

2. Which of the following statements is **FALSE**?
   A. The power of test = 1 − P(Type II error).
   B. A two-tailed test with a significance level of 5 percent has z-critical values of ±1.96.
   C. If the computed z-statistic = −2 and the critical z-value = −1.96, the null hypothesis is rejected.
   D. The calculated z-statistic for a test of a sample mean when the population variance is known is

   $$z = \frac{\overline{X} - \mu}{\dfrac{\sigma^2}{\sqrt{n}}}.$$

Use the following data to answer Questions 3 through 7.

Austin Roberts wants to determine whether the mean price of houses in the area is greater than $145,000. A random sample of 36 houses in the area has a mean price of $149,750. The population standard deviation is $24,000, and Roberts wants to conduct hypothesis testing at a 1 percent level of significance.

3. The appropriate alternative hypothesis is:
    A. H_a: μ < $145,000.
    B. H_a: μ ± $145,000.
    C. H_a: μ ≥ $145,000.
    D. H_a: μ > $145,000.

4. The value of the calculated test statistic is *closest* to:
    A. z = 0.67.
    B. z = 1.19.
    C. z = 4.00.
    D. z = 8.13.

5. Which of the following *most accurately* describes the appropriate test structure?
    A. F-test.
    B. Two-tailed test.
    C. One-tailed test.
    D. Chi-square test.

6. The critical value of the z-statistic is:
    A. z = ±1.96.
    B. z = +2.33.
    C. z = −2.33.
    D. z = ±2.33.

7. At a 1 percent level of significance, Roberts should:
    A. accept the null hypothesis.
    B. reject the null hypothesis.
    C. fail to reject the null hypothesis.
    D. neither reject nor fail to reject the null hypothesis.

Use the following data to answer Questions 8 through 13.

An analyst is conducting a hypothesis test to determine if the mean time spent on investment research is different from 3 hours per day. The test is performed at the 5 percent level of significance and uses a random sample of 64 portfolio managers, where the mean time spent on research is found to be 2.5 hours. The population standard deviation is 1.5 hours.

8. The appropriate null hypothesis for the described test is:
    A. H_0: μ = 3 hours.
    B. H_0: μ ≠ 3 hours.
    C. H_0: μ ≤ 3 hours.
    D. H_0: μ ≥ 3 hours.

9. This is a:
   A. one-tailed test.
   B. two-tailed test.
   C. chi-square test.
   D. paired comparisons test.

10. The calculated z-statistic is:
    A. −2.13.
    B. −2.67.
    C. +0.33.
    D. +2.67.

11. The critical z-value(s) of the test statistic is (are):
    A. −1.96.
    B. +1.96.
    C. ±1.96.
    D. ±2.58.

12. The 95 percent confidence interval for the population mean is:
    A. $\{1.00 < \mu < 3.50\}$.
    B. $\{0.54 < \mu < 4.46\}$.
    C. $\{2.13 < \mu < 2.87\}$.
    D. $\{-1.96 < \mu < 1.96\}$.

13. Which of the following decisions is the **CORRECT** decision for this study?
    A. Reject the null hypothesis.
    B. Fail to reject the null hypothesis.
    C. The sample size is too small, so increase the sample size.
    D. No decision is possible because the sample standard deviation was not given.

14. A study was conducted to determine whether the standard deviation of monthly maintenance costs of a Pepper III aircraft is $300. A sample of 30 Pepper IIIs had a mean monthly maintenance cost of $3,025 and a standard deviation of $325. Using a 5 percent level of significance, which of the following is the most appropriate conclusion regarding the *difference* between the hypothesized value of the population variance and the sample variance?
    A. The difference is not meaningful.
    B. The population and sample variances are significantly different.
    C. The population and sample variances are not significantly different.
    D. There are no tests that may be used to test variance differences in small samples.

Use the following data to answer Questions 15 through 20.

Two samples were drawn from a normally distributed population. For the first sample, the mean was $50 and the standard deviation was $5. For the second sample, the mean was $55 and the standard deviation was $6. The first sample consists of 25 observations and the second sample consists of 36 observations. (Note: In the questions below, the subscripts "1" and "2" indicate the first and second sample, respectively.)

15. Consider the hypotheses structured as H_0: $\mu_1 = \$48$ versus H_a: $\mu_1 \neq \$48$. At a 1 percent level of significance, the null hypothesis:
    A. cannot be rejected.
    B. should be rejected.
    C. should neither be rejected nor failed to be rejected.
    D. cannot be tested using this sample information provided.

16. Using a 5 percent level of significance and a hypothesis test structure of H_0: $\sigma^2_1 \leq 24$ versus H_a: $\sigma^2_1 > 24$, the null hypothesis:
    A. cannot be rejected.
    B. should be rejected.
    C. should neither be rejected nor failed to be rejected.
    D. cannot be tested using this sample information provided.

17. Consider the hypotheses structured as H_0: $\mu_1 \leq \$48$ versus H_a: $\mu_1 > \$48$. At a 5 percent level of significance, the null hypothesis:
    A. cannot be rejected.
    B. should be rejected.
    C. should neither be rejected nor failed to be rejected.
    D. cannot be tested using the sample information provided.

18. Using a 5 percent level of significance for a test of the null of H_0: $\sigma_1 = \sigma_2$ versus the alternative of H_a: $\sigma_1 \neq \sigma_2$, the null hypothesis:
    A. cannot be rejected.
    B. should be rejected.
    C. should neither be rejected nor failed to be rejected.
    D. cannot be tested using the sample information provided.

19. If the significance level of a test is 0.05 and the probability of a Type II error is 0.15, what is the power of the test?
    A. 0.015.
    B. 0.950.
    C. 0.975.
    D. 0.850.

20. All of the following are true about the F-distribution and chi-square distribution EXCEPT they:
    A. are both asymmetrical.
    B. are both bound by zero on the left.
    C. are both defined by degrees of freedom.
    D. both have means that are less than their standard deviations.

21. The appropriate test statistic for a test of the equality of variances for two normally distributed random variables, based on two independent random samples, is the:
    A. t-test.
    B. F-test.
    C. χ^2 test.
    D. z-test.

22. The appropriate test statistic for a test that the variance of a normally distributed population is equal to 13, is the:
    A. t-test.
    B. F-test.
    C. χ^2 test.
    D. z-test.

23. William Adams wants to test whether the mean monthly returns over the last 5 years are the same for two stocks. If he assumes that the returns distributions are normal and have equal variances, the type of test and test statistic are best described as:
    A. paired comparisons test, *t*-statistic.
    B. paired comparisons test, *F*-statistic.
    C. difference in means test, *t*-statistic.
    D. difference in means test, *F*-statistic.

24. Which of the following assumptions is **NOT** required for the difference in means test based on two samples?
    A. The two samples are independent.
    B. The two populations are normally distributed.
    C. The sample means are approximately normally distributed.
    D. The two populations have equal variances.

25. For a hypothesis test with a probability of a Type II error of 60 percent and a probability of a Type I error of 5 percent, which of the following statements is **TRUE**?
    A. The power of the test is 40% and there is a 5% probability that the test statistic will exceed the critical value(s).
    B. There is a 95% probability that the test statistic will be between the critical values if this is a two-tail test.
    C. The power of the test is 55%, and the confidence level is 95%.
    D. There is a 5% probability that the null hypothesis will be rejected when actually true, and the probability of rejecting the null when it is false is 40%.

ANSWERS – EXAM FLASHBACKS

1. **B**   When the alternative hypothesis, H_a, contains a less-than sign (<) or greater-than sign (>), a one-tailed test is appropriate. When the alternative hypothesis contains a not-equal sign ($\neq$), a two-tailed test is in order. This is the situation for the second hypothesis in the question.

2. **D**   A Type II error is to not reject the null when it is false. A Type I error is to reject the null when it is true.

    Professor's Note: It might help you to just remember one of the error types and know that the other is the opposite. Also note that it is statistically incorrect to say "accept" the null hypothesis, as it can only be supported or rejected.

ANSWERS – CONCEPT CHECKERS: HYPOTHESIS TESTING

1. **D**   Rejecting the null when it is actually true is a Type I error. A Type II error is failing to reject the null hypothesis when it is false. The significance level equals the probability of a Type I error. If the alternative hypothesis is $H_a: \mu > \mu_0$, then the test is a one-tailed test. A two-tailed test would have an alternative hypothesis of $H_a: \mu \neq \mu_0$.

2. **D** $z = \dfrac{\overline{X} - \mu}{\dfrac{\sigma}{\sqrt{n}}}$ (σ^2 is the variance)

3. **D** $H_a: \mu > \$145,000$

4. **B** $z = \dfrac{149,750 - 145,000}{24,000 / \sqrt{36}} = 1.1875$

5. **C**   The alternative hypothesis, $H_a: \mu > \$145,000$, only allows for values greater than the hypothesized value. Thus, this is a one-sided (one-tailed) test.

6. **B**   For a one-tailed z-test at the 1% level of significance, the critical z-value is $z_{0.01} = 2.33$. Since the test is one-tailed on the upper end (i.e., $H_a: \mu > 145,000$), we use a positive z-critical value.

7. **C**   The decision rule is to reject H_0 if z-computed > z-critical. Since 1.1875 < 2.33, Roberts will fail to reject the null.

8. **A** $H_0: \mu = 3$ hours

9. **B**   This is a two-sided (tailed) test. We want to test if the mean "differs from" 3 hours (i.e., $H_a: \mu \neq 3$ hours).

10. **B**   The normally distributed test statistic = $z = \dfrac{(2.5 - 3.0)}{1.5 / \sqrt{64}} = -2.67$.

11. **C**   At $\alpha/2 = 0.025$, the critical z-values are: $\pm z_{\alpha/2} = \pm z_{0.025} = \pm 1.96$.

12. **C**   The 95% confidence interval is $\{2.5 \pm (1.96)(0.1875)\} = \{2.5 \pm 0.3675\} \rightarrow \{2.1325 < \mu < 2.8675\}$.

13. **A**   Decision rule: reject H_0 if $z_{computed} < -1.96$ or if $z_{computed} > +1.96$. Since $-2.67 < -1.96$, reject H_0.

14. **C** The wording of the proposition is a little tricky, but the test structure is $H_0: \sigma^2 = 300^2$ versus $H_a: \sigma^2 \neq 300^2$. The appropriate test is a two-tailed chi-square test. The decision rule is to reject H_0 if the test statistic is outside the range defined by the critical chi-square values at $\alpha/2 = 0.025$ with df = 29. The test statistic is

$$\chi^2 = \frac{(n-1)s^2}{\sigma_0^2} = \frac{(29)(105,625)}{90,000} = 34.035.$$ The critical chi-square values are 16.047 on the left and 45.722 on the right.

Since the χ^2 falls between these two values, we fail to reject the null hypothesis. This means the population standard deviation is not significantly different than \$300.

15. **A** A two-tailed t-test is appropriate. The decision rule is to reject H_0 if the t-statistic is outside the range defined by $\pm t$

at $\alpha = 0.01$ with df = 24. The t-statistic $= t_{24} = \frac{\overline{x} - \mu_0}{s/\sqrt{n}} = \frac{50-48}{5/\sqrt{25}} = 2.0$. $\pm t_{24}$ at $\alpha = 0.01 = \pm 2.797$; therefore, H_0

cannot be rejected.

16. **A** The chi-square test is used to test hypotheses concerning a single population variance. Since this is a one-tailed test, the decision rule is to reject H_0 if $\chi^2 >$ the critical chi-square value at $\alpha = 0.05$ with df = 24.

$$\chi_{n-1}^2 = \frac{(n-1)s^2}{\sigma_0^2} = \frac{(24)(25)}{24} = 25.0.$$ The right-tail critical chi-square value is 36.415. Since $\chi^2 = 25 \leq 36.415$,

H_0 cannot be rejected.

17. **B** A one-tailed t-test is appropriate. The decision rule is to reject H_0 if the computed t-statistic $> t$-critical at $\alpha = 0.05$

with df = 24. The computed value of the t-statistic $= \frac{\overline{x} - \mu_0}{s/\sqrt{n}} = \frac{50-48}{5/\sqrt{25}} = 2.0$, and t-critical $= t_{24} = 1.711$.

Since $t > t$-critical, H_0 should be rejected.

18. **A** The F-test is appropriate to the equality of population variances. The decision rule is to reject H_0 if the computed

test statistic, F, exceeds the critical F-value at $\alpha/2$. For the information provided, $F = s_1^2 / s_2^2 = 36/25 = 1.44$. At a
0.025 level of significance with $d_1 = 35$ and $d_2 = 24$, F-critical = 2.18. Since F < F-critical (1.44 < 2.18), we fail to reject the null hypothesis.

Professor's Note: Many F-tables do not contain numerator df of 35. On the exam, CFA Institute will design problems such that the df are contained directly in the tables that you will be given on the exam. If the tables do not contain the exact df that you need, pick the df that is closest to what you need.

19. **D** The power of a test is 1 – P(Type II error) = 1 – 0.15 = 0.85.

20. **D** There is no consistent relationship between the mean and standard deviation of the chi-square distribution or F-distribution.

21. **B** The F-test is the appropriate test.

22. **C** A test of $\sigma^2 = \sigma_0^2$ is a χ^2 test.

23. **A** Since the observations are likely dependent (both related to market returns), a paired comparisons (mean differences) test is appropriate and is based on a t-statistic.

24. **D** When the variances are assumed to be unequal, we just calculate the denominator (standard error) differently and use both sample variances to calculate the t-statistic. The distribution of sample means from a normally distributed population will be at least approximately normal.

25. **D** A Type I error is rejecting the null hypothesis when it's true. The probability of rejecting a false null is [1 – Prob Type II] = [1 – 0.60] = 40%, which is called the power of the test. A and B are not necessarily true, since the null may be false and the probability of rejection unknown.

CORRELATION AND REGRESSION

EXAM FOCUS

Correlation measures the direction and extent of linear association between two variables. Regression is used to summarize the relationship between a dependent variable and one or more independent variables. In addition to calculating and interpreting correlation coefficients and regression estimates, you should be able to test for the statistical significance of these measures. A *t*-test may be used to assess the statistical significance of an estimated parameter, such as a correlation coefficient. An *F*-test may be used to assess the statistical significance of an estimated equation. You will undoubtedly find regression analysis to be more complex than correlation analysis. Focus first on the basic interpretation of the regression equation, then move on to the statistical assessment of the equation.

LOS 13.a: Define and interpret a scatter plot.

A scatter plot is a collection of points on a graph where each point represents the values of two variables (i.e., an X/Y pair). Suppose that we wish to graphically represent the data for the returns on Stock A and returns on a market index, over the last six months, shown in Figure 1. Figure 2 shows the data graphically with the returns on Stock A shown on the Y-axis and the returns on the market index on the X-axis. Each point of the scatter plot in Figure 2 represents one month of the six in our sample. The rightmost point in the scatter plot is for the month of March, a 2.0 percent return on the market index and a 1.8 percent return on Stock A.

Figure 1: Monthly Returns Data

Month	Return on Stock A	Return on Market Index
Jan	+0.8%	+1.2%
Feb	+0.6%	+0.5%
Mar	+1.8%	+2.0%
Apr	-0.7%	-0.9%
May	+0.3%	+0.2%
June	-0.1%	-0.5%

Figure 2: A Scatter Plot of Returns
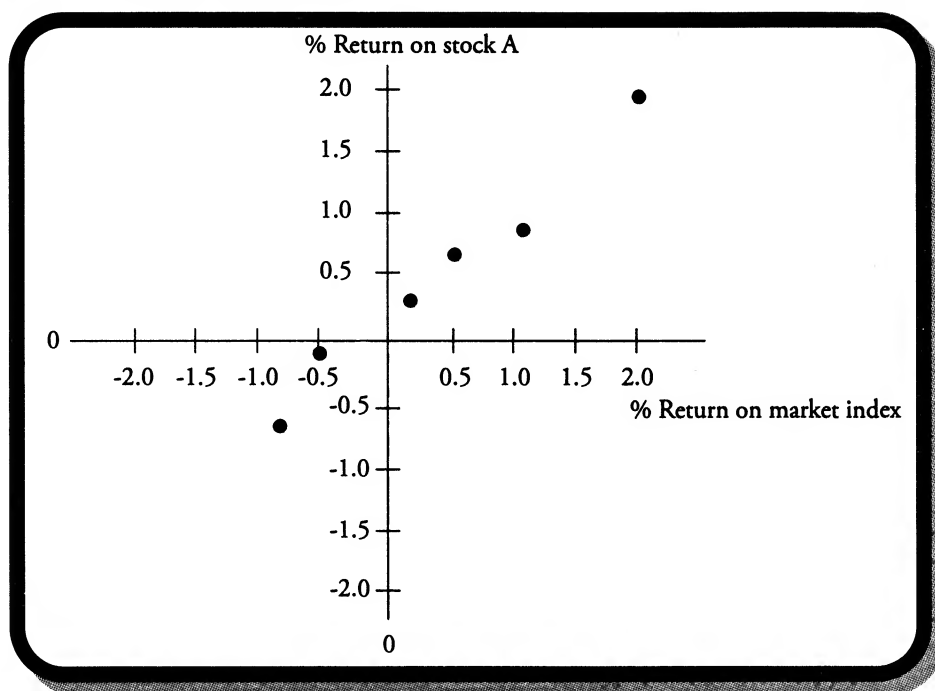
LOS 13.b: Calculate and interpret a sample covariance and a sample correlation coefficient.

The **covariance** between two random variables is a statistical measure of the degree to which the two variables move together. The covariance captures the linear relationship between one variable and another. A positive covariance indicates that the variables tend to move together; a negative covariance indicates that the variables tend to move in opposite directions. The **sample covariance** is calculated as:

$$cov_{xy} = \frac{\sum_{i=1}^{n}(X_i - \overline{X})(Y_i - \overline{Y})}{n-1}$$

where:

n	=	sample size
X_i	=	*i*th observation on variable X
$\overline{X}$	=	mean of the variable X observations
Y_i	=	*i*th observation on variable Y
$\overline{Y}$	=	mean of the variable Y observations

The actual value of the covariance is not very meaningful because its measurement is extremely sensitive to the scale of the two variables. Also, the covariance may range from negative to positive infinity and its computation often results in squared units (e.g., percent squared). For these reasons, we calculate the correlation coefficient, which converts the covariance into a measure that is easier to interpret.

Sample Correlation Coefficient

The **correlation coefficient**, *r*, is a measure of the strength of the linear relationship (correlation) between two variables. The correlation coefficient has no unit of measurement; it is a "pure" measure of the tendency of two variables to move together.

The **sample correlation coefficient** for two variables, X and Y, is calculated as:

$$r_{XY} = \frac{\text{covariance of X and Y}}{(\text{standard deviation of X})(\text{standard deviation of Y})} = \frac{\text{Cov}(X, Y)}{(\sigma_X)(\sigma_Y)}$$

The correlation coefficient is bounded by positive and negative one (i.e., $-1 \le r \le +1$), where a correlation coefficient of $+1$ indicates that there is a one-for-one movement in the variables. On the other hand, if the correlation coefficient is -1, the variables move exactly opposite of each other.

The table in Figure 3 provides the data for two variables, X and Y, and shows the calculation of the correlation between X and Y.

Figure 3: Procedure for Computing Correlation

Obs.	X	Y	$X - \bar{X}$	$(X - \bar{X})^2$	$Y - \bar{Y}$	$(Y - \bar{Y})^2$	$(X - \bar{X})(Y - \bar{Y})$
1	12	50	−1.50	2.25	8.40	70.56	−12.60
2	13	54	−0.50	0.25	12.40	153.76	−6.20
3	10	48	−3.50	12.25	6.40	40.96	−22.40
4	9	47	−4.50	20.25	5.40	29.16	−24.30
5	20	70	6.50	42.25	28.40	806.56	184.60
6	7	20	−6.50	42.25	−21.60	466.56	140.40
7	4	15	−9.50	90.25	−26.60	707.56	252.70
8	22	40	8.50	72.25	−1.60	2.56	−13.60
9	15	35	1.50	2.25	−6.60	43.56	−9.90
10	23	37	9.50	90.25	−4.60	21.16	−43.70
Sum	135	416	0.00	374.50	0.00	2,342.40	445.00
$\bar{X}$ = 135/10 = 13.5				s_X^2 = 374.5/9 = 41.611			
$\bar{Y}$ = 416/10 = 41.6				s_Y^2 = 2,342.4/9 = 260.267			

Using the information in Figure 3, the sample correlation coefficient for variables X and Y may be calculated as:

$$r_X = \frac{\dfrac{445}{9}}{\sqrt{41.611}\sqrt{260.267}} = \frac{49.444}{(6.451)(16.133)} = 0.475$$

The interpretation of the possible correlation values is summarized in Figure 4.

Figure 4: Interpretation of Correlation Coefficients

Correlation Coefficient (r)	Interpretation
$r = +1$	perfect positive correlation
$0 < r < +1$	a positive linear relationship
$r = 0$	no linear relationship
$r = -1$	perfect negative correlation
$-1 < r < 0$	a negative linear relationship

Figure 5 shows several scatter plots for the two random variables X and Y and the corresponding interpretation of correlation. As shown, an upward sweeping scatter plot indicates a positive correlation between the two variables, while a downward sweeping plot implies a negative correlation. Also illustrated in Figure 5 is that as we move from left to right in the rows of scatter plots, the extent of the linear relationship between the two variables deteriorates, and the correlation gets closer to zero.

Figure 5: Interpretations of Correlation
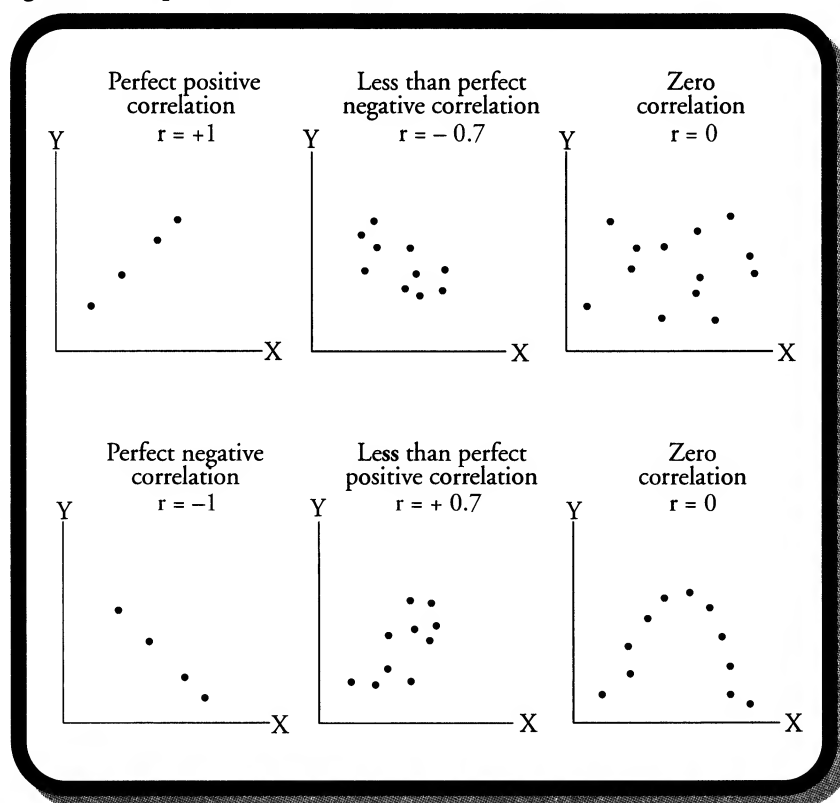
LOS 13.c: Formulate a test of the hypothesis that the population correlation coefficient equals zero and determine whether the hypothesis is rejected at a given level of significance.

As indicated earlier, the closer the correlation coefficient is to plus or minus one, the stronger the correlation. With the exception of these extremes (i.e., r = ±1.0), we cannot really speak of the strength of the relationship indicated by the correlation coefficient without a statistical test of significance. Thus, a hypothesis test is in order.

For our purposes, we want *to test whether the correlation between the population of two variables is equal to zero.* Using the lower case Greek letter rho, ρ, to represent the population parameter, the appropriate null and alternative hypotheses can be structured as a two-tailed test as follows:

H_0: $\rho = 0$ versus H_a: $\rho \neq 0$

Assuming that the two populations are normally distributed, we can use a *t*-test to determine whether the null hypothesis should be rejected. The test statistic is computed using the sample correlation, *r*, with n − 2 degrees of freedom (*df*):

$$t = \frac{r\sqrt{n-2}}{\sqrt{1-r^2}}$$

To make a decision, the calculated *t*-statistic is compared with the critical *t*-value for the appropriate degrees of freedom and level of significance. Bearing in mind that we are conducting a two-tailed test, the decision rule can be stated as:

Reject H_0 if $+t_{critical} < t$, or $t < -t_{critical}$

Example: Test of significance for the correlation coefficient

Using the information from the table in Figure 3, **determine** if the sample correlation is significant at the 5 percent level of significance.

Answer:

For the sample data in Figure 3, n = 10 and r = 0.475. Using this information, the test statistic can be computed as:

$$t = \frac{0.475\sqrt{8}}{\sqrt{1-0.475^2}} = \frac{1.3435}{0.88} = 1.5267$$

The two-tailed critical *t*-values at a 5 percent level of significance with df = 8 (n − 2) are found in the *t*-table to be ±2.306. (Look in the df = 8 row and match that with the p = 0.05 two-tailed probability column, or p = 0.025 one-tailed probability column.)

Since −2.306 ≤ 1.5267 ≤ 2.306 (i.e., $-t_{critical} \leq t \leq +t_{critical}$), the null cannot be rejected. We conclude that the correlation between variables *X* and *Y* is not significantly different than zero at a 5 percent significance level.

Example: Test of significance for the correlation coefficient

Suppose the sample correlation between variables *X* and *Y* is 0.2, and the number of sample observations is 32. Using a 5 percent level of significance, **determine** if this correlation is significantly different from zero.

Answer:

The hypotheses are structured as H_0: $\rho = 0$ versus H_a: $\rho \neq 0$.

The calculated t-statistic is $t = \dfrac{0.2\sqrt{32-2}}{\sqrt{1-0.04}} = \dfrac{0.2\sqrt{30}}{\sqrt{0.96}} = 1.11803$.

The critical t-values at a 5 percent level of significance with 30 degrees of freedom (32 − 2 = 30) are ±2.042.

Since −2.042 < 1.11803 < 2.042 (i.e., $-t_{critical} < t < +t_{critical}$), the null cannot be rejected. We conclude that the correlation between variables X and Y is not significantly different than zero at a 5 percent level of significance.

Example: Test of significance for the correlation coefficient

Determine if the correlation of two random variables is significantly different than zero at a 1 percent level of significance. Assume that a sample correlation has been determined to be 0.80 using a sample with 12 observations.

Answer:

The hypotheses are structured as H_0: $\rho = 0$ versus H_a: $\rho \neq 0$.

The calculated t-statistic is $t = \dfrac{0.80\sqrt{12-2}}{\sqrt{1-0.64}} = \dfrac{0.80\sqrt{10}}{\sqrt{0.36}} = \dfrac{2.529822}{0.6} = 4.21637$.

The critical t-values at a 1 percent level of significance with 10 degrees of freedom (12 − 2) are ±3.169.

Since 4.21637 > 3.169 (i.e., $t > t_{critical}$), the null hypothesis is rejected, and we conclude that there is a significant correlation between the two variables.

WARM-UP: LINEAR REGRESSION

Regression analysis may be used to summarize and explain the nature of the relationship between one variable (a dependent variable) in terms of one or more other variables (independent variables). In this topic review, we will learn how to apply regression techniques to the analysis of the linear relationship between one dependent variable and only one independent variable. This type of application of regression analysis is often referred to as simple linear regression.

The overall purpose of linear regression is to explain the variation in a dependent variable in terms of the variation in the independent variables. Here the term "variation" is interpreted as the degree to which a variable differs from its mean value. Don't confuse variation with variance; they are related but are not the same.

As you progress through the remainder of this review, pay close attention to the following two issues:

- Understanding and interpreting a regression model.
- Using an estimated regression equation to predict a value for a dependent variable.

Professor's Note: Linear Regression is an important topic and is very likely to appear on the exam.

LOS 13.d: Differentiate between the dependent and independent variables in a linear regression and explain the assumptions underlying linear regression.

- The **dependent variable** is the variable whose variation is explained by the other variable(s). The dependent variable is also referred to as the *explained variable*, the *endogenous variable*, or the *predicted variable*.
- The **independent variable** is the variable whose variation is used to explain the variation of the dependent variable. The independent variable is also referred to as the *explanatory variable*, the *exogenous variable*, or the *predicting variable*.

Example: Dependent vs. independent variables

Suppose that you want to predict stock returns with GDP growth. Which variable is the independent variable?

Answer:

Since GDP is going to be used as a predictor of stock returns, stock returns are being *explained* by GDP. Hence, stock returns are the dependent (explained) variable, and GDP is the independent (explanatory) variable.

The Slope and the Intercept Terms in a Regression

The following linear regression model is used to describe the relationship between two variables, X and Y:

$$Y_i = b_0 + b_1 X_i + \varepsilon_i$$

where:
Y_i = ith observation of the dependent variable, Y
X_i = ith observation of the independent variable, X
b_0 = **intercept with the Y-axis**
b_1 = **slope coefficient**
ε_i = the residual for the ith observation (also referred to as the disturbance term or error term)

The linear regression model says that the value of the dependent variable, Y, is equal to the intercept, b_0, plus the product of the slope coefficient, b_1, and the value of the independent variable, X, plus an error term, ε.

Based on the regression model stated above, the regression process estimates an equation for a line through a scatter plot of the data that best explains the observed values for Y in terms of the observed values for X. The linear equation, often called the line of best fit or regression line, takes the following form:

$$\hat{Y}_i = \hat{b}_0 + \hat{b}_1 X_i$$

where:
$\hat{Y}_i$ = the estimated value of Y_i given X_i
$\hat{b}_0$ = the estimated intercept term
$\hat{b}_1$ = the estimated slope coefficient

Professor's Note: The hat "^" above a variable or parameter indicates an estimated value.

The regression line is just one of the many possible lines that can be drawn through the scatter plot of X and Y. In fact, the criteria used to estimate this line forms the very essence of linear regression. The regression line is chosen so that the sum of the squared differences (vertical distances) between the Y-values predicted by the regression equation $(\hat{Y}_i = \hat{b}_0 + \hat{b}_1 X_i)$ and actual Y-values, Y_i, is minimized. The sum of the squared vertical distances between the estimated and actual Y-values is referred to as the sum of the squared errors, or SSE, which is expressed as:

$$SSE = \sum_{i=1}^{n}\left(Y_i - \hat{Y}\right)^2 = \sum_{i=1}^{n}\left(Y_i - \hat{b}_0 - \hat{b}_1 X_i\right)^2 = \sum_{i=1}^{n}\left(\hat{\varepsilon}_i\right)^2$$

Thus, the regression line is the line that minimizes SSE. This explains why simple linear regression is frequently referred to as *ordinary least squares* (OLS) regression, and the values estimated by the estimated regression equation, $\hat{Y}_i$, are called least squares estimates. Figure 6 illustrates the concept behind the OLS regression method.

Figure 6: Least Squares Regression Line
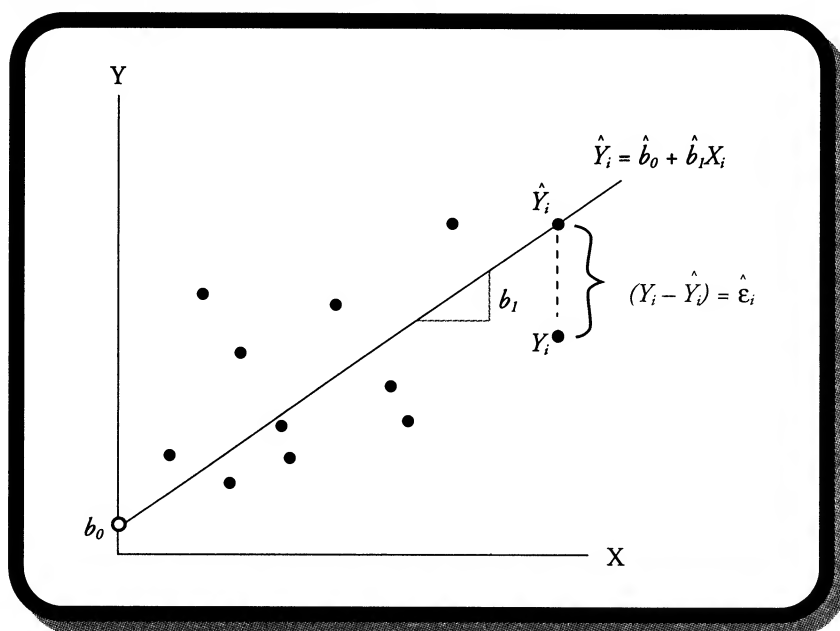
The estimated *slope coefficient* ($\hat{b}_1$) for the regression line describes the change in *Y* for a given change in *X*. It can be positive, negative, or zero, depending on the relationship between the regression variables. The slope term is calculated as:

$$\hat{b}_1 = \frac{cov(X,Y)}{var(X)} = \frac{\sum_{i=1}^{n}(Y_i - \overline{Y})(X_i - \overline{X})}{\sum_{i=1}^{n}(X_i - \overline{X})^2}$$

Professor's Note: For the exam, know that the slope equals covariance divided by variance.

The *intercept* term ($\hat{b}_0$) is the line's intersection with the Y-axis at X = 0. It can be positive, negative, or zero. A property of the least squares method is that the intercept term may be expressed as:

$$\hat{b}_0 = \overline{Y} - \hat{b}_1\overline{X}$$

where:
$\overline{Y}$ = the mean of Y
$\overline{X}$ = the mean of X

The intercept equation highlights the fact that the regression line passes through a point with coordinates equal to the mean of the independent and dependent variables (i.e., the point $\overline{X}, \overline{Y}$).

Professor's Note: It is unlikely that you will be asked to calculate a slope coefficient from raw data on the exam, but it is extremely likely that you will be expected to interpret it. The same goes for the intercept term.

Example: Computing the slope coefficient

Compute the slope coefficient and intercept term for the least squares regression equation using the following information:

$$\sum_{i=1}^{n}\left(Y_i - \overline{Y}\right)\left(X_i - \overline{X}\right) = 445 \qquad \overline{X} = 25$$

$$\sum_{i=1}^{n}\left(X_i - \overline{X}\right)^2 = 374.5 \qquad \overline{Y} = 75$$

Answer:

Hence, the slope coefficient is calculated as $\hat{b}_1 = 445/374.5 = 1.188$.

The intercept term is $\hat{b}_0 = \overline{Y} - \hat{b}_1\overline{X} = 75 - 1.188(25) = 45.3$.

Assumptions Underlying Linear Regression

Linear regression makes a number of assumptions. Fortunately, the validity of the model is fairly insensitive to minor violations of these assumptions. Most of the major assumptions pertain to the regression model's error term, ε, which is commonly called the residual term, or residual. Memorize the following list.

- A *linear relationship* exists between the dependent and independent variables.
- The *independent variable is uncorrelated with the error term.*
- The expected value of the *error term is zero* ($E(\varepsilon_i) = 0$).
- There is a *constant variance* of the error term (ε_i). In other words, the error terms are homoskedastic. (A violation of this is referred to as *heteroskedasticity*.)
- The *error term is independently distributed*; that is, the error term for one observation is not correlated with that of another observation. (A violation of this is referred to as *autocorrelation*.)
- The *error term is normally distributed.*

(See Exam Flashback #1.)

LOS 13.e: Define, calculate, and interpret the standard error of estimate and the coefficient of determination.

The **standard error of the estimate** (SEE) measures the uncertainty about the accuracy of the predicted values of the dependent variable, $\hat{Y}_i = \hat{b}_0 + \hat{b}_1 X_i$. In some regressions, the relationship between the independent and dependent variables is very strong [e.g., the relationship between 10-year Treasury bond (T-bond) yields and mortgage rates]. In other cases, the relationship is much weaker (e.g., the relationship between stock returns and inflation). SEE will be low (relative to total variability) if the relationship is very strong and high if the relationship is weak.

Formally, SEE is the standard deviation of the predicted values for the dependent variable about the regression line. Equivalently, it is the standard deviation of the error terms in the regression. As such, SEE is also referred to as the standard error of the residual, or standard error of the regression, and often specified as s_e.

The SEE is easy to calculate. Recall that regression minimizes the sum of the squared vertical distances between the predicted value and actual value for each observation (i.e., prediction errors). Also recall that the sum of the squared prediction errors, $\sum_{i=1}^{n}\left(Y_i - \hat{Y}_i\right)^2$, is called the sum of squared errors, SSE (not to be confused with SEE).

If the relationship between the variables in the regression is very strong (actual values are close to the line), the prediction errors and the SSE will be small. Thus, as shown in the following equations, the standard error of the estimate (SEE = s_e) is a function of the SSE:

Professor's Note: We are starting to get into some alphabet soup here, so you may want to make a note card to keep it all straight until the terms become familiar to you.

$$SEE = \sqrt{s_e^2} = \sqrt{\frac{SSE}{n-2}}$$

$$=\sqrt{\frac{\sum_{i=1}^{n}\left(Y_i - \hat{b}_0 - \hat{b}_i X_i\right)^2}{n-2}} = \sqrt{\frac{\sum_{i=1}^{n}(Y_i - \hat{Y}_i)^2}{n-2}} = \sqrt{\frac{\sum_{i=1}^{n}\hat{\varepsilon}_i^2}{n-2}}$$

where:

$\sum_{i=1}^{n}\left(Y_i - \hat{Y}_i\right)^2$ = SSE = the sum of squared errors

$\hat{Y}_i = \hat{b}_0 + \hat{b}_i X_i$ = a point on the regression line corresponding to a value of X_i. It is the expected (predicted) value of Y, given the estimated relation between X and Y.

Professor's Note: Before you panic and seek a life in the arts, I think that the only way this could show up on the exam is if they give you SSE (sum of squared errors), in which case the calculation is quite easy: SEE = $s_e = \sqrt{\dfrac{SSE}{n-2}}$.

Similar to the standard deviation for a single variable, SEE measures the degree of variability of the actual Y-values relative to the estimated Y-values. The SEE gauges the "fit" of the regression line. *The smaller the standard error of the estimate, the better the fit.*

Professor's Note: As you will see in the discussion of ANOVA tables, SEE = $\sqrt{MSE}$, where MSE = mean square error. MSE is reported directly in most ANOVA tables.

The Coefficient Of Determination

The **coefficient of determination** (R^2) is formally defined as the percentage of the total variation in the dependent variable explained by the independent variable. For example, an R^2 of 0.63 indicates that the variation of the independent variable explains 63 percent of the variation in the dependent variable.

For simple linear regression (i.e., one independent variable), the coefficient of determination may be computed by simply squaring the correlation coefficient, r. In other words, $R^2 = r^2$ for regression with one independent variable. Unfortunately, this approach is not appropriate when more than one independent variable is used in the regression, as is the case with the multiple regression techniques presented at Level 2.

We now look at a method for measuring the coefficient of determination that may be used for regressions with any number of independent variables. Let's define some terms before we move on.

- Total variation = $\sum_{i=1}^{n}(Y_i - \overline{Y})^2$ = SST. The *total* variation of the dependent variable is the sum of the squared differences between the actual Y-values and $\overline{Y}$, the mean of Y. It is as if the mean, $\overline{Y}$, is the best estimate of the dependent variable, Y, given a value for the independent variable X. To risk stating the obvious, SST stands for sum of the squared total variations.

- Unexplained variation = $\sum_{i=1}^{n}(Y_i - \widehat{Y_i})^2$ = SSE. The unexplained variation is simply the sum of the squared errors we have been discussing. It is the sum of the squared vertical distances between the actual Y-values, Y_i, and the predicted Y-values, $\hat{Y_i}$, on the regression line.

- Explained variation = $\sum_{i=1}^{n}(\widehat{Y_i} - \overline{Y})^2$ = SSR. The explained variation is the sum of the squared distances between the predicted Y-values and the mean of Y. The explained variation is commonly referred to as the sum of the squares regression (SSR).

Thus, total variation = unexplained variation + explained variation, or:

SST = SSE + SSR

Figure 7 illustrates the total variation in the dependent variable.

Figure 7: Components of the Total Variation
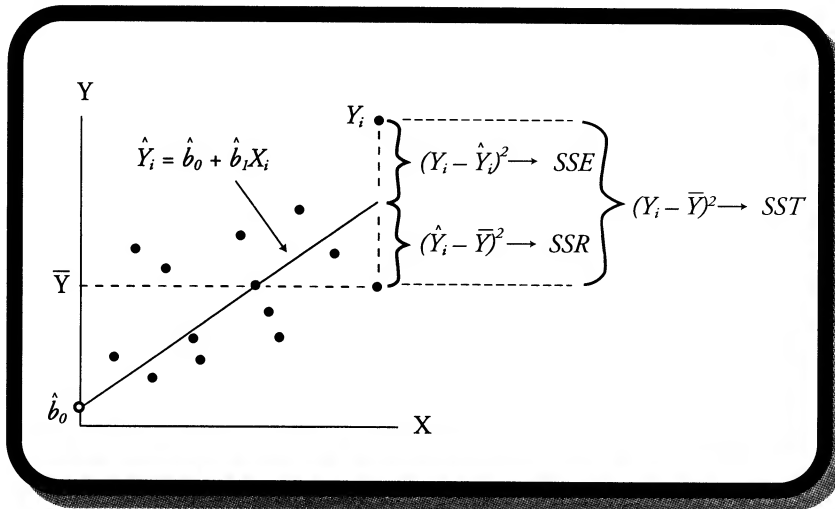
The coefficient of determination can be expressed as:

$$R^2 = \frac{\text{total variation} - \text{unexplained variation}}{\text{total variation}} = \frac{\text{explained variation}}{\text{total variation}}$$

$$R^2 = \frac{\sum_{i=1}^{n}(Y_i - \overline{Y})^2 - \sum_{i=1}^{n}(Y_i - \hat{Y}_i)^2}{\sum_{i=1}^{n}(Y_i - \overline{Y})^2} = \frac{SST - SSE}{SST} = \frac{SSR}{SST}$$

Example: Coefficient of determination

Given the data in Figure 8, **calculate** the coefficient of determination.

Figure 8: Components of the Coefficient of Determination R^2

Observation	X_i	Y_i	SST $\left(Y_i - \overline{Y}\right)^2$	$\hat{Y}_i$	$Y_i - \hat{Y}_i$	SSR $\left(\hat{Y}_i - \overline{Y}\right)^2$	SSE $\left(Y_i - \hat{Y}\right)^2$
1	12	50	70.56	39.82	10.18	3.17	103.68
2	13	54	153.76	41.01	12.99	0.35	168.85
3	10	48	40.96	37.44	10.56	17.30	111.49
4	9	47	29.16	36.25	10.75	28.59	115.50
5	20	70	806.56	49.32	20.68	59.65	427.51
6	7	20	466.56	33.88	−13.88	59.65	192.55
7	4	15	707.56	30.31	−15.31	127.43	234.45
8	22	40	2.56	51.70	−11.70	102.01	136.89
9	15	35	43.56	43.38	−8.38	3.18	70.26
10	23	37	21.16	52.89	−15.89	127.43	252.44
Total	135	416	2,342.40	416.00	0.00	528.76	1,813.63

Answer:

$$R^2 = \frac{SSR}{SST} = \frac{528.76}{2,342.40} = 0.2257 \text{ or } 22.57\%$$

Alternatively, we can compute R^2 as:

$$R^2 = \frac{SST - SSE}{SST} = 1 - \frac{SSE}{SST} = 1 - \frac{1,813.63}{2,342.40} = 0.2257 \text{ or } 22.57\%$$

An R^2 of 0.2257 indicates that the variation of X explains 22.57 percent of the variation in Y.

Professor's Note: For the exam, know that SSE + SSR = SST and that $R^2 = \dfrac{SSR}{SST} = \dfrac{SST - SSE}{SST} = 1 - \left(\dfrac{SSE}{SST}\right)$.

WARM-UP: TESTING THE SIGNIFICANCE OF REGRESSION COEFFICIENTS

Hypothesis tests for the statistical significance of estimated regression coefficients may be conducted using confidence intervals or a *t*-test.

Hypothesis testing for a regression coefficient may use the confidence interval for the coefficient being tested. For instance, a frequently asked question is whether an estimated slope coefficient is statistically different from zero. In other words, the null hypothesis is H_0: $b_1 = 0$, and the alternative hypothesis is H_a: $b_1 \neq 0$. If the confidence interval at the desired level of significance does not include zero, the null is rejected, and the coefficient is said to be statistically different from zero.

The confidence interval for the regression coefficient, b_1, is calculated as:

$$\hat{b}_1 \pm t_c s_{\hat{b}_1}, \text{ or } \left\{ \hat{b}_1 - t_c s_{\hat{b}_1} < b_1 < \hat{b}_1 + t_c s_{\hat{b}_1} \right\}$$

In this expression, t_c is the critical t-value for the selected confidence level with the appropriate number of degrees of freedom, which is equal to the number of sample observations, n, minus the number of regression parameters estimated. Equivalently, df = n – k – 1, where k is the number of independent variables in the regression. In the case of simple linear regression, there are two estimated parameters, $\hat{b}_0$ and $\hat{b}_1$, and one independent variable, so df = n – 2 for $t_{critical}$. The term $s_{\hat{b}_1}$ represents the *standard error of the coefficient* (or the coefficient standard error). It is the square root of the ratio of the variance of the regression to the variance in the independent variable. The standard error of the coefficient is computed as:

$$s_{\hat{b}_1} = \sqrt{\frac{s_e^2}{\sum_{i=1}^{n}(X_i - \overline{X})^2}}$$

Professor's Note: It is unlikely that you will need to know the equation for $s_{\hat{b}_1}$ on the exam. The standard error of the coefficient is included in the output of all statistical software packages and, if needed, should be given to you on the exam.

Although the confidence interval for regression parameters looks slightly different from what we've seen before, it is precisely the same concept. All confidence intervals take the predicted value, then add and subtract the critical test statistic times the variability of the statistic.

Note that as the standard error of the estimate, s_e, rises, the confidence interval widens. This makes sense since s_e measures the variability of the data about the regression line, and the more variable the data, the less confidence there is in the regression model to estimate a coefficient.

LOS 13.f: Calculate a confidence interval for a regression coefficient.

The **confidence interval for b_1** is:

$$\hat{b}_1 \pm t_c s_{\hat{b}_1} = \left\{ \hat{b}_1 - t_c s_{\hat{b}_1} < b_1 < \hat{b}_1 + t_c s_{\hat{b}_1} \right\}$$

Example: Hypothesis test for significance of regression coefficients—confidence intervals

Compute the 95 percent confidence interval for a slope coefficient that is estimated to be 0.78 based on a simple linear regression with 26 observations when the standard error of the estimate is 0.32.

Answer:

The critical t-value with 24 df and a one-tail probability of 0.025 is 2.064.

The 95 percent confidence interval is computed as:

$$0.78 \pm (0.32)(2.064) = 0.78 \pm 0.660 = \{0.120 < b_1 < 1.44\}$$

Since this confidence interval does not include zero, the null hypothesis is rejected.

Note that the t-test and the confidence interval lead to the same conclusion.

LOS 13.g: Formulate a null and an alternative hypothesis about a population value of a regression coefficient, select the appropriate test statistic, and determine whether the null hypothesis is rejected at a given level of significance.

A t-test may also be used to test the hypothesis that the true slope coefficient, b_1, is equal to some hypothesized value. Letting $\hat{b}_1$ be the point estimate for b_1, the appropriate test statistic is:

$$t_b = \frac{\hat{b}_1 - b_1}{s_{\hat{b}_1}}$$

To test whether an independent variable explains the variation in the dependent variable, the **hypothesis that is tested is whether the true slope is zero.** The appropriate test structure for the null and alternative hypotheses is:

$H_0: b_1 = 0$ versus $H_a: b_1 \neq 0$

The decision rule for tests of significance for regression coefficients is:

Reject H_0 if $t > t_{critical}$ or $-t < -t_{critical}$

Rejection of the null means that the slope coefficient is statistically different from the hypothesized value of b_1.

> **Example: Hypothesis test for significance of regression coefficients—t-test**
>
> Suppose a regression line has an estimated slope coefficient of 0.78 with a standard error equal to 0.32. Assuming that the sample had 26 observations, **determine** if the estimated slope coefficient is significantly different from zero at a 5 percent level of significance.
>
> **Answer:**
>
> The calculated test statistic is $t = \dfrac{\hat{b}_1 - b_1}{s_{\hat{b}_1}} = \dfrac{0.78 - 0}{0.32} = 2.4375$.
>
> The critical t-values are ± 2.064 (from the t-table with df $= 26 - 2 = 24$ and one-tail probability of 0.025). Since $t > t_{critical}$ (i.e., $2.4375 > 2.064$), we reject the null hypothesis and conclude that the slope coefficient is different from zero.

LOS 13.h: Interpret a regression coefficient.

As indicated earlier, the **estimated intercept,** $\hat{b}_0$, represents the value of the dependent variable at the point of intersection of the regression line and the axis of the dependent variable (usually the vertical axis). In other words, the intercept is an estimate of the dependent variable when the independent variable takes on a value of zero. We also mentioned earlier that the **estimated slope coefficient,** $\hat{b}_1$, is interpreted as the change in the

dependent variable for a given one-unit change in the independent variable. For example, an estimated slope coefficient of 1 would indicate that the dependent variable will change one unit for every one-unit change in the independent variable.

Keep in mind, however, that any conclusions regarding the importance of an independent variable in explaining a dependent variable require determining the statistical significance of the slope coefficient. Simply looking at the magnitude of the slope coefficient does not address the issue of the importance of the variable. A hypothesis test must be conducted or a confidence interval must be formed to assess the importance of the variable.

LOS 13.j: Calculate and interpret a predicted value and a confidence interval for the predicted value for the dependent variable given an estimated regression model and a value for the independent variable.

Predicted values are values of the dependent variable based on the estimated regression coefficients and a prediction about the values of the independent variables. They are the values that are predicted by the equation for the regression, often called the prediction equation, given an expected value for the independent variable.

For a simple regression, the predicted or forecasted value of Y is:

$$\hat{Y} = \hat{b}_0 + \hat{b}_1 X_p$$

where:
$\hat{Y}$ = predicted value of the dependent variable
X_p = predicted value of the independent variable (input)

Example: Forecasting

Professor's Note: It is almost certain that you will see something like the next two examples on the exam!

Suppose you estimate the following regression equation:

$$\hat{Y} = 1.50 + 2.5 X_1$$

What is the predicted value of Y if the predicted value of the independent variable is 20?

Answer:

The forecasted value for Y is determined as follows:

$$\hat{Y} = 1.50 + 2.50(20) = 1.50 + 50 = 51.5$$

(See Exam Flashbacks #2 and #3.)

Confidence intervals for the predicted value of a dependent variable are calculated in a manner similar to the confidence interval for the regression coefficients. The equation for the confidence interval for a predicted value of Y is:

$$\hat{Y} \pm t_c s_f = \hat{Y} - t_c s_f < Y < \hat{Y} + t_c s_f$$

where:
t_c = critical t-value at the desired level of significance with df = n − 2
s_f = standard error of the forecast

Professor's Note: It's unlikely that you will have to calculate the standard error of the forecast (it will probably be provided if you need to compute a confidence interval on the dependent variable). The following is the formula for the variance of the forecast (s_f^2) :

$$s_f^2 = s_e^2 \left[1 + \frac{1}{n} + \frac{(X - \bar{X})^2}{(n-1)s_x^2} \right]$$

s_e^2 *is the variance of the regression (or SEE²),* s_x^2 *is the variance of the independent variable, and X is the value of the independent variable for which the forecast was made. This equation implies that the standard error of the forecast,* s_f, *is larger than the standard error of the regression,* s_e.

Example: Confidence interval for a predicted value

Suppose an analyst uses least squares regression to explain the variation in Y in terms of X. The regression produced the following output:

$$\hat{b}_0 = 0.01, \ \hat{b}_1 = 1.2, \ SEE = s_e = 0.23, \ s_x = 0.16, \ n = 32, \ \bar{X} = 0.06, \ s_f^2 = 0.05456$$

Calculate the predicted value of the dependent variable given that the forecasted X-value is X = 0.05, and **calculate** a 95 percent prediction interval on the forecasted value of Y.

Answer:

Given the estimated parameters, the prediction equation can be stated as:

$$\hat{Y} = 0.01 + 1.2X$$

For a forecasted value of X = 0.05, the *predicted value of the dependent variable* is:

$$\hat{Y} = 0.01 + 1.2(0.05) = 0.07$$

The critical t-value at the 5 percent level of significance (95 percent confidence level) with df = 32 − 2 = 30 is 2.042.

The standard error of the forecast, s_f, is $\sqrt{0.05456}$ or 0.23358. Hence, the prediction interval at the 95 percent confidence level is:

$$\hat{Y} \pm t_c s_f = \hat{Y} - t_c s_f < Y < \hat{Y} + t_c s_f$$

$$= [0.07 - (2.042)\,(0.23358)] < Y < [0.07 + (2.042)\,(0.23358)], \text{ or } -0.40697 < Y < 0.54697$$

This range can be interpreted such that with 95 percent confidence, the forecast value of Y *given* a forecast value for the independent variable of 0.05 will be between −0.40697 and 0.54697.

LOS 13.i: Describe the use of analysis of variance (ANOVA) in regression analysis and interpret ANOVA results.

Analysis of variance (ANOVA) is a statistical procedure for analyzing the total variability of a data set. The output of the ANOVA procedure is an ANOVA table, which is a summary of the variation in the dependent variable. ANOVA tables are included in the regression output of many statistical software packages. You can think of the ANOVA table as the source of the data for the computation of many of the regression concepts discussed in this topic review. For instance, the data to compute the R^2 and the standard error of the estimate (SEE) come directly from the ANOVA table. A generic ANOVA table for a simple linear regression (one independent variable) is presented in Figure 9.

Figure 9: General Form of Analysis of Variance (ANOVA) Table

Source of Variation	Degrees of Freedom	Sum of Squares	Mean Square
Regression (explained)	1 = k	Sum of squares regression (SSR)	Mean sum of squares (MSR) regression = SSR / k
Error (unexplained)	n − 2	Sum of squared errors (SSE)	Mean square error (MSE) = $\dfrac{SSE}{n-2}$
Total	n − 1	Sum of squares total (SST)	

* Note: k is the number of slope parameters estimated, and n is the number of observations. In general, the regression df = k, and the error df = (n − k − 1). Since we are limited to simple linear regressions in this review (one independent variable), we use k = 1 for the regression *df* and n − 1 − 1 = n − 2 for the error *df*.

There are many ways the data from the ANOVA table can be used in the statistical inference process (most are beyond the scope of the Level 1 CFA curriculum). From the representative ANOVA table shown in Figure 10, we can see how to calculate the R^2 and the SEE of an estimated equation.

Figure 10: Computing R^2 and SEE From an ANOVA Table

Source of Variation	df	Sum of Squares	Mean Square
Regression (explained)	1	5,050	5,050
Error (unexplained)	28	600	21.429
Total	29	5,650	

$$R^2 = \frac{SSR}{SST} = \frac{5,050}{5,650} = 0.8938 = 89.38\% \text{ or } R^2 = \frac{SST - SSE}{SST} = \frac{5,650 - 600}{5,650} = 89.38\%$$

$$SEE = \sqrt{\frac{SSE}{n-2}} = \sqrt{\frac{600}{28}} = \sqrt{21.429} = 4.629$$

LOS 13.k: Discuss the limitations of regression analysis and identify problems with a particular regression analysis or its associated results and any conclusions drawn from them.

Limitations of regression analysis include the following:

- Regression relations change over time. This means that the estimation equation based on data from a specific time period may not be relevant for forecasts or predictions in another time period. This is referred to as non-stationarity.
- If the assumptions of regression analysis are not valid, the interpretation and tests of hypotheses are not valid. For example, if the data is heteroskedastic (non-constant variance of the error terms) or exhibits autocorrelation (error terms are not independent), it is very difficult to use the regression to forecast the dependent variable given information about the independent variables.
- In general, when any of the assumptions underlying linear regression are violated, we cannot rely on the parameter estimates, test statistics, or point and interval forecasts from the regression.

KEY CONCEPTS

1. Covariance, $COV(X,Y)$, measures the linear relationship between two random variables and is calculated as:

$$\frac{\sum_{i=1}^{N}\left(X_i - \overline{X}\right)\left(Y_i - \overline{Y}\right)}{N-1}$$

2. Sample correlation is a measure of the relationship between two variables: $r_{x,y} = \dfrac{COV(X,Y)}{(\sigma_X)(\sigma_Y)}$, which takes on values from −1.0 to +1.0.

3. A t-test is used to determine if a correlation coefficient, r, is statistically significant:

   $t_{n-2} = \dfrac{r\sqrt{n-2}}{\sqrt{1-r^2}}$, significance is supported if the test statistic is less than $-t_{critical}$ or greater than $t_{critical}$ with n − 2 degrees of freedom.

4. The general form of the linear *regression model* is $Y_i = b_0 + b_1 X_i + \varepsilon_i$.
   - Y_i and X_i are the ith observation of the dependent and independent variable, respectively.
   - b_0 = intercept = the value of Y if X is zero.
   - b_1 = slope coefficient = the change in Y for a one-unit change in X.
   - ε_i = residual error for the ith observation.

5. Assumptions made with simple linear regression include:
   - The dependent variable, Y, and independent variable, X, are linearly related.
   - The independent variable is uncorrelated with the error term.
   - The expected value of the error term is zero ($E(\varepsilon_i) = 0$).
   - The variance of the error term is constant for all observations (i.e., $\sigma^2_{\varepsilon 1} = \sigma^2_{\varepsilon 2} = \sigma^2_{\varepsilon 3}$).
   - The error terms are independent.
   - The error term is normally distributed.

6. The regression line, $\hat{Y}_i = \hat{b}_0 + \hat{b}_1 X_i$, is the line through the data that minimizes the squared vertical distances between Y_i and $\hat{Y}_i$ (i.e., minimize $\sum_{i=1}^{n} (Y_i - \hat{Y}_i)^2$, or SSE).

7. The standard error of the estimate in a simple linear regression is calculated as:

$$\sqrt{\frac{SSE}{n-2}}$$

8. The *coefficient of determination*, R^2, is the proportion of the total variation of the dependent variable explained by the regression $R^2 = \frac{SSR}{SST} = \left[1 - \left(\frac{SSE}{SST}\right)\right] = \frac{SST - SSE}{SST}$.

9. A confidence interval for an estimated regression coefficient is calculated as:

$\hat{b} \pm t_c s_{\hat{b}}$, where $s_{\hat{b}}$ is the standard error for the parameter being tested.

10. A *t*-test is used to test whether an estimated regression parameter is statistically significantly different from a hypothesized value.

$$t_b = \frac{\hat{b}_1 - b_1}{s_{\hat{b}_1}}$$

11. A *predicted value* of the dependent variable, $\hat{Y}$, is determined by inserting the predicted value of the independent variable, X_p, in the regression equation and calculating $\hat{Y} = \hat{b}_0 + \hat{b}_1 X_p$.

12. The *confidence interval* for a predicted Y-value is $\{\hat{Y} - (t_{critical})s_f < Y < \hat{Y} + (t_{critical})s_f\}$, where s_f is the standard error of the forecast.

13. An analysis of variance (ANOVA) table is a source of values for the computation of many statistics used in regression analysis.

14. The *limitations* of regression analysis include non-stationarity of the relationships over time (especially when dealing with economic and financial variables) and the possibility of violating the assumptions underlying regression analysis (heteroskedasticity and autocorrelation).

EXAM FLASHBACKS

Required CFA Institute disclaimer: Due to CFA curriculum changes from year to year, published sample exam questions and guideline answers prior to the current year may not reflect the current curriculum.

Exam Flashback # 1
Source: Question #26 from '99, '00 and '02 sample exams.

Least squares regression assumes that the relationship between the dependent and independent variables is:
A. casual.
B. random.
C. straight-line.
D. economically significant.

Exam Flashback # 2
Source: Question #57 from '93 actual exam.

Reg Reshin, a CFA candidate, has recently concluded that weekend beverage sales are linearly related to average hourly wages. The regression has an intercept of 10 liters, a correlation coefficient of 0.55, and an R^2 of 0.30. If the average hourly wage is predicted to be $20, what would be the associated prediction of weekend beverage sales in liters?
A. 11 liters.
B. 16 liters.
C. 21 liters.
D. Cannot be calculated from the information provided.

Exam Flashback # 3
Source: Question #67 from '89 actual exam.

Assume you perform two simple regressions. The first regression analysis has an R-squared of 0.25 and a beta coefficient of 0.90. The second regression analysis has an R-squared of 0.75 and a beta coefficient of –0.20. Which *one* of the following statements is TRUE?
A. Given a one-unit increase in the independent variable of the second analysis, you would expect a 0.20 unit increase in the dependent variable of that analysis.
B. Results of the first analysis are more reliable than the second analysis.
C. Results of the second analysis are more reliable than the first analysis.
D. The beta coefficient of 0.90 in the first analysis indicates that changes in the value of the independent variable explain 90% of changes in the value of the dependent variable.

CONCEPT CHECKERS: CORRELATION AND REGRESSION

Use the following data to answer Questions 1 through 9.

An analyst is given the data in Table 1 for the annual sales for Company XYZ, a maker of toilet tissue, and toilet tissue industry sales. The analyst has been asked to develop a model to aid in the prediction of future sales levels for Company XYZ. Regression estimates and analysis of variance results are presented in Table 2 and Table 3, respectively.

Table 1: Company XYZ Annual Sales

Year	X = Industry Sales ($ millions)	Y = Company Sales ($ millions)
1	$3,000	$750
2	3,200	800
3	3,400	850
4	3,350	825
5	3,500	900
Total	$16,450	$4,125

Table 2: Regression Output

Predictor	Coefficient	Standard Error of the Coefficient
Intercept	–94.88	32.97
Slope (industry sales)	0.2796	0.0363

Table 3: ANOVA Table

Source	Degrees of Freedom (df)	Sum of Squares	Mean Square (SS/df)
Regression	1	11,899.50	11,899.50
Error	3	600.50	200.17
Total	4	12,500.00	

1. Which of the following reports the **CORRECT** value and interpretation of the R^2 for this regression? The R^2 is:
   A. 0.048, indicating that the variability of industry sales explains about 4.8 percent of the variability of company sales.
   B. 0.048, indicating that the variability of company sales explains about 4.8 percent of the variability of industry sales.
   C. 0.952, indicating that the variability of industry sales explains about 95.2 percent of the variability of company sales.
   D. 0.952, indicating that the variability of company sales explains about 95.2 percent of the variability of industry sales.

2. Which of the following is the **CORRECT** value of the correlation coefficient between industry sales and company sales?
    A. 0.0023.
    B. 0.2192.
    C. 0.9062.
    D. 0.9757.

3. Which of the following **CORRECTLY** represents the t-statistic and the statistical significance of the correlation coefficient with 95 percent confidence?

	t-Statistic	Correlation Coefficient
A.	7.71	not significantly different from zero
B.	7.71	significantly different from zero
C.	60.93	not significantly different from zero
D.	60.93	significantly different from zero

4. Which of the following **CORRECTLY** represents the t-statistic and the statistical significance of the intercept with 95 percent confidence?

	t-Statistic	Intercept
A.	−2.878	significantly different from zero
B.	−2.878	not significantly different from zero
C.	−0.347	significantly different from zero
D.	−0.347	not significantly different from zero

5. Which of the following **CORRECTLY** represents the t-statistic and the statistical significance of the slope coefficient with 95 percent confidence?

	t-Statistic	Slope
A.	0.130	significantly different from zero
B.	0.130	not significantly different from zero
C.	7.702	significantly different from zero
D.	7.702	not significantly different from zero

6. Which of the following **CORRECTLY** describes the economic significance and interpretation of the slope coefficient? The slope coefficient is:
    A. economically significant, indicating that for each $1 increase in company sales, industry sales are expected to increase by $0.2796.
    B. economically significant, indicating that for each $1 increase in industry sales, company sales are expected to increase by $0.2796.
    C. not economically significant because it suggests that for each $1 increase in company sales, industry sales are expected to increase by $0.2796.
    D. not economically significant because it suggests that for each $1 increase in industry sales, company sales are expected to increase by $0.2796.

7. Which of the following is **NOT** an assumption of simple linear regression analysis?
    A. The residuals are normally distributed.
    B. There is a constant variance of the disturbance term.
    C. The independent variable is uncorrelated with the residuals.
    D. The dependent variable is uncorrelated with the residuals.

8. Based on the estimated regression equation, what level of sales for Company XYZ is forecast if the forecast for industry sales is $3,600?
   A. $911.68.
   B. $1,006.56.
   C. $1,101.44.
   D. $3,505.40.

9. Disregarding the potential problems associated with the small number of observations used to fit the regression line, which of the following *best* describes the overall reliability of the regression equation? The regression:
   A. appears to lack reliability because the slope coefficient is negative, the intercept is positive, and the standard error of the regression model is relatively high.
   B. appears to be highly reliable because the slope coefficient is positive, the intercept is negative, and the standard error of the regression model is relatively low.
   C. appears to lack reliability because the R^2 is low, the *t*-statistic for the slope coefficient is not significant at the 95 percent level, and the standard error of the regression model is relatively high.
   D. model appears to be highly reliable because the R^2 is high, the *t*-statistic for the slope coefficient is significant at the 95 percent level, and the standard error of the regression model is relatively low.

Use the following information for Questions 10 and 11.

A study was conducted by the British Department of Transportation to estimate urban travel time between locations in London, England. Data was collected for motorcycles and passenger cars. Simple linear regression was conducted using data sets for both types of vehicles, where Y = urban travel time in minutes and X = distance between locations in kilometers. The following results were obtained:

Regression Results for Travel Times Between Distances in London			
Passenger cars:	$\hat{Y} = 1.85 + 3.86X$	$R^2 = 0.758$	*p*-value < 0.01
Motorcycles:	$\hat{Y} = 2.50 + 1.93X$	$R^2 = 0.676$	*p*-value < 0.05

10. Which of the following is the *most accurate* estimate of the mean urban travel time for all motorcycles traveling a distance of 3 kilometers on London's streets?
    A. 5.71 minutes.
    B. 8.29 minutes.
    C. 17.13 minutes.
    D. 21.15 minutes

11. Predict the urban travel time for a particular passenger car traveling a distance of 5 kilometers on London's streets.
    A. 12.15 minutes.
    B. 21.15 minutes.
    C. 22.15 minutes.
    D. 28.55 minutes.

COMPREHENSIVE PROBLEMS: CORRELATION AND REGRESSION

1. Use the following data to:
    A. calculate the mean, variance, and standard deviation of X and Y.
    B. calculate the sample covariance and correlation coefficient for X and Y.
    C. calculate the intercept and slope coefficient for a regression of Y on X.
    D. calculate the R^2 for the regression.
    E. calculate the t-stat for the correlation coefficient.
    F. interpret the R^2 and t-stat calculated in parts D and E.

Obs.	X	Y
1	0.4	20
2	0.3	23
3	0.42	18
4	0.34	31
5	0.35	33
6	0.19	19

2. The following output was produced from a regression:

OBS = 6

ANOVA

	df	SS	MS
Regression	1	163.0435	163.0435
Residual	4	44.95652	11.23913
Total	5	208	

	Coeff.	Std. Error
Intercept	–4.26087	7.545107
X Variable	3.26087	0.856146

A. What is the R^2 for the regression, and how would you interpret it? What is the correlation between X and Y?
B. Evaluate the significance of the coefficients from the regression.
C. If the value of the independent variables is 9 next period, what is the estimated value of the dependent variable?
D. What is the standard error of the estimate?

ANSWERS – EXAM FLASHBACKS

1. **C** Linear regression assumes that a *linear relationship* exists between the dependent and independent variables. A standard·least squares regression equation is Y = α + bX, which is the equation of a straight line with intercept α and slope *b*.

2. **D** The general form of the least squares regression equation is Y = α + bX, which is the equation of a straight line with intercept α and slope *b*. We are given α = 10, X = 20, correlation = 0.55, and R^2 = 0.30. We are not given beta, however, so it is not possible to solve the equation.

3. **C** The R^2 indicates the percentage of the variation in the dependent variable that is explained by the variation in the independent variable. Since the second regression's R^2 of 0.75 is greater than the first regression's R^2 of 0.25, it can be said that the second regression is more reliable in explaining the dependent variable in terms of the independent variable.

ANSWERS – CONCEPT CHECKERS: CORRELATION AND REGRESSION

1. **C** The R^2 is computed as:

    $$R^2 = \frac{SS_{Regression}}{SS_{Total}} = \frac{11,899.50}{12,500} = 0.952$$

    The interpretation of this R^2 is that 95.2% of the variation in Company XYZ's sales is explained by the variation in tissue industry sales. Answer D is incorrect because it is the independent variable (industry sales) that explains the variation in the dependent variable (company sales).

2. **D** The correlation coefficient, *r*, is the square root of the R^2 in a simple linear regression, so *r* is $\sqrt{0.952} = 0.9757$. (*Note:* If the slope had been negative, we would multiply the square root of R^2 by −1.)

3. **B** The test of significance for the correlation coefficient is evaluated using the following *t*-statistic:

    $$t = \frac{r\sqrt{n-2}}{\sqrt{1-r^2}} = \frac{0.9757\sqrt{3}}{\sqrt{1-0.952}} = \frac{1.69}{0.219} = 7.72$$

    From the *t*-table, we find that with df = 3 and 95% significance, the two-tailed critical *t* values are ±3.182 (recall that for the *t*-test the degrees of freedom = n − 2). Since the computed *t* is greater than +3.182, the correlation coefficient is significantly different from zero.

4. **B** The *t*-statistic is computed as follows:

    $$t = \frac{\hat{b}_1 - b_1}{s_{\hat{b}_1}} = \frac{-94.88}{32.97} = -2.878$$

    From the *t*-table, we know that the two-tailed critical *t*-values at 95% significance and df = 3 are ±3.182. Since the computed *t* for the intercept is between the critical values of *t*, the intercept is not considered statistically significantly different from zero.

5. C   The t-statistic is computed as $t = \dfrac{\hat{b}_1 - b_1}{s_{\hat{b}_1}} = \dfrac{0.2796}{0.0363} = 7.70$.

Once again, we already know that the critical t-values at 95% significance with df = 3 are ±3.182. Since the computed t for the slope coefficient exceeds +3.182, the slope coefficient is significantly different from zero.

6. B   The interpretation of the slope is that for each $1 increase in industry sales, Company XYZ sales are expected to rise by $0.2796. Hence, an increase in industry sales has an economically significant impact on the sales of Company XYZ.

7. D   The model does not assume that the dependent variable is uncorrelated with the residuals.

8. A   The regression equation is defined as company sales = −94.88 + 0.2796(industry sales).

If industry sales are $3,600, our forecast is company sales = −94.88 + 0.2796($3,600) = $911.68 (don't overlook the minus sign of the intercept).

9. D   This regression appears to be a highly reliable forecasting tool. The R^2 is extremely high, indicating good model fit to the data; the t-statistic is significant at the 95% level, and the confidence interval is relatively narrow due to the relatively low standard error. However, one must be cautious whenever such a low number of observations is used in a regression analysis. Confidence in the model would be increased if more data points were included.

10. B   Motorcycles: $\hat{Y}_i = 2.50 + 1.93X = 2.50 + 1.93(3) = 8.29$ minutes.

11. B   Passenger cars: $\hat{Y}_i = 1.85 + 3.86X = 1.85 + 3.86(5) = 21.15$ minutes.

COMPREHENSIVE PROBLEMS: CORRELATION AND REGRESSION

1. A $\dfrac{\sum X_i}{6} = \dfrac{2}{6} = 0.33 = \bar{X}$

$\dfrac{\sum (X_i - \bar{X})^2}{N-1} = \dfrac{0.033933}{5} = 0.006787 = \sigma_x^2$

$\sigma_x = 0.08238$

$\dfrac{\sum Y_i}{6} = \dfrac{144}{6} = 24 = \bar{Y}$

$\dfrac{\sum (Y_i - \bar{Y})^2}{N-1} = \dfrac{208}{5} = 41.6 = \sigma_y^2$

$\sigma_y = 6.4498$

    B $\dfrac{\sum (X_i - \bar{X})(Y_i - \bar{Y})}{N-1} = \dfrac{0.16}{5} = 0.032 = COV_{x,y}$

$CORR_{x,y} = \dfrac{COV_{x,y}}{\sigma_x \sigma_y} = \dfrac{0.032}{6.4498(0.08238)} = 0.0602$

C $\quad \hat{B}_1 = \dfrac{COV_{x,y}}{VAR_x} = \dfrac{0.032}{0.006787} = 4.715$

The intercept, $\hat{B}_0$, $= \overline{Y} - \hat{B}_1 \overline{X} = 24 - 4.715(0.333) = 22.430$

D In a simple regression, R^2 is just $(CORR_{x,y})^2 = 0.0602^2 = 0.003624$.

E t-stat for test of significance of $CORR_{x,y}$ is $\dfrac{CORR_{x,y}\sqrt{N-2}}{\sqrt{1 - CORR_{x,y}{}^2}} = \dfrac{0.0602\sqrt{6-2}}{\sqrt{1 - 0.0602^2}} = 0.1206$.

F The R^2 is low, and the independent variable only explains approximately 3.6% of the variation in the dependent variable. The t-stat of 0.1206 is far less than the critical value at the 5% level of significance (df = 4 prob. = 0.025) of 2.776. We cannot reject the hypothesis that $B_1 = 0$ (i.e., there is no linear relationship between X and Y).

2. A $R^2 = \dfrac{SSR}{SST} = \dfrac{163.0435}{208} = 0.783863$

Approximately 78% of the variation in Y is explained by variation in X. The correlation$_{x,y}$ is the square root of $R^2 = \sqrt{0.783863} = 0.88536$.

B The t-stats for the estimated regression coefficients are $\dfrac{-4.26087}{7.545107} = -0.56472$ for the intercept and

$\dfrac{3.26087}{0.856146} = 3.80878$ for the slope coefficient.

The critical value at a 95% level of significance and 6 – 2 = 4 df is 2.776. The intercept is not statistically significant; we can't reject the hypothesis that it is zero. The slope coefficient is highly significant.

C The estimated regression equation is $-4.26 + 3.26X_i = Y_i$, so with a value of 9 for X_i, we have $-4.26 + 3.26(9) = 25.08$.

D The standard error of the estimate, SEE, is the square root of MSE, which is given in the ANOVA table as 11.23913: $\sqrt{11.23913} = 3.35248$. Note that the MSE is $\dfrac{SSE}{N-2} = \dfrac{44.95652}{4} = 11.23913$.

FORMULAS

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nominal risk-free rate = real risk-free rate + expected inflation rate

required interest rate on a security
$\quad$ = nominal risk-free rate
$\quad$ + default risk premium
$\quad$ + liquidity premium
$\quad$ + maturity risk premium

EAR = $(1 + \text{periodic rate})^m - 1$

continuous compounding: $e^r - 1 = \text{EAR}$

$$PV_{perpetuity} = \frac{PMT}{I/Y}$$

$$FV = PV(1 + I/Y)^N$$

$$NPV = \sum_{t=0}^{N} \frac{CF_t}{(1+r)^t}$$

general formula for the IRR: $0 = CF_0 + \dfrac{CF_1}{1+IRR} + \dfrac{CF_2}{(1+IRR)^2} + \cdots + \dfrac{CF_N}{(1+IRR)^N}$

bank discount yield: $r_{BD} = \dfrac{D}{F} \times \dfrac{360}{t}$

$$HPY = \frac{P_1 - P_0 + D_1}{P_0} = \frac{P_1 + D_1}{P_0} - 1$$

EAY = $(1 + HPY)^{365/t} - 1$

money market yield: $r_{MM} = \dfrac{360 \times r_{BD}}{360 - (t \times r_{BD})}$

position of the observation at a given percentile, y: $L_y = (n+1)\dfrac{y}{100}$

population mean: $\mu = \dfrac{\sum_{i=1}^{N} X_i}{N}$

sample mean: $\overline{X} = \dfrac{\sum\limits_{i=1}^{n} X_i}{n}$

geometric mean return (R_G): $1 + R_G = \sqrt[n]{(1+R_1) \times (1+R_2) \times \ldots \times (1+R_n)}$

range = maximum value − minimum value

semivariance $= \dfrac{\sum\limits_{\text{All } X_i < \overline{X}} \left(X_i - \overline{X}\right)^2}{\left(\text{\# of Xs less than } \overline{X}\right) - 1}$

coefficient of variation: $CV = \dfrac{s_x}{\overline{X}} = \dfrac{\text{standard deviation of x}}{\text{average value of x}}$

Sharpe ratio $= \dfrac{\overline{r_p} - \overline{r_f}}{\sigma_p}$

excess kurtosis = sample kurtosis − 3

weighted mean: $\overline{X}_W = \sum\limits_{i=1}^{n} w_i X_i$

harmonic mean: $\overline{X}_H = \dfrac{N}{\sum\limits_{i=1}^{N} \dfrac{1}{x_i}}$

$MAD = \dfrac{\sum\limits_{i=1}^{n} \left|X_i - \overline{X}\right|}{n}$

population variance $= \sigma^2 = \dfrac{\sum\limits_{i=1}^{N}(X_i - \mu)^2}{N}$, where μ = population mean and N = number of possible outcomes

sample variance $= s^2 = \dfrac{\sum\limits_{i=1}^{n}(X_i - \overline{X})^2}{n-1}$, where $\overline{X}$ = sample mean and n = sample size

joint probability: $P(AB) = P(A \mid B) \times P(B)$

$P(A \text{ or } B) = P(A) + P(B) - P(AB)$

unconditional probability: $P(R) = P(R \mid S_1) \times P(S_1) + P(R \mid S_2) \times P(S_2) + \ldots + P(R \mid S_N) \times P(S_N)$

expected value: $E(X) = \Sigma P(x_i)x_i = P(x_1)x_1 + P(x_2)x_2 + \ldots + P(x_n)x_n$

$$\text{Corr}\left(R_i, R_j\right) = \frac{\text{Cov}\left(R_i, R_j\right)}{\sigma\left(R_i\right)\sigma\left(R_j\right)}$$

$$w_i = \frac{\text{market value of investment in asset i}}{\text{market value of the portfolio}}$$

portfolio expected return: $E\left(R_p\right) = \sum_{i=1}^{N} w_i E\left(R_i\right) = w_1 E(R_1) + w_2 E(R_2) + \ldots + w_n E(R_n)$

portfolio variance: $\text{Var}\left(R_p\right) = \sum_{i=1}^{N}\sum_{j=1}^{N} w_i w_j \text{Cov}\left(R_i, R_j\right)$

Bayes' formula: $\text{updated probability} = \frac{\text{probability of new information for a given event}}{\text{unconditional probability of new information}} \times \text{prior probability of event}$

$$_n C_r = \frac{n!}{(n-r)!r!}$$

$$_n P_r = \frac{n!}{(n-r)!}$$

binomial probability: $p\left(x\right) = \frac{n!}{(n-x)!x!} p^x \left(1-p\right)^{n-x}$

for a binomial random variable:

    expected value of X = E(X) = np

    variance of X = Var(X) = np(1 − p)

90 percent confidence interval for X is $\overline{X} - 1.65s$ to $\overline{X} + 1.65s$

95 percent confidence interval for X is $\overline{X} - 1.96s$ to $\overline{X} + 1.96s$

99 percent confidence interval for X is $\overline{X} - 2.58s$ to $\overline{X} + 2.58s$

$$z = \frac{\text{observation} - \text{population mean}}{\text{standard deviation}} = \frac{x - \mu}{\sigma}$$

$$\text{SFRatio} = \frac{\left[E\left(R_p\right) - R_L\right]}{\sigma_p}$$

continuously compounded rate of return: $\ln\left(\frac{S_1}{S_0}\right) = \ln\left(1 + \text{HPR}\right)$

for a uniform distribution: $P(x_1 \leq X \leq x_2) = \dfrac{(x_2 - x_1)}{(b - a)}$

sampling error of the mean = sample mean − population mean = $\overline{x} - \mu$

standard error of the sample mean: $\sigma_{\overline{x}} = \dfrac{\sigma}{\sqrt{n}}$

standard deviation of the sample mean: $s_{\overline{x}} = \dfrac{s}{\sqrt{n}}$

confidence interval: point estimate ± (reliability factor × standard error)

confidence interval for the population mean: $\overline{x} \pm z_{\alpha/2} \dfrac{\sigma}{\sqrt{n}}$

Type I error: the rejection of the null hypothesis when it is actually true

Type II error: the failure to reject the null hypothesis when it is actually false

test for a population mean: z-statistic $= \dfrac{\overline{x} - \mu_0}{\sigma / \sqrt{n}}$

to test equality of variances: $F = \dfrac{s_1^2}{s_2^2}$

test of r (correlation) = 0: $t = \dfrac{r\sqrt{n-2}}{\sqrt{1 - r^2}}$

linear regression model: $Y_i = b_0 + b_1 X_i + \varepsilon_i$

regression line: $\hat{Y}_i = \hat{b}_0 + \hat{b}_1 X_i$

$SSE = \sum\limits_{i=1}^{n} \left(\hat{\varepsilon}_i \right)^2$

$\hat{b}_1 = \dfrac{\text{cov}(X,Y)}{\text{var}(X)}$

intercept term: $\hat{b}_0 = \overline{Y} - \hat{b}_1 \overline{X}$

$SEE = \sqrt{s_e^2} = \sqrt{\dfrac{SSE}{n-2}}$

total variation $= \sum\limits_{i=1}^{n} (Y_i - \overline{Y})^2 = SST$

unexplained variation = $\sum_{i=1}^{n}(Y_i - \widehat{Y}_i)^2$ = SSE

explained variation = $\sum_{i=1}^{n}(\widehat{Y}_i - \overline{Y})^2$ = SSR

$R^2 = \dfrac{\text{total variation} - \text{unexplained variation}}{\text{total variation}} = \dfrac{\text{explained variation}}{\text{total variation}}$

test statistic for slope = b_1: $t_b = \dfrac{\hat{b}_1 - b_1}{s_{\hat{b}_1}}$

predicted or forecasted value of Y: $\widehat{Y} = \hat{b}_0 + \hat{b}_1 X_p$

APPENDIX A:
AREAS UNDER THE NORMAL CURVE

Most of the examples in this book have used one version of the z-table to find the area under the normal curve. This table provides the cumulative probabilities (or the area under the entire curve to the left of the z-value).

Probability Example

Assume that the annual earnings per share (EPS) for a large sample of firms is normally distributed with a mean of $5.00 and a standard deviation of $1.50. What is the approximate probability of an observed EPS value falling between $3.00 and $7.25?

If *EPS* = x = $7.25, then $z = (x - \mu)/\sigma = (\$7.25 - \$5.00)/\$1.50 = +1.50$

If *EPS* = x = $3.00, then $z = (x - \mu)/\sigma = (\$3.00 - \$5.00)/\$1.50 = -1.33$

Solving Using The Cumulative Z-Table

For z-value of 1.50: Use the row headed 1.5 and the column headed 0 to find the value 0.9332. This represents the area under the curve to the left of the critical value 1.50.

For z-value of –1.33: Use the row headed 1.3 and the column headed 3 to find the value 0.9082. This represents the area under the curve to the left of the critical value +1.33. The area to the left of –1.33 is 1 – 0.9082 = 0.0918.

The area between these critical values is 0.9332 – 0.0918 = 0.8414, or 84.14%.

Hypothesis Testing – One-Tailed Test Example

A sample of a stock's returns on 36 non-consecutive days results in a mean return of 2.0 percent. Assume the population standard deviation is 20.0 percent. Can we say with 95 percent confidence that the mean return is greater than zero percent?

H_0: $\mu \leq 0.0\%$, H_A: $\mu > 0.0\%$. The test statistic = z-statistic = $\dfrac{\bar{x} - \mu_0}{\sigma/\sqrt{n}}$ = (2.0 – 0.0) / (20.0 / 6) = 0.60.

The significance level = 1.0 – 0.95 = 0.05, or 5%. Since we are interested in a return greater than 0.0 percent, this is a one-tailed test.

Using The Cumulative Z-Table

Since this is a one-tailed test with an alpha of 0.05, we need to find the value 0.95 in the cumulative z-table. The closest value is 0.9505, with a corresponding critical z-value of 1.65. Since the test statistic is less than the critical value, we fail to reject H_0.

Hypothesis Testing – Two-Tailed Test Example

Using the same assumptions as before, suppose that the analyst now wants to determine if he can say with 99% confidence that the stock's return is not equal to 0.0 percent.

H_0: μ = 0.0%, H_A: $\mu \neq$ 0.0%. The test statistic (z-value) = (2.0 – 0.0) / (20.0 / 6) = 0.60. The significance level = 1.0 – 0.99 = 0.01, or 1%. Since we are interested in whether or not the stock return is nonzero, this is a two-tailed test.

Using The Cumulative Z-Table

Since this is a two-tailed test with an alpha of 0.01, there is a 0.005 rejection region in both tails. Thus, we need to find the value 0.995 (1.0 – 0.005) in the table. The closest value is 0.9951, which corresponds to a critical z-value of 2.58. Since the test statistic is less than the critical value, we fail to reject H_0 and conclude that the stock's return equals 0.0 percent.
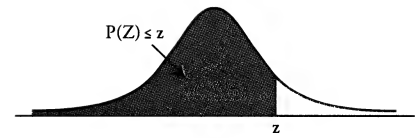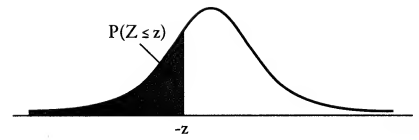
CUMULATIVE Z-TABLE

P(Z) ≤ z

STANDARD NORMAL DISTRIBUTION
$P(Z \le z) = N(z)$ FOR $z \ge 0$

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990

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CUMULATIVE Z-TABLE (CONT.)

STANDARD NORMAL DISTRIBUTION

$P(Z \le z) = N(z)$ FOR $z \le 0$

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641
-0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
-0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
-0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
-0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
-0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
-0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
-0.7	0.2420	0.2389	0.2358	0.2327	0.2297	0.2266	0.2236	0.2207	0.2177	0.2148
-0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
-0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
-1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
-1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
-1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1057	0.1038	0.1020	0.1003	0.0985
-1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
-1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681
-1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
-1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
-1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
-1.8	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
-1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
-2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
-2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
-2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
-2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
-2.4	0.0082	0.0080	0.0078	0.0076	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
-2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
-2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
-2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
-2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
-2.9	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
-3.0	0.0014	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010

ALTERNATIVE Z-TABLE
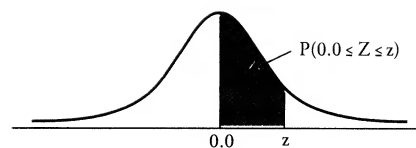
$P(0.0 \le Z \le z) = N(z)$ FOR $z \ge 0$

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0199	0.0239	0.0279	0.0319	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0753
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.1480	0.1517
0.4	0.1554	0.1591	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844	0.1879
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2088	0.2123	0.2157	0.2190	0.2224
0.6	0.2257	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2517	0.2549
0.7	0.2580	0.2611	0.2642	0.2673	0.2704	0.2734	0.2764	0.2794	0.2823	0.2852
0.8	0.2881	0.2910	0.2939	0.2967	0.2995	0.3023	0.3051	0.3078	0.3106	0.3133
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3340	0.3356	0.3389
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3531	0.3554	0.3577	0.3599	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319
1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4429	0.4441
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4761	0.4767
2.0	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.4890
2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
2.4	0.4918	0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
2.5	0.4939	0.4940	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
2.6	0.4953	0.4955	0.4956	0.4957	0.4959	0.4960	0.4961	0.4962	0.4963	0.4964
2.7	0.4965	0.4966	0.4967	0.4968	0.4969	0.4970	0.4971	0.4972	0.4973	0.4974
2.8	0.4974	0.4975	0.4976	0.4977	0.4977	0.4978	0.4979	0.4979	0.4980	0.4981
2.9	0.4981	0.4982	0.4982	0.4983	0.4984	0.4984	0.4985	0.4985	0.4986	0.4986
3.0	0.4987	0.4987	0.4987	0.4988	0.4988	0.4989	0.4989	0.4989	0.4990	0.4990

APPENDIX B:
STUDENT'S T-DISTRIBUTION

STUDENT'S T-DISTRIBUTION

	Level of Significance for One-Tailed Test					
df	0.100	0.050	0.025	0.01	0.005	0.0005

	Level of Significance for Two-Tailed Test					
df	0.20	0.10	0.05	0.02	0.01	0.001
1	3.078	6.314	12.706	31.821	63.657	636.619
2	1.886	2.920	4.303	6.965	9.925	31.599
3	1.638	2.353	3.182	4.541	5.841	12.294
4	1.533	2.132	2.776	3.747	4.604	8.610
5	1.476	2.015	2.571	3.365	4.032	6.869
6	1.440	1.943	2.447	3.143	3.707	5.959
7	1.415	1.895	2.365	2.998	3.499	5.408
8	1.397	1.860	2.306	2.896	3.355	5.041
9	1.383	1.833	2.262	2.821	3.250	4.781
10	1.372	1.812	2.228	2.764	3.169	4.587
11	1.363	1.796	2.201	2.718	3.106	4.437
12	1.356	1.782	2.179	2.681	3.055	4.318
13	1.350	1.771	2.106	2.650	3.012	4.221
14	1.345	1.761	2.145	2.624	2.977	4.140
15	1.341	1.753	2.131	2.602	2.947	4.073
16	1.337	1.746	2.120	2.583	2.921	4.015
17	1.333	1.740	2.110	2.567	2.898	3.965
18	1.330	1.734	2.101	2.552	2.878	3.922
19	1.328	1.729	2.093	2.539	2.861	3.883
20	1.325	1.725	2.086	2.528	2.845	3.850
21	1.323	1.721	2.080	2.518	2.831	3.819
22	1.321	1.717	2.074	2.508	2.819	3.792
23	1.319	1.714	2.069	2.500	2.807	3.768
24	1.318	1.711	2.064	2.492	2.797	3.745
25	1.316	1.708	2.060	2.485	2.787	3.725
26	1.315	1.706	2.056	2.479	2.779	3.707
27	1.314	1.703	2.052	2.473	2.771	3.690
28	1.313	1.701	2.048	2.467	2.763	3.674
29	1.311	1.699	2.045	2.462	2.756	3.659
30	1.310	1.697	2.042	2.457	2.750	3.646
40	1.303	1.684	2.021	2.423	2.704	3.551
60	1.296	1.671	2.000	2.390	2.660	3.460
120	1.289	1.658	1.980	2.358	2.617	3.373
∞	1.282	1.645	1.960	2.326	2.576	3.291

APPENDIX C:
F-TABLE AT 5 PERCENT

F-TABLE AT 5 PERCENT

Critical values of the F-distribution at a 5 percent level of significance
Degrees of freedom for the numerator along top row
Degrees of freedom for the denominator along side row

	1	2	3	4	5	6	7	8	9	10	12	15	20	24	30	40
1	161	200	216	225	230	234	237	239	241	242	244	246	248	249	250	251
2	18.5	19.0	19.2	19.2	19.3	19.3	19.4	19.4	19.4	19.4	19.4	19.4	19.4	19.5	19.5	19.5
3	10.1	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81	8.79	8.74	8.70	8.66	8.64	8.62	8.59
4	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6.00	5.96	5.91	5.86	5.80	5.77	5.75	5.72
5	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77	4.74	4.68	4.62	4.56	4.53	4.50	4.46
6	5.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.10	4.06	4.00	3.94	3.87	3.84	3.81	3.77
7	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68	3.64	3.57	3.51	3.44	3.41	3.38	3.34
8	5.32	4.46	4.07	3.84	3.69	3.58	3.50	3.44	3.39	3.35	3.28	3.22	3.15	3.12	3.08	3.04
9	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18	3.14	3.07	6.01	2.94	2.90	2.86	2.83
10	4.96	4.10	3.71	3.48	3.33	3.22	3.14	3.07	3.02	2.98	2.91	2.85	2.77	2.74	2.70	2.66
11	4.84	3.98	3.59	3.36	3.20	3.09	3.01	2.95	2.90	2.85	2.79	2.72	2.65	2.61	2.57	2.53
12	4.75	3.89	3.49	3.26	3.11	3.00	2.91	2.85	2.80	2.75	2.69	2.62	2.54	2.51	2.47	2.43
13	4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.71	2.67	2.60	2.53	2.46	2.42	2.38	2.34
14	4.60	3.74	3.34	3.11	2.96	2.85	2.76	2.70	2.65	2.60	2.53	2.46	2.39	2.35	2.31	2.27
15	4.54	3.68	3.29	3.06	2.90	2.79	2.71	2.64	2.59	2.54	2.48	2.40	2.33	2.29	2.25	2.20
16	4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.54	2.49	2.42	2.35	2.28	2.24	2.19	2.15
17	4.45	3.59	3.20	2.96	2.81	2.70	2.61	2.55	2.49	2.45	2.38	2.31	2.23	2.19	2.15	2.10
18	4.41	3.55	3.16	2.93	2.77	2.66	2.58	2.51	2.46	2.41	2.34	2.27	2.19	2.15	2.11	2.06
19	4.38	3.52	3.13	2.90	2.74	2.63	2.54	2.48	2.42	2.38	2.31	2.23	2.16	2.11	2.07	2.03
20	4.35	3.49	3.10	2.87	2.71	2.60	2.51	2.45	2.39	2.35	2.28	2.20	2.12	2.08	2.04	1.99
21	4.32	3.47	3.07	2.84	2.68	2.57	2.49	2.42	2.37	2.32	2.25	2.18	2.10	2.05	2.01	1.96
22	4.30	3.44	3.05	2.82	2.66	2.55	2.46	2.40	2.34	2.30	2.23	2.15	2.07	2.03	1.98	1.94
23	4.28	3.42	3.03	2.80	2.64	2.53	2.44	2.37	2.32	2.27	2.20	2.13	2.05	2.01	1.96	1.91
24	4.26	3.40	3.01	2.78	2.62	2.51	2.42	2.36	2.30	2.25	2.18	2.11	2.03	1.98	1.94	1.89
25	4.24	3.39	2.99	2.76	2.60	2.49	2.40	2.34	2.28	2.24	2.16	2.09	2.01	1.96	1.92	1.87
30	4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.21	2.16	2.09	2.01	1.93	1.89	1.84	1.79
40	4.08	3.23	2.84	2.61	2.45	2.34	2.25	2.18	2.12	2.08	2.00	1.92	1.84	1.79	1.74	1.69
60	4.00	3.15	2.76	2.53	2.37	2.25	2.17	2.10	2.04	1.99	1.92	1.84	1.75	1.70	1.65	1.59
120	3.92	3.07	2.68	2.45	2.29	2.18	2.09	2.02	1.96	1.91	1.83	1.75	1.66	1.61	1.55	1.50
∞	3.84	3.00	2.60	2.37	2.21	2.10	2.01	1.94	1.88	1.83	1.75	1.67	1.57	1.52	1.46	1.39

APPENDIX D:
F-TABLE AT 2.5 PERCENT

F-TABLE AT 2.5 PERCENT

Critical values of the F-distribution at a 2.5 percent level of significance
Degrees of freedom for the numerator along top row
Degrees of freedom for the denominator along side row

	1	2	3	4	5	6	7	8	9	10	12	15	20	24	30	40
1	648	799	864	900	922	937	948	957	963	969	977	985	993	997	1001	1006
2	38.51	39.00	39.17	39.25	39.30	39.33	39.36	39.37	39.39	39.40	39.41	39.43	39.45	39.46	39.46	39.47
3	17.44	16.04	15.44	15.10	14.88	14.73	14.62	14.54	14.47	14.42	14.34	14.25	14.17	14.12	14.08	14.04
4	12.22	10.65	9.98	9.60	9.36	9.20	9.07	8.98	8.90	8.84	8.75	8.66	8.56	8.51	8.46	8.41
5	10.01	8.43	7.76	7.39	7.15	6.98	6.85	6.76	6.68	6.62	6.52	6.43	6.33	6.28	6.23	6.18
6	8.81	7.26	6.60	6.23	5.99	5.82	5.70	5.60	5.52	5.46	5.37	5.27	5.17	5.12	5.07	5.01
7	8.07	6.54	5.89	5.52	5.29	5.12	4.99	4.90	4.82	4.76	4.67	4.57	4.47	4.41	4.36	4.31
8	7.57	6.06	5.42	5.05	4.82	4.65	4.53	4.43	4.36	4.30	4.20	4.10	4.00	3.95	3.89	3.84
9	7.21	5.71	5.08	4.72	4.48	4.32	4.20	4.10	4.03	3.96	3.87	3.77	3.67	3.61	3.56	3.51
10	6.94	5.46	4.83	4.47	4.24	4.07	3.95	3.85	3.78	3.72	3.62	3.52	3.42	3.37	3.31	3.26
11	6.72	5.26	4.63	4.28	4.04	3.88	3.76	3.66	3.59	3.53	3.43	3.33	3.23	3.17	3.12	3.06
12	6.55	5.10	4.47	4.12	3.89	3.73	3.61	3.51	3.44	3.37	3.28	3.18	3.07	3.02	2.96	2.91
13	6.41	4.97	4.35	4.00	3.77	3.60	3.48	3.39	3.31	3.25	3.15	3.05	2.95	2.89	2.84	2.78
14	6.30	4.86	4.24	3.89	3.66	3.50	3.38	3.29	3.21	3.15	3.05	2.95	2.84	2.79	2.73	2.67
15	6.20	4.77	4.15	3.80	3.58	3.41	3.29	3.20	3.12	3.06	2.96	2.86	2.76	2.70	2.64	2.59
16	6.12	4.69	4.08	3.73	3.50	3.34	3.22	3.12	3.05	2.99	2.89	2.79	2.68	2.63	2.57	2.51
17	6.04	4.62	4.01	3.66	3.44	3.28	3.16	3.06	2.98	2.92	2.82	2.72	2.62	2.56	2.50	2.44
18	5.98	4.56	3.95	3.61	3.38	3.22	3.10	3.01	2.93	2.87	2.77	2.67	2.56	2.50	2.44	2.38
19	5.92	4.51	3.90	3.56	3.33	3.17	3.05	2.96	2.88	2.82	2.72	2.62	2.51	2.45	2.39	2.33
20	5.87	4.46	3.86	3.51	3.29	3.13	3.01	2.91	2.84	2.77	2.68	2.57	2.46	2.41	2.35	2.29
21	5.83	4.42	3.82	3.48	3.25	3.09	2.97	2.87	2.80	2.73	2.64	2.53	2.42	2.37	2.31	2.25
22	5.79	4.38	3.78	3.44	3.22	3.05	2.93	2.84	2.76	2.70	2.60	2.50	2.39	2.33	2.27	2.21
23	5.75	4.35	3.75	3.41	3.18	3.02	2.90	2.81	2.73	2.67	2.57	2.47	2.36	2.30	2.24	2.18
24	5.72	4.32	3.72	3.38	3.15	2.99	2.87	2.78	2.70	2.64	2.54	2.44	2.33	2.27	2.21	2.15
25	5.69	4.29	3.69	3.35	3.13	2.97	2.85	2.75	2.68	2.61	2.51	2.41	2.30	2.24	2.18	2.12
30	5.57	4.18	3.59	3.25	3.03	2.87	2.75	2.65	2.57	2.51	2.41	2.31	2.20	2.14	2.07	2.01
40	5.42	4.05	3.46	3.13	2.90	2.74	2.62	2.53	2.45	2.39	2.29	2.18	2.07	2.01	1.94	1.88
60	5.29	3.93	3.34	3.01	2.79	2.63	2.51	2.41	2.33	2.27	2.17	2.06	1.94	1.88	1.82	1.74
120	5.15	3.80	3.23	2.89	2.67	2.52	2.39	2.30	2.22	2.16	2.05	1.94	1.82	1.76	1.69	1.61
∞	5.02	3.69	3.12	2.79	2.57	2.41	2.29	2.19	2.11	2.05	1.94	1.83	1.71	1.64	1.57	1.48

APPENDIX E:
CHI-SQUARED TABLE

Values of χ^2 (Degrees of Freedom, Level of Significance)
Probability in Right Tail

Degrees of Freedom	0.99	0.975	0.95	0.9	0.1	0.05	0.025	0.01	0.005
1	0.000157	0.000982	0.003932	0.0158	2.706	3.841	5.024	6.635	7.879
2	0.020100	0.050636	0.102586	0.2107	4.605	5.991	7.378	9.210	10.597
3	0.1148	0.2158	0.3518	0.5844	6.251	7.815	9.348	11.345	12.838
4	0.297	0.484	0.711	1.064	7.779	9.488	11.143	13.277	14.860
5	0.554	0.831	1.145	1.610	9.236	11.070	12.832	15.086	16.750
6	0.872	1.237	1.635	2.204	10.645	12.592	14.449	16.812	18.548
7	1.239	1.690	2.167	2.833	12.017	14.067	16.013	18.475	20.278
8	1.647	2.180	2.733	3.490	13.362	15.507	17.535	20.090	21.955
9	2.088	2.700	3.325	4.168	14.684	16.919	19.023	21.666	23.589
10	2.558	3.247	3.940	4.865	15.987	18.307	20.483	23.209	25.188
11	3.053	3.816	4.575	5.578	17.275	19.675	21.920	24.725	26.757
12	3.571	4.404	5.226	6.304	18.549	21.026	23.337	26.217	28.300
13	4.107	5.009	5.892	7.041	19.812	22.362	24.736	27.688	29.819
14	4.660	5.629	6.571	7.790	21.064	23.685	26.119	29.141	31.319
15	5.229	6.262	7.261	8.547	22.307	24.996	27.488	30.578	32.801
16	5.812	6.908	7.962	9.312	23.542	26.296	28.845	32.000	34.267
17	6.408	7.564	8.672	10.085	24.769	27.587	30.191	33.409	35.718
18	7.015	8.231	9.390	10.865	25.989	28.869	31.526	34.805	37.156
19	7.633	8.907	10.117	11.651	27.204	30.144	32.852	36.191	38.582
20	8.260	9.591	10.851	12.443	28.412	31.410	34.170	37.566	39.997
21	8.897	10.283	11.591	13.240	29.615	32.671	35.479	38.932	41.401
22	9.542	10.982	12.338	14.041	30.813	33.924	36.781	40.289	42.796
23	10.196	11.689	13.091	14.848	32.007	35.172	38.076	41.638	44.181
24	10.856	12.401	13.848	15.659	33.196	36.415	39.364	42.980	45.558
25	11.524	13.120	14.611	16.473	34.382	37.652	40.646	44.314	46.928
26	12.198	13.844	15.379	17.292	35.563	38.885	41.923	45.642	48.290
27	12.878	14.573	16.151	18.114	36.741	40.113	43.195	46.963	49.645
28	13.565	15.308	16.928	18.939	37.916	41.337	44.461	48.278	50.994
29	14.256	16.047	17.708	19.768	39.087	42.557	45.722	49.588	52.335
30	14.953	16.791	18.493	20.599	40.256	43.773	46.979	50.892	53.672
50	29.707	32.357	34.764	37.689	63.167	67.505	71.420	76.154	79.490
60	37.485	40.482	43.188	46.459	74.397	79.082	83.298	88.379	91.952
80	53.540	57.153	60.391	64.278	96.578	101.879	106.629	112.329	116.321
100	70.065	74.222	77.929	82.358	118.498	124.342	129.561	135.807	140.170

APPENDIX F:
ADDITIONAL STUDY TIPS

This appendix contains a compendium of study tips that we have collected from our faculty members and successful CFA candidates. In many cases, there are several similar comments for each topic area. The purpose of including these comments is to provide you with the perspectives of several different individuals so that you may gain from their experience (or learn from their mistakes).

Calculators

- Get the TI calculator ASAP and begin using it for work, bill paying, studies, etc. The more you use the calculator, the quicker you will be able to solve problems during the exam. This can be an important time saver.
- **Choose the calculator you plan to use on the exam, and use it throughout your studying. Don't switch calculators mid-stream.**

Focus

- Expect the unexpected on exam day and don't get rattled when strange stuff happens. We've all heard and read about exam day horror stories: booklets are misprinted, proctors are slow, exams aren't handed out on time, birds are flying in the rafters, etc. Keep your focus despite the distractions; that's what makes Tiger Woods the best golfer in the world.
- Study whenever and wherever you can. Don't count on being able to always study in long, leisurely blocks of time. We're all busy with family and careers—daily 4-hour study sessions are a luxury most of us can't afford. Take advantage of 10 minutes at your desk at lunch to review one LOS; listen to Schweser audio CDs in the car on the way to work; go over a few flash cards for 30 minutes in the airport when your flight is delayed.
- Try to study a bit each day, even if it is only for an hour. Again, in the interest of repetition, it may feel like you've forgotten what you studied in February by May, but it is in there somewhere.
- Remember why you're doing this (to expand your knowledge and advance your career) and try to make it a positive experience. Learning new things is exciting; keep reminding yourself of that when you're lost in derivatives or accounting.
- **Stay Focused**—develop a study program and *stick to it...*
- **Identify those topics you're weakest in and study them hard.** Objective: to get as many points as you can in these areas.
- Work on your weakest area first—that way you can't procrastinate on this area.
- **Identify those topics you're strongest in and study them hard, too.** Objective: to generate *excess returns* that can be used to offset some of the points you may lose in other parts of the exam.
- Don't forget to study your strong areas. You don't want to lose those points.
- **Don't Get Frustrated**: When you come across something in your study program that you just don't understand (and you will), *don't get upset!* Give it your best shot, learn as much as you can, and then move on to the next topic.
- Focus on the big picture; don't get bogged down in minutia. If you understand the larger issues, the smaller ones usually take care of themselves.
- **Know thyself.** Know how you learn and choose an approach that fits your learning style.
- **Assume that you will not be able to study the two weeks prior to the exam.** Life has a way of throwing you a curve-ball just when you had planned to use that last two weeks to cram for the exam.

The Day Before The Exam

- Visiting the test center prior to exam day is extremely important. In doing so, you will be more focused on taking the exam when you wake up on test day.
- Find your test center and testing room before the day of the exam. There have been many horror stories from people who got lost or could not find the test center on exam day.
- You must visit the exam center in advance for familiarity and go to the exam center well on time. Any additional anxiety due to a delay in the commute is totally unnecessary.
- Check out the site in advance. I took the level 3 exam in Madrid in June. It's not a city I know well, and it took me about an hour to find the exam hall. Luckily, I checked it out the evening before the exam. It's reassuring to know where to go, how long the subway will take, etc.
- Don't try to cram the night before the exam. Being sharp and well-rested will help you far more than a stressful last minute cram session.
- Rest, rest, rest the night before. The better you feel day of the exam, the more likely you are to do well.
- **Get a good night's sleep the night before the exam.** Easy to say, but …

Exam-Day Tips

- Since we always feel that we could have done better than we actually do, avoid scrutinizing your performance in the morning session during the lunch break.
- Try to avoid talking to other candidates about the exam during breaks. You don't want to enter the next session feeling intimidated or overconfident. It is easier to stay focused without comparing notes.
- Use the time immediately before each session to commit formulas to short-term memory.
- The first few minutes in the exam can be great confidence boosters. For that, I suggest that you start the exam by picking the section that you are most comfortable with.
- To build confidence, start with the section of the exam with which you are most comfortable.
- To build confidence, work the problems that are easiest in each section first.
- Budget your time between sections and do not go over the budget for the first pass. Come back to any sections that have not been completed at the end of the exam.
- Watch the time carefully. You may do a practice exam in an hour and half and find that you're running out of time on the actual exam. I believe this is because you are more likely to check and recheck your answers on the actual exam, which you don't feel compelled to do on the practice exam. One of the WORST things you can do is run out of time on the actual exam.
- As my friend Richard says, "fight for every point on the exam. That last point you get might be the one that pushes you over the top." Don't get discouraged by what you don't know while taking the exam, just keep fighting for points.
- Don't necessarily answer questions in the order they are given. Skim all questions in the session, and begin by answering the ones that you know very well. This will build confidence.
- **Expect the Unexpected**: You're sure to come across questions that you don't have a clue where they come from; when that happens, just *guess and move on* to the next question.

Exam-Day Time Constraints

- Time yourself very carefully. You do not want to be in a situation where you run out of time on the questions that you know well.
- Expect CFA Institute to include questions and situations designed to throw you off. If you are well prepared, don't let them intimidate you! Chances are, these situations will affect other candidates as well—remember, CFA Institute grades on a curve.
- Some questions CFA Institute gives you will be straightforward. Don't waste time making something more difficult than it actually is.
- The exam is long. You have 1½ minutes to answer each of 240 questions. You need to work quickly and recognize that there are some questions you should guess at rather than spending time contemplating or

calculating. In some cases, there could be a question that you know will take 4 minutes of calculations. Consider coming back to that question after you have answered the others. You must keep moving forward.

Guessing

- **Guess if you must**: There are no penalties for guessing, and it won't count against you; what's more, you might even guess correctly.
- If you are not completely sure about a question, go with your initial gut feeling about it. Do not second-guess yourself.
- Don't leave anything blank. Guess on the multiple-choice questions that you don't know.

The Passing Score

- **You don't need to know it all**: *Remember the number 75; that's the approximate number of questions you can miss and still pass the exam.*
- **You Don't Need a Passing Grade on Each Topic**: It's the *overall grade* that matters.

After the Exam

- After the exam, be sure to thank those friends and family members who supported you as you prepared to earn the Chartered Financial Analyst designation.
- Plan to treat yourself immediately after the exam. This could be taking a trip, doing something special with your family, or just going to that special restaurant.

Ethics

- Read the CFA Institute Standards of Practice Handbook twice—once at the beginning of your study schedule and again shortly before the exam.
- Learn the ethics curriculum early on in your study program, and then review the ethics curriculum as your last topic in your study program.
- In studying prior and sample item set Ethics questions, be sure you understand *not* just why the correct answer is correct, but also understand *why the other answers are wrong*. Ethics questions are notorious for drawing fine distinctions that you will have to wade through on the exam.
- Study ethics every day starting in January. If you have flash cards, look at a few of them each day. Otherwise, read at least a few pages from the Schweser notes or source material each day.
- Some people feel that ethics is common sense, and they take it for granted (they do not spend much time studying this material). However, the ethics questions on the exam are difficult because of the detail candidates must know. Study the details! Much of the material you learn at Level 1 will be useful to know at Levels 2 and 3, so spend the time now to learn it well.
- Spend a lot of time working practice ethics questions. Practice questions may be more valuable in the ethics area than in any other area.
- The Code of Ethics is short and is likely to be addressed at all three levels. Learn the four components (or at least the key words) of the Code of Ethics.
- When working ethics questions on the exam that have long preambles, underline key concepts as you read so you can refer back to these concepts quickly after reading the question. This is particularly true when several questions are asked about one set of circumstances.
- Don't rely fully on the Schweser notes in the ethics area. Thoroughly read the original Standards of Practice Handbook and focus on the examples. You're a CFA candidate – you should own your own copy anyway!

Practice Exams / SchweserPro

- Use Schweser online tests and practice tests to help you assess your strengths and weaknesses. It is also important to practice taking exams under exam-like situations. Take a practice exam 4, 3, 2, and 1 week before the exam. If you are not seeing notable improvement, increase your study time.

- Take as many practice exams as possible before the actual exam. There is no substitute for answering actual questions to prepare you for what you will face on exam day.
- Don't take practice exams until May.
- Work the practice questions under time constraints.
- Take all of Schweser mock exams seriously—like the actual exam.
- Put pencil to paper! Don't just read a numerical problem and answer, convincing yourself that you know how to solve the problem just because the answer "makes sense." Going through the steps to actually solve the problem may uncover stumbling blocks that you didn't see upon your cursory review. These stumbling blocks can cost you points and time on the exam.
- **Learn the material,** don't just learn the old test questions. The exam has been getting more demanding in recent years, and pass rates are falling. You must be able to understand and apply the concepts from the curriculum to new situations. Simply studying the old test questions and practicing on sample exams will not be enough. There are no shortcuts to passing the exam.
- You should take as many practice exams as you can. If you run out of practice exams, you should work problems. Active learning is effective learning.
- **Practice using the most challenging problems you can find.** The more difficult the better. Working easy problems may make preparation easier and more enjoyable, but hey—the exam is difficult.

Flashcards

- Begin implementing flashcards into your study regimen as early as March. Then, during the last week before the exam, you should read through the flashcards with learning objectives that you find to be the most difficult. The goal is to continually make this stack smaller and smaller.
- The day before the exam should have a short stack of problem flash cards that you can go through in an hour.
- I am a firm believer in flashcards. Whether you compose your own or purchase those made by Schweser, it is a very efficient means of memorizing material. You can take a stack with you the next time you visit the post office and read them as you wait in line. At the end of a day at work, as you walk to your car or train, you can look at a few and start getting your mind into study mode. (Don't read them when driving!) When you take practice exams, make a study card for every question you miss.
- Use flash cards, either purchased or homemade. Again, they help you focus on your weak areas, and they are more portable than study notes. Carry them and look at them while you're stuck in traffic, waiting for someone in a restaurant, etc. Repetition is key for most of us.

Seminars

- Take a prep course if you possibly can. They jump start your study if you're having a difficult time getting into it, and, more importantly, they help you focus your energy on what is important, rather than the minutia that "grabs" so many of us.
- If you take a course, listen to the tips. For example, one of the instructors at Windsor mentioned that if you put a different currency in the denominator and multiplied them together, and you didn't end up with 1, an arbitrage opportunity exists. On the exam, this "tip" allowed me to quickly narrow the answers down to two possible choices from four.

General Comments

- Have a firm reminder of why you are studying for the exam. If it is to earn a higher standard of living for your family, have a picture of your family along with the CFA® logo displayed in a prominent place. If it is to prove to some supervisor that you have what it takes, have his/her picture displayed next to the logo. If it is because you enjoy challenges, have the logo displayed next to the picture of you in the last marathon that you ran.
- Answer questions *exclusively* in terms of the curriculum. You may well know a more "correct" answer, you may only know a "wrong" answer, but there is only one **correct** answer, and it is what AIMR has defined *in*

terms of the curriculum. Hence the timeless exam preparation advice: There is a right answer, a wrong answer, and the CFA Institute answer.

- Think carefully about how you learn best, and utilize those methods of study. Some people learn best through self-study (the Schweser Study Notes and flash cards) and others learn better by having concepts explained to them (Schweser Video Series, Audio CDs, Live Seminars). You may want to vary your learning methods to stay fresh.
- **Don't Fool Yourself:** You *cannot* successfully bluff or guess your way through this exam.
- **Do Your Homework:** You need to study the material in order to pass the exam.
- Make sure to master the obvious. There are some things that you just "know" are going to show up on the exam. For example: time value of money computations, the use of effective duration, the use of the dividend discount model. As you study each reading, think about what these obvious issues are and write them down.
- Tell yourself that it is only an exam and is not the end of the world. I know of many candidates who do poorly because they are under tremendous anxiety to pass.
- **Use the current year's notes.** The curriculum changes enough to require you to use the current curriculum. Don't scrimp by using last year's notes.

INDEX

A

a priori probability 164
absolute frequency 131
addition rule for probabilities 166
alternative hypothesis 242
amortization 95
analysis of variance 294
annuity 87
annuity due 87, 89
ANOVA 294
arithmetic mean 136
autocorrelation 285

B

bank discount yield 118
Bayes' formula 180
biased estimator 142
binomial distribution 198
binomial formula. See combination formula 183
binomial random variable 197, 198
bond equivalent yield 121

C

cash flow additivity principle 101
cdf. See cumulative distribution function 197
central limit theorem 224
Chebyshev's inequality 144
chi-square distribution 261
coefficient of determination 287
coefficient of variation 144
combination 183
combination formula. See binomial formula 183
compound interest 79
compound value 86
compounding frequencies 83
compounding periods 83
conditional expectation 172
conditional expected values 172
conditional probability 165
confidence interval 202, 227, 290
confidence interval for a regression coefficient 290
confidence interval for the population mean 228

consistent estimator 227
continuous distribution 196
continuous random variable 195
continuous uniform distribution 200
continuously compounded rates of return 211
correlation 175, 201
correlation coefficient 278
correlation matrix 178
cost of capital 87
covariance 173, 278
covariance matrix 179
cross-sectional data 224
cumulative absolute frequency 133
cumulative distribution function 197
cumulative relative frequency 133

D

data mining 232
data-mining bias 232
decile 135
decision rule 243, 247
default risk premium 81
degrees of freedom 228
dependent events 167
dependent variable 283
descriptive statistics 129
discount factor 87
discount rate 87, 98
discounting 86
discrete distribution 195
discrete random variable 195
discrete uniform random variable 197
dispersion 140
distribution function 197

E

EAR. See effective annual rate
EAY. See effective annual yield
economic decision 249
effective annual rate 82
effective annual yield 119
efficient 227
empirical probability 164

T

U

V

W

Z